Introduction to AI - assignment 6

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Contents

Exercise 1

```
I({A,B},{C,D}|{})
I({A,B},{C,D,G,H}|{})
I({E,F},{C,D,G,H}|{})
I({G},{C}|{D})

not I({A},{B}|{})
not I({C},{G}|{})
not I({E},{F}|{})
not I({C},{G}|{D,H})
```

1.1 Construct a Bayes network that is consistent with the above set of independence statements

Since A and B are closed and are independent from all other nodes, they can be separated from the rest of the tree and just connected between themselves (since they are dependent).

Same goes for nodes E and F.

Since C and G and dependent unless D is in evidence this suggests that D is a parent of both C and G. In addition, since C and G are dependent in the presence of H this suggests that C and G are parents of H.

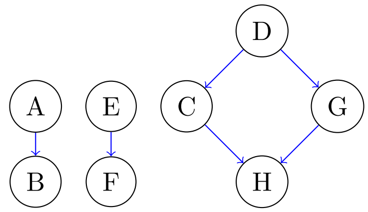

Figure 1: Bayesian Network

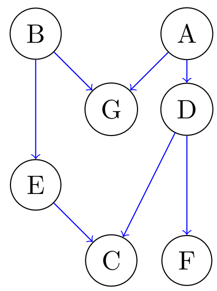

Figure 2: Network tree

Fig. 1 proposes a Bayes Network tree that supports all of the statements in the requirements.

1.2 Is the answer above unique?

No. As mentioned above H has to be the child of C and G, and D has to be their parents. However, A can be the parent of B, or B can be the parent of A. Same goes for E and F. Thus, there are 2^2 combinations, which add up to **4 possible BN that will hold the statements**.

Exercise 2

2.1 Is this network a poly-tree?

2.2 Is this network (directed-path) singly connected?

2.3 Determine the truth of the following independence statements (using d-separation)

1. `I({D}, {E} | {    }) - False`
2. `I({D}, {E} | {  A }) - True`
3. `I({D}, {E} | {A, C}) - False`
4. `I({B}, {F} | {  G }) - True`
5. `I({B}, {F} | {  D }) - True`
6. `I({B}, {F} | {A, C}) - False`

2.4 Find P(E=true, A=True | B=true)

```
P(A=true) = 0.2, P(B=true)= 0.3
P(G=true|A=true,B=true) = 0.9, P(G=true|A=false,B=true) = 0.2
P(G=true|A=true,B=false) = 0.5, P(G=true|A=false,B=false) = 0.1
P(E=true|B=true) = 0.8, P(E=true|B=false) = 0.1
P(C=true|D=true, E=true) = 0
```

Since G and C are not in evidence and B is in evidence, we can safely say that A and E are statistically independent. Moreover, A and B are also independent since G and C are not in evidence. Hence
$P(E, A|B) = P(E|B) * P(A|B) = P(E|B) * P(A) = 0.8 * 0.2 = 0.16$

Exercise 3

3.1 Can this be done without hidden units?

No. It is impossible with no hidden layers to predict a non-linear function such as modulu. However, for this simple example, a single hidden layer will suffice as shown in the following subsection.

3.2 Show a network using threshold elements that performs the required computation

Consider the weights

$$M1 = \begin{bmatrix} -2.6035163 & 1.0730004 & 0.73517716 & 0.7892835 & 2.0436563 & -3.0340612 \\ -2.8252845 & 1.1691521 & 1.3764652 & 1.5624126 & -3.209769 & -2.571492 \\ -2.5988533 & 1.1376183 & 0.9316116 & 0.563356 & 2.0437527 & -3.031866 \\ -2.5722306 & 1.2495581 & 0.7956654 & 0.7881598 & 2.0363252 & -3.0355027 \\ -2.603346 & 1.1103705 & 0.60637933 & 0.61684054 & 2.0353322 & -3.025518 \\ -2.5890596 & 1.0401373 & 0.26387808 & 1.0831383 & 2.0149522 & -3.0376844 \end{bmatrix} \tag{1}$$

$$b1 = [8.979317, -5.988329, 0.26482144, -0.250453, -1.1340998, 6.665368]^T \tag{2}$$

$$M2 = \begin{bmatrix} 6.996323 \\ 4.3360047 \\ -0.807298 \\ -0.6610357 \\ -1.4745305 \\ -10.0931015 \end{bmatrix} \tag{3}$$

$$b2 = -2.8664327 \tag{4}$$

$$X1 = tanh(M1 \times X + b1) \tag{5}$$

$$\hat{y} = round(sigmoid(M2 \times X1) + b2) \tag{6}$$