Logistic Loss

ou will:

- explore the reason why the squared error loss is not appropriate for logisti regression
- explore the logistic loss function

Squared error for logistic regression

ecall for **Linear** Regression we have used the **squared error cost function**: The quation for the squared error cost with one variable is:

$$J(w,b) = \frac{1}{2m} \sum_{i=0}^{m-1} (f_{w,b}(x^{(i)}) - y^{(i)})^2 \quad (:$$

here

$$f_{w,b}(x^{(i)}) = wx^{(i)} + b \quad (:$$

his cost function worked well for linear regression.

lowever, in logistic regression as we have arleady seen $f_{w,b}(x)$ has a non-linear omponent, the sigmoid function: $f_{w,b}(x^{(i)}) = sigmoid(w \cdot x^{(i)} + b).

et's take a look of an example.

```
import numpy as np
import matplotlib.pyplot as plt
```

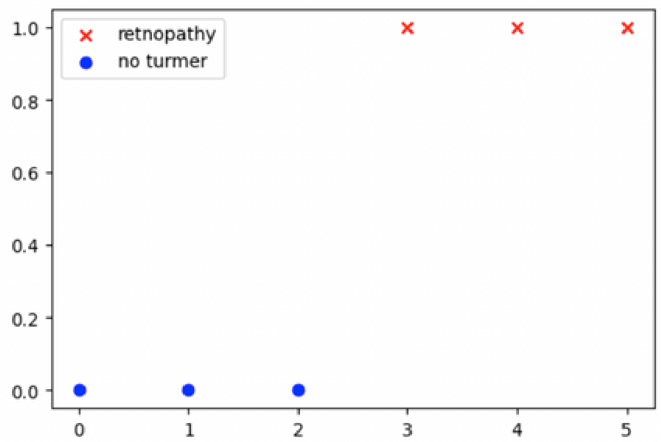
Pythor

```
x_train=np.array([0., 1, 2, 3, 4, 5], dtype=np.longdouble)
y_train = np.array([0, 0, 0, 1, 1, 1], dtype=np.longdouble)
```

Pythor

```
plt.figure(figsize=(6,4))
plt.scatter(x_train[y_train==1], y_train[y_train==1], color='red', marker='
plt.scatter(x_train[y_train==0], y_train[y_train==0], color='blue', marker=
plt.legend()
plt.show()
```

Pythor



From the above plot you can see clearly that even our datadestribution is non-linear.

So we need to have other form of cost function .

Logistic Loss Function

Logistic regression uses a loss fucntion more suitable for the task of categorization where the target is 0 or 1 rather than any number.

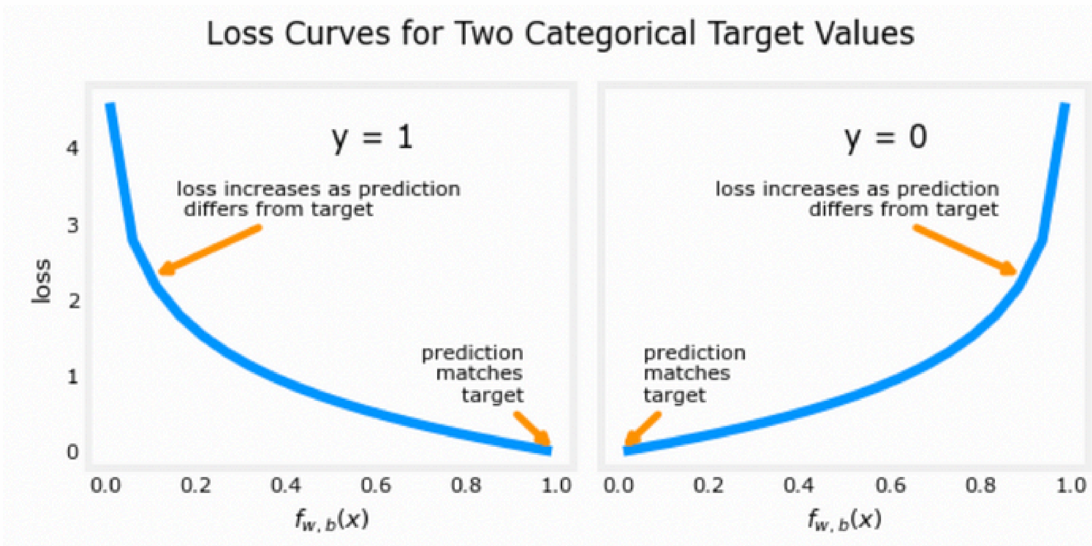
Loss

- this is the measure of the difference of a single example to it's target value, $loss(f_{w,b}(\mathbf{x}^{(i)}, y^{(i)}))$

$$loss(f_{\mathbf{w},b}(\mathbf{x}^{(i)}), y^{(i)}) = \begin{cases} -\log\left(f_{\mathbf{w},b}\left(\mathbf{x}^{(i)}\right)\right) & \text{if } y^{(i)} = 1 \\ -\log\left(1 - f_{\mathbf{w},b}\left(\mathbf{x}^{(i)}\right)\right) & \text{if } y^{(i)} = 0 \end{cases} \quad (1)$$

- where:
  - $f_{w,b}(\mathbf{x}^{(i)})$ is the model prediction.
  - $y^{(i)}$ is the target value.
  - $f_{w,b}(\mathbf{x}^{(i)}) = g(\mathbf{w} \cdot \mathbf{x}^{(i)} + b)$ is the sigmoid function.

The defining feature of this loss function is the fact that it uses two separate curves. One for the case when the target is zero or ($y = 0$) and another for when the target is one ($y = 1$). Combined, these curves provide the behavior useful for a loss function, namely, being zero when the prediction matches the target and rapidly increasing in value as the prediction differs from the target. Consider the curves below:



Combined, the curves are similar to the quadratic curve of the squared error loss. Note, the x-axis is $f_{\mathbf{w},b}$ which is the output of a sigmoid. The sigmoid output is strictly between 0 and 1.

Now, the loos function can be re-written to be easier to implement.

$$loss(f_{\mathbf{w},b}(\mathbf{x}^{(i)}), y^{(i)}) = (-y^{(i)} \log\left(f_{\mathbf{w},b}\left(\mathbf{x}^{(i)} \right) \right) - \left(1 - y^{(i)}\right) \log \left(1 - f_{\mathbf{w},b}\left(\mathbf{x}^{(i)} \right) \right)$$

where:

- if $y^{(i)} = 0$.
  - then

$$\begin{align} loss(f_{\mathbf{w},b}(\mathbf{x}^{(i)}), 0) &= (-(0) \log\left(f_{\mathbf{w},b}\left(\mathbf{x}^{(i)} \right) \right) - \left(1 - 0\right) \log \left(1 - f_{\mathbf{w},b}\left(\mathbf{x}^{(i)} \right) \right) \tag{1}\\ &= -\log \left(1 - f_{\mathbf{w},b}\left(\mathbf{x}^{(i)} \right) \right) \tag{2} \end{align}$$

- if $y^{(i)} = 0$.
  - then

$$\begin{align} loss(f_{\mathbf{w},b}(\mathbf{x}^{(i)}), 1) &= (-(1) \log\left(f_{\mathbf{w},b}\left(\mathbf{x}^{(i)} \right) \right) - \left(1 - 1\right) \log \left(1 - f_{\mathbf{w},b}\left(\mathbf{x}^{(i)} \right) \right) \tag{1}\\ &= -\log \left(f_{\mathbf{w},b}\left(\mathbf{x}^{(i)} \right) \right) \tag{2} \end{align}$$

cost

- this is the measure of the difference of a training set example to it's target value, $\mathbf{J}(\mathbf{w},b)$

$$\mathbf{J}(\mathbf{w},b) = \frac{1}{m} \sum_{i=0}^{m-1} \left[loss(f_{w,b}(\mathbf{x}^{(i)}), y^{(i)}) \right] \tag{1}$$

where

- $loss(f_{\mathbf{w},b}(\mathbf{x}^{(i)}), y^{(i)})$ is the cost for a single data point, which is:

$$loss(f_{\mathbf{w},b}(\mathbf{x}^{(i)}), y^{(i)}) = -y^{(i)} \log\left(f_{\mathbf{w},b}\left(\mathbf{x}^{(i)} \right) \right) - \left(1 - y^{(i)}\right) \log \left(1 - f_{\mathbf{w},b}\left(\mathbf{x}^{(i)} \right) \right) \tag{2}$$

- where m is the number of training examples in the data set and:

$$\begin{align} f_{\mathbf{w},b}(\mathbf{x^{(i)}}) &= g(z^{(i)})\tag{3} \\ z^{(i)} &= \mathbf{w} \cdot \mathbf{x}^{(i)}+ b\tag{4} \\ g(z^{(i)}) &= \frac{1}{1+e^{-z^{(i)}}}\tag{5} \end{align}$$

Code

Python - imports

```python
import numpy as np
import matplotlib.pyplot as plt
```

Python

Python - sigmoid() function

```python
def sigmoid(z_i):
    """
    Compute sigmoid

    Args:
        z_i (scalar ):  target values

    Returns:
        sigmoid (scalar ):  sigmoid
    """

    return 1/(1 + np.exp(-z_i))
```

Python

C - sigmoid() function

```c
float sigmoid(float z_i){

    /*
        Compute sigmoid

        Args:
            z_i (scalar ):  target values

        Returns:
            sigmoid (scalar ):  sigmoid

    */

    return (float)1/(1 + exp(-z_i));
}

```

Python - cost() function

```python
def compute_cost_logistic(X, y, w, b):
    """
    Compute cost

    Args:
        X (ndarray (m,n)):  Data, m exapmles with n features
        y (ndarray (m, )):  target values
        w (ndarray (n, )):  model parameters
        b (scalar) .     :  model parameter

    Returns:
        cost (scalar):  cost
    """

    m=X.shape[0]
    cost=0.0
    for i in range(m):
        z_i = np.dot(X[i], w) + b
        f_wb_i = sigmoid(z_i)
        cost += -y[i]*np.log(f_wb_i) - (1 - y[i])*np.log(1 - f_wb_i)
    cost /=m

    return cost
```

Python

C - cost() function

```c
float compute_cost_logistic(float X[][nx], int y[m], int w[][ny], int b){
    /*
        Compute cost

        Args:
            X (ndarray (m,n)):  Data, m exapmles with n features
            y (ndarray (m, )):  target values
            w (ndarray (n, )):  model parameters
            b (scalar) .     :  model parameter

        Returns:
            cost (scalar):  cost
    */

    float cost=0;
    float z_i;
    float f_wb_i;

    for (int i = 0; i < m; i++)
    {
     for (int j = 0; j < ny; j++)
     {
        z_i=0;
        f_wb_i=0;
        for (int k = 0; k < nx; k++)
        {
            z_i += X[i][k] * w[k][j];
        }
        z_i += b;
        f_wb_i = sigmoid(z_i);
        cost +=  (-y[i]*log(f_wb_i)) - ((1 - y[i]) * log(1 - f_wb_i));
     }


    }
    cost /=m;

    return cost;

}
```

Python - main() function

```python
def main():


    X_train = np.array([[0.5, 1.5], [1,1], [1.5, 0.5], [3, 0.5], [2, 2],
    y_train = np.array([0, 0, 0, 1, 1, 1])

    w_tmp = np.array([1, 1])
    b_tmp = -3

    cost=compute_cost_logistic(X_train, y_train, w_tmp, b_tmp)
    print("Cost for b = -3 : ",cost)

if __name__ == "__main__":
    main()
```

✓ 0.0s Python

```
Cost for b = -3 :  0.36686678640551745
```

C - main() function

```c
#include<stdio.h>
#include<math.h>
#include<stdlib.h>

#define m 6          //number of examples
#define nx 2         //number of input features
#define ny 1         //number of output features

float sigmoid(float );
float compute_cost_logistic( float X[][nx], int y[m], int w[][ny], int );

int main(){

    //input and output datasets
    float X_train[][nx] = {{0.5, 1.5}, {1, 1}, {1.5, 0.5}, {3, 0.5}, {2, 2}, {1, 2.5}};        //(m,n)
    int y_train[m] = {0, 0, 0 , 1, 1, 1};

    //w and b parameters
    int w_init[][ny] = {1, 1};                  //(n, )
    int b1_init = -3;                           //scalar
    int b2_init = -4;                           //scalar

    double cost_b1 = compute_cost_logistic(X_train, y_train, w_init, b1_init);
    double cost_b2 = compute_cost_logistic(X_train, y_train, w_init, b2_init);

    printf("Cost for b = %d : %le\n", b1_init, cost_b1);

    printf("Cost for b = %d : %le\n", b2_init, cost_b2);



    return 0;
}
```

```
venvsuzanodero@suzans-MacBook-Air Logistic_Loss % gcc logistic_loss.c
venvsuzanodero@suzans-MacBook-Air Logistic_Loss % ./a.out
Cost for b = -3 : 3.668667e-01
Cost for b = -4 : 5.036809e-01
```

Let's try to see what will be the cost for a different value of b

- if $w_0 = 1$, $w_1 = 1$ and $b = -4$

```python
def main():


    X_train = np.array([[0.5, 1.5], [1,1], [1.5, 0.5], [3, 0.5], [2, 2],
    y_train = np.array([0, 0, 0, 1, 1, 1])

    w_tmp = np.array([1, 1])
    b_tmp = -4

    cost=compute_cost_logistic(X_train, y_train, w_tmp, b_tmp)
    print("Cost for b = -3 : ",cost)

if __name__ == "__main__":
    main()
```

✓ 0.0s Python

```
Cost for b = -3 :  0.5036808636748461
```

Let us plot the decision boundary for these two different values of b.

- For $b = -3$, $w_0 = 1$ and $w_1 = 1$, we will plot $-3 + x_0 + x_1 = 0$ (shown in blue)
- For $b = -4$, $w_0 = 1$ and $w_1 = 1$, we will plot $-4 + x_0 + x_1 = 0$ (shown in magenta)

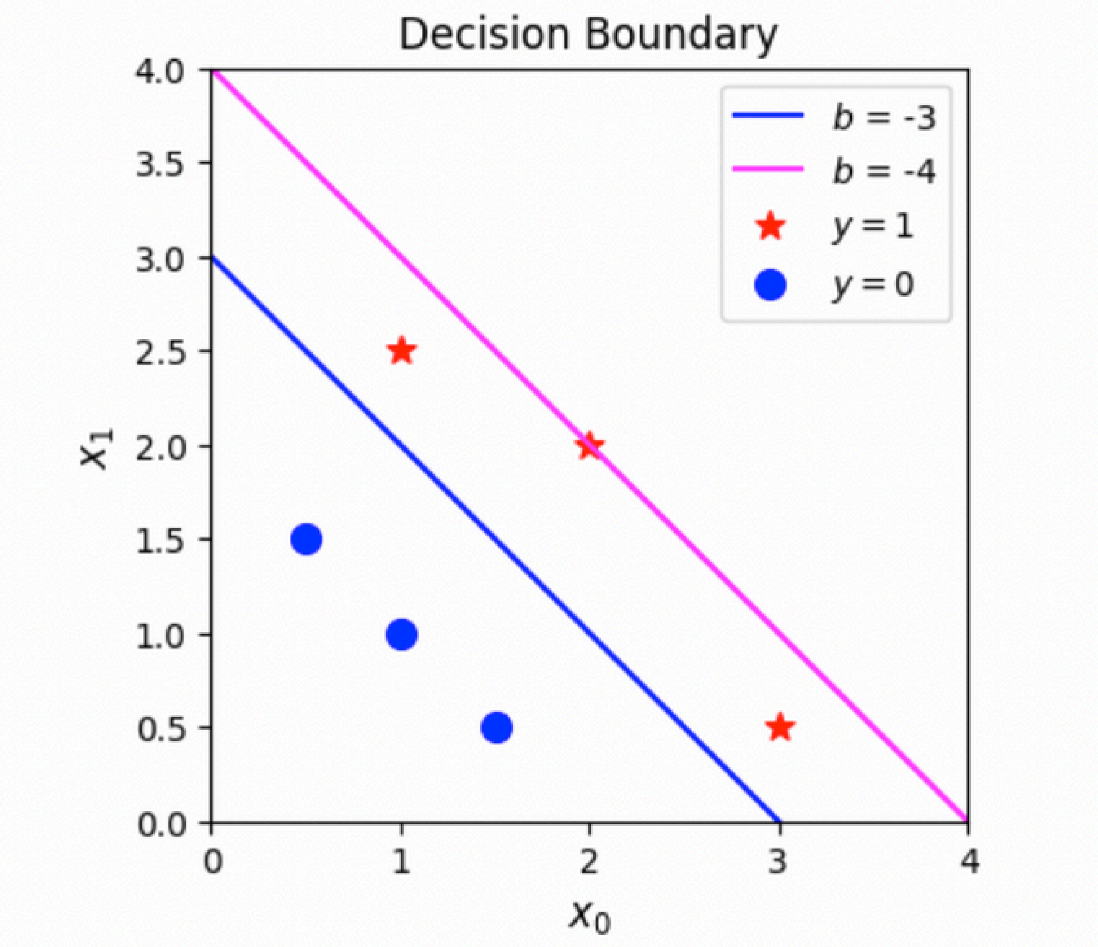
```python
# Choose values between 0 and 6
x0 = np.arange(0,6)

# Plot the two decision boundaries
x1 = 3 - x0
x1_other = 4 - x0

fig,ax = plt.subplots(1, 1, figsize=(4,4))
# Plot the decision boundary
ax.plot(x0,x1, color="blue", label="$b$ = -3")
ax.plot(x0,x1_other, color="magenta", label="$b$ = -4")
ax.axis([0, 4, 0, 4])

# Plot the original data
plt.scatter(X_train[y_train==1, 0], X_train[y_train==1, 1], marker='*', color='red', s=70, label='$y = 1$')
plt.scatter(X_train[y_train==0, 0], X_train[y_train==0, 1], marker='o', color='blue', s=70, label='$y = 0$')
ax.axis([0, 4, 0, 4])
ax.set_ylabel('$x_1$', fontsize=12)
ax.set_xlabel('$x_0$', fontsize=12)
plt.legend(loc="upper right")
plt.title("Decision Boundary")
plt.show()
```

Python



Since

- Cost for b = -3 : 0.36686678640551745
- Cost for b = -4 : 0.5036808636748461

We can see the cost function behaves as expected and the cost for $w_0 = 1, w_1 = 1$ and $b = -3$ is indeed higher than the cost for $w_0 = 1, w_1 = 1$ and $b = -4$