

# Entailment

Entailment means that one thing **follows from** another:

$$KB \models \alpha$$

Knowledge base  $KB$  entails sentence  $\alpha$

if and only if

$\alpha$  is true in all worlds where  $KB$  is true

E.g., the KB containing “the Giants won” and “the Reds won” entails “Either the Giants won or the Reds won”

E.g.,  $x + y = 4$  entails  $4 = x + y$

Entailment is a relationship between sentences (i.e., **syntax**) that is based on **semantics**

Note: brains process **syntax** (of some sort)

# Models

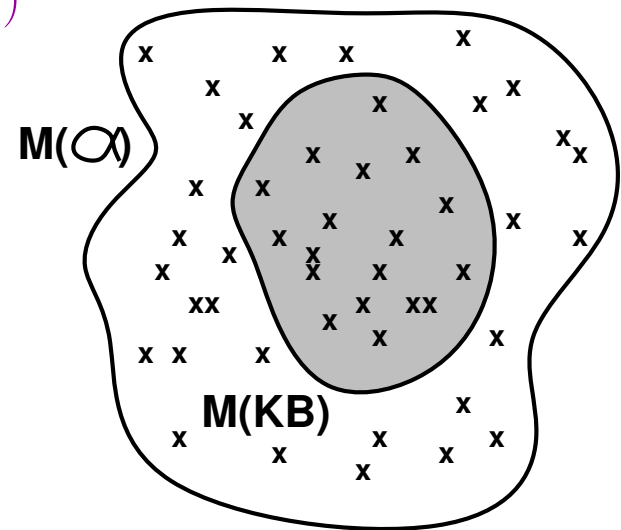
Logicians typically think in terms of **models**, which are formally structured worlds with respect to which truth can be evaluated

We say  $m$  is a model of a sentence  $\alpha$  if  $\alpha$  is true in  $m$

$M(\alpha)$  is the set of all models of  $\alpha$

Then  $KB \models \alpha$  if and only if  $M(KB) \subseteq M(\alpha)$

E.g.  $KB = \text{Giants won and Reds won}$   
 $\alpha = \text{Giants won}$



# Inference

$KB \vdash_i \alpha$  = sentence  $\alpha$  can be derived from  $KB$  by procedure  $i$

Consequences of  $KB$  are a haystack;  $\alpha$  is a needle.

Entailment = needle in haystack; inference = finding it

**Soundness:**  $i$  is sound if

whenever  $KB \vdash_i \alpha$ , it is also true that  $KB \models \alpha$

**Completeness:**  $i$  is complete if

whenever  $KB \models \alpha$ , it is also true that  $KB \vdash_i \alpha$

Preview: we will define a logic (first-order logic) which is expressive enough to say almost anything of interest, and for which there exists a sound and complete inference procedure.

That is, the procedure will answer any question whose answer follows from what is known by the  $KB$ .

# Logical equivalence

Two sentences are **logically equivalent** iff true in same models:

$\alpha \equiv \beta$  if and only if  $\alpha \models \beta$  and  $\beta \models \alpha$

$(\alpha \wedge \beta) \equiv (\beta \wedge \alpha)$	commutativity of $\wedge$
$(\alpha \vee \beta) \equiv (\beta \vee \alpha)$	commutativity of $\vee$
$((\alpha \wedge \beta) \wedge \gamma) \equiv (\alpha \wedge (\beta \wedge \gamma))$	associativity of $\wedge$
$((\alpha \vee \beta) \vee \gamma) \equiv (\alpha \vee (\beta \vee \gamma))$	associativity of $\vee$
$\neg(\neg\alpha) \equiv \alpha$	double-negation elimination
$(\alpha \Rightarrow \beta) \equiv (\neg\beta \Rightarrow \neg\alpha)$	contraposition
$(\alpha \Rightarrow \beta) \equiv (\neg\alpha \vee \beta)$	implication elimination
$(\alpha \Leftrightarrow \beta) \equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha))$	biconditional elimination
$\neg(\alpha \wedge \beta) \equiv (\neg\alpha \vee \neg\beta)$	De Morgan
$\neg(\alpha \vee \beta) \equiv (\neg\alpha \wedge \neg\beta)$	De Morgan
$(\alpha \wedge (\beta \vee \gamma)) \equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma))$	distributivity of $\wedge$ over $\vee$
$(\alpha \vee (\beta \wedge \gamma)) \equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma))$	distributivity of $\vee$ over $\wedge$