

Point Process Modeling of Limit Order Books

Oden Petersen

UNSW Mathematics and Statistics

Graph normally modeled with time series models. eg autoregression, state models, garch these models have had some success but don't describe the process of price formation from first principles

Limit Order Book

Markets are venues for buyers and sellers to come together to exchange financial assets
Limit order book diagram
Order components
Modeling individual order arrivals, cancellations, and trades allows us to simulate the evolution of the limit order book

Order Book Events

For order insertions, cancellations we need to model size, price, side, and time. First three can be modeled with standard time series tools. But modeling arrival time process is nontrivial. Some parts of the day have lots of arrivals, others don't. graph of intraday activity bucketed by minute

Inhomogenous Poisson Process Model

graph of intraday activity bucketed by minute overlay maxlikelihood
spline intensity
counting process overlaid with integral $\bar{\Lambda}$ of intensity

Likelihood function for Point Process Models

Explain why the loglikelihood formula is reasonable

Why integral of intensity should be iid exponential Show that it isnt
→ motivate autoregressive intensity

Hawkes Processes

Formula for intensity kernel is linear combination of exponential functions. can approximate analytic functions on $[0, t]$ uniformly well with enough exponential components by weierstrass approximation theorem. Can treat events as coming from a mixture of multiple point processes. As is common with mixture models, use an EM algorithm to fit. Explain how to get branching matrix.

State Dependence

Book imbalance vs time until next buy event / time until next sell event Motivate state dependence Formula for state dependence - discrete and continuous Maybe some more details on book imbalance

Point Process Simulation

Detail Ogata algorithm Original paper uses markov state switching.
I can use time series model for orders and limit order book simulation to make this a bit more sophisticated.

Model Results

IC by model crossvalidation loglikelihood actual vs expected counts
residual distribution and autocorrelation and ks pval table
replication number and

Price Impact Function

Graph of impact vs size VWAP vs TWAP

Temporary page!

\LaTeX was unable to guess the total number of pages correctly. There was some unprocessed data that should have been added to the final page; this extra page has been added to receive it. If you rerun the document (without altering it) this surplus page will go away, because \LaTeX now knows how many pages to expect for this document.