In [5]:

```
%matplotlib inline
import sys
sys.path.insert(0,'..')
from IPython.display import HTML,Image,SVG,YouTubeVideo
```

Image restoration

Image acquisition is rarely perfect, physics or external condition may deform the image being acquired, here are some example of typicall problems:

Deformation model

The original image is $f(x, y)$ undergoes a deformation, given by H, and an additive noise $\eta(x, y)$ the acquired image is $g(x, y)$.

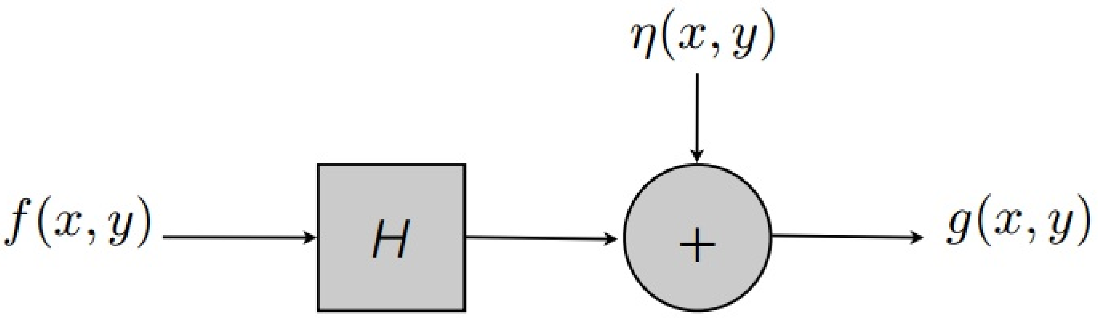
The restoration problem can be stated as follow:

how to recover a good approximation of $f(x, y)$ from $g(x, y)$?

In [6]:

```
Image('http://homepages.ulb.ac.be/~odebeir/data/restauration.png')
```

Out[6]:



Some examples:

$g(x, y) = H[f(x, y)] + \eta(x, y)$

H properties:

- linear

$H[k_1 f_1(x, y) + k_2 f_2(x, y)] = k_1 H[f_1(x, y)] + k_2 H[f_2(x, y)]$

- additive

$H[f_1(x, y) + f_2(x, y)] = H[f_1(x, y)] + H[f_2(x, y)]$

- homogeneous

$H[k_1 f_1(x, y)] = k_1 H[f_1(x, y)]$

- spatially invariant

$H[f(x - \alpha, y - \beta)] = g(x - \alpha, y - \beta)$

Point Spread Function (PSF)

We can rewrite f(x, y) as a sum of Dirac function.
$f(x, y) = \int_{\infty - \infty} \int_{\infty - \infty} f(\alpha, \beta)\delta(x - \alpha, y - \beta) d\alpha d\beta$

if there is no addiditive noise:
$g(x, y) = H[f(x, y)] = H\left[\int_{\infty - \infty} \int_{\infty - \infty} f(\alpha, \beta)\delta(x - \alpha, y - \beta) d\alpha d\beta\right]$

by linearity property:
$g(x, y) = \int_{\infty - \infty} \int_{\infty - \infty} H\left[f(\alpha, \beta)\delta(x - \alpha, y - \beta)\right] d\alpha d\beta$

$f(\alpha, \beta)$ is independant of x and y:
$g(x, y) = \int_{\infty - \infty} \int_{\infty - \infty} f(\alpha, \beta) H\left[\delta(x - \alpha, y - \beta)\right] d\alpha d\beta$

the impulse response of H, also known as point spread function (PSF), is:
$h(x, \alpha, y, \beta) = H\left[\delta(x - \alpha, y - \beta)\right]$

after substitution we have:
$g(x, y) = \int_{\infty - \infty} \int_{\infty - \infty} f(\alpha, \beta) h(x, \alpha, y, \beta) d\alpha d\beta$

this expression means that, if the response H of an impulse is known, the response of any input $f(\alpha, \beta)$ is known.

if h is spatially invariant (see above):
$g(x, y) = \int_{\infty - \infty} \int_{\infty - \infty} f(\alpha, \beta) h(x - \alpha, y - \beta) d\alpha d\beta$

by adding the noise:
$g(x, y) = \int_{\infty - \infty} \int_{\infty - \infty} f(\alpha, \beta) h(x - \alpha, y - \beta) d\alpha d\beta + \eta(x, y)$

> see also:
>
> - {% cite gonzalez1977digital %} p254

Restoration

inverse filtering

If noise is negligeable and PSF is known (in the Fourier domain):
$\hat{}F(u, v) = G(u, v)H(u, v) \hat{}f(x, y) = F^{-1}\left[\hat{}F(u, v)\right] = F^{-1}\left[G(u, v)H(u, v)\right]$

else:
$\hat{}F(u, v) = F(u, v) + N(u, v)H(u, v)$

and noise is increasing when H is low, restoration is limited where H is big.

if define a restoration transform M(u, v) as:
$\hat{}F(u, v) = (G(u, v) + N(u, v))M(u, v)$

one can use:
$M(u, v) = \begin{cases} 1/H(u, v), & u^2 + v^2 \le w_{20} \\ 1, & u^2 + v^2 > w_{20} \end{cases}$

with w_0 being a distance to the origin in the Fourier space.

> see also:
>
> - {% cite rosenfeld1976digital%} p276

Wiener filtering

To avoid arbitrary setting of a parameter for theinverse transform such as w_0, one can use the Wiener approach wich consists in minimizing by least square some error function.

Without going into the details, the restoration transform is:

$M(u, v) = H^{*}(u, v) | H(u, v) |^{2} + S_{\nu\nu}(u,v) S_{gg}(u,v)$

where $H^{*}(u, v)$ is the complex conjugate of $H(u, v)$,

$S_{\nu\nu}(u, v)$ is the spectral density of the noise and

$S_{gg}(u, v)$ is the spectral density of the degraded image.

> see also:
>
> - {% cite sonka2014image %} p107

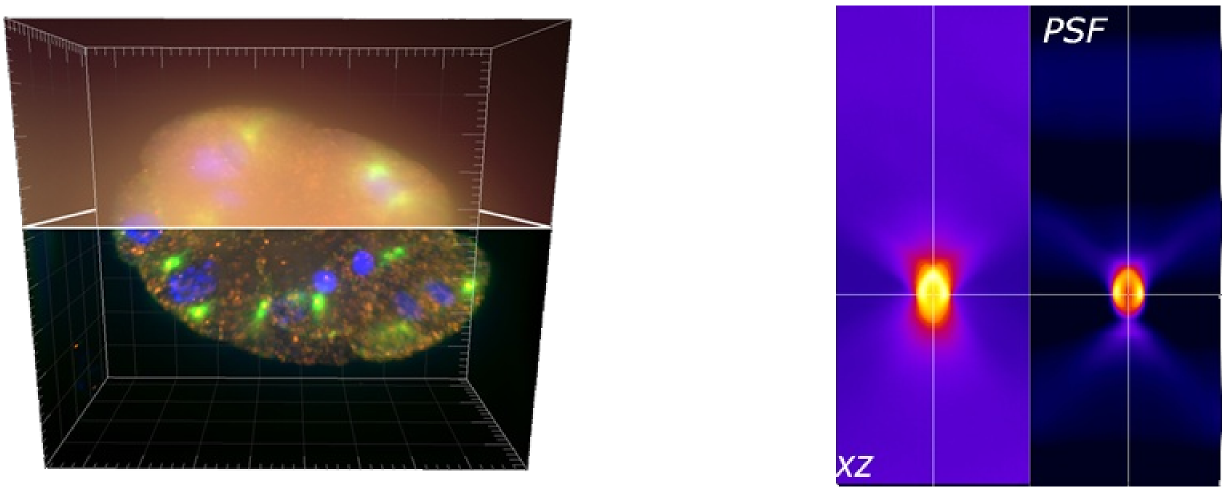
Blind deconvolution

If PSF in unknown, is has to be estimated. This is called blind deconvolution.

In [7]:

```
Image('http://bigwww.epfl.ch/algorithms/deconvolutionlab/meta/splash.png')
```

Out[7]:



[image source (http://bigwww.epfl.ch/algorithms/deconvolutionlab/)](http://bigwww.epfl.ch/algorithms/deconvolutionlab/)

References

{% bibliography --cited %}