Balanced Parentheses (hard)

Problem Statement

For a given number 'N', write a function to generate all combination of 'N' pairs of balanced parentheses.

Example 1:

```
Input: N=2
Output: (()), ()()
```

Example 2:

```
Input: N=3
Output: ((())), (()()), (())(), ()(()), ()()()
```

Try it yourself

Try solving this question here:

Java | Python3 | JS | C++

```
const generate_valid_parentheses = function(num) {
  result = [];
  // TODO: Write your code here
  return result;
};


console.log(`All combinations of balanced parentheses are: ${generate_valid_parentheses(2)}`)
console.log(`All combinations of balanced parentheses are: ${generate_valid_parentheses(3)}`)

```

RUN SAVE RESET

Solution

This problem follows the Subsets pattern and can be mapped to Permutations. We can follow a similar BFS approach.

Let's take Example-2 mentioned above to generate all the combinations of balanced parentheses. Following a BFS approach, we will keep adding open parentheses `(` or close parentheses `)`. At each step we need to keep two things in mind:

- We can't add more than 'N' open parenthesis.
- To keep the parentheses balanced, we can add a close parenthesis `)` only when we have already added enough open parenthesis `(`. For this, we can keep a count of open and close parenthesis with every combination.

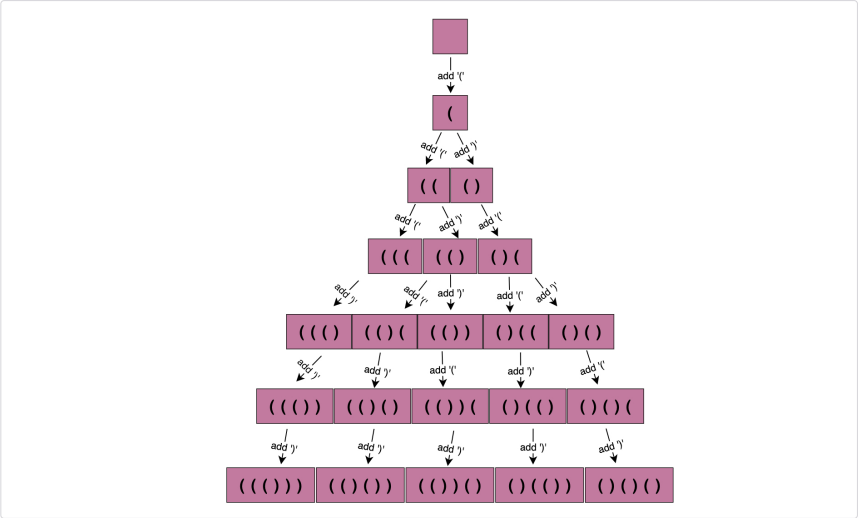
Following this guideline, let's generate parentheses for N=3:

1. Start with an empty combination: ""
2. At every step, let's take all combinations of the previous step and add `(` or `)` keeping the above-mentioned two rules in mind.
3. For the empty combination, we can add `(` since the count of open parenthesis will be less than 'N'. We can't add `)` as we don't have an equivalent open parenthesis, so our list of combinations will now be: "("
4. For the next iteration, let's take all combinations of the previous set. For "(" we can add another `(` to it since the count of open parenthesis will be less than 'N'. We can also add `)` as we do have an equivalent open parenthesis, so our list of combinations will be: "((", "()"
5. In the next iteration, for the first combination "((", we can add another `(` to it as the count of open parenthesis will be less than 'N', we can also add `)` as we do have an equivalent open parenthesis. This gives us two new combinations: "(((" and "(()". For the second combination "()", we can add another `(` to it since the count of open parenthesis will be less than 'N'. We can't add `)` as we don't have an equivalent open parenthesis, so our list of combinations will be: "(((", "(()", ()("
6. Following the same approach, next we will get the following list of combinations: "(((()", "(()(", "(())", ")((",

"()()"

7. Next we will get: "(())", "()()", "(())(", "()()", "()()("
8. Finally, we will have the following combinations of balanced parentheses: "((()))", "(()())", "(())()", "()(())", "()()()"
9. We can't add more parentheses to any of the combinations, so we stop here.

Here is the visual representation of this algorithm:



Code

Here is what our algorithm will look like:

Java Python3 C++ JS

```
const Deque = require('./collections/deque'); //http://www.collectionsjs.com

class ParenthesesString {
  constructor(str, openCount, closeCount) {
    this.str = str;
    this.openCount = openCount;
    this.closeCount = closeCount;
  }
}


function generate_valid_parentheses(num) {
  let result = [],
    queue = new Deque();
  queue.push(new ParenthesesString('', 0, 0));
  while (queue.length > 0) {
    const ps = queue.shift();
    // if we've reached the maximum number of open and close parentheses, add to the result
    if (ps.openCount === num && ps.closeCount === num) {
      result.push(ps.str);
    } else {
      if (ps.openCount < num) { // if we can add an open parentheses, add it
        queue.push(new ParenthesesString(`${ps.str}(`, ps.openCount + 1, ps.closeCount));
      }
      if (ps.openCount > ps.closeCount) { // if we can add a close parentheses, add it
        queue.push(new ParenthesesString(`${ps.str})`, ps.openCount, ps.closeCount + 1));
      }
    }
```

RUN SAVE RESET

Close

Output 6.785s

```
All combinations of balanced parentheses are: (()),()()
All combinations of balanced parentheses are: ((())),(()()),(())(),()(()),()()()
```

Time complexity

Let's try to estimate how many combinations we can have for 'N' pairs of balanced parentheses. If we don't care for the ordering - *that* `)` *can only come after* `(` - then we have two options for every position, i.e., either put open parentheses or close parentheses. This means we can have a maximum of 2^N combinations. Because of the ordering, the actual number will be less than 2^N.

If you see the visual representation of Example-2 closely you will realize that, in the worst case, it is equivalent to a binary tree, where each node will have two children. This means that we will have 2^N leaf nodes and $2^N - 1$ intermediate nodes. So the total number of elements pushed to the queue will be $2^N + 2^N - 1$, which is asymptotically equivalent to $O(2^N)$. While processing each element, we do need to concatenate the current string with `(` or `)`. This operation will take $O(N)$, so the overall time complexity of our algorithm will be $O(N * 2^N)$. This is not completely accurate but reasonable enough to be presented in the interview.

The actual time complexity ($O(4^n/\sqrt{n})$) is bounded by the Catalan number and is beyond the scope of a coding interview. See more details here.

Space complexity

All the additional space used by our algorithm is for the output list. Since we can't have more than $O(2^N)$ combinations, the space complexity of our algorithm is $O(N * 2^N)$.

Recursive Solution

Here is the recursive algorithm following a similar approach:

Java | Python3 | C++ | JS

```
function generate_valid_parentheses(num) {
  result = [];
  const parenthesesString = Array(2 * num);
  generate_valid_parentheses_rec(num, 0, 0, parenthesesString, 0, result);
  return result;
}


function generate_valid_parentheses_rec(num, openCount, closeCount, parenthesesString, index, result) {
  // if we've reached the maximum number of open and close parentheses, add to the result
  if (openCount === num && closeCount === num) {
    result.push(parenthesesString.join(''));
  } else {
    if (openCount < num) { // if we can add an open parentheses, add it
      parenthesesString[index] = '(';
      generate_valid_parentheses_rec(num, openCount + 1, closeCount, parenthesesString, index + 1, result
    }
    if (openCount > closeCount) { // if we can add a close parentheses, add it
      parenthesesString[index] = ')';
      generate_valid_parentheses_rec(num, openCount, closeCount + 1, parenthesesString, index + 1, result
    }
  }
}

console.log(`All combinations of balanced parentheses are: ${generate_valid_parentheses(2)}`);
console.log(`All combinations of balanced parentheses are: ${generate_valid_parentheses(3)}`);
```

RUN | SAVE | RESET

Close

Output 2.090s

```
All combinations of balanced parentheses are: (()),()()
All combinations of balanced parentheses are: ((())),(()()),(())(),()(()),()()()
```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