Problem Set #3

Sarah Odeh - odesmodes

1 Question 1

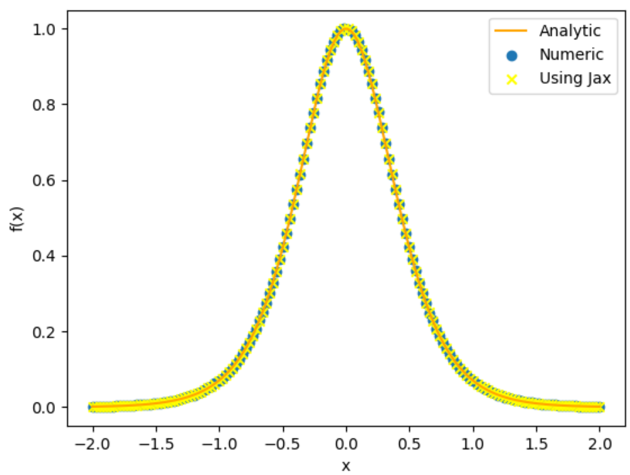

Figure 1: Plot of f(x) vs x using the analytic, numeric, and jax methods, as shown in the legend. All of the methods yield basically the same result.

2 Question 2

- Parts a-d are on the next page I couldn't figure out how to get it to move where I wanted it to be so I just gave up.
- Part e:
  Value of gamma(3/2) = 0.8862272081548264
- Part f:
  gamma(3) = 2.0000000000000657
  gamma(6) = 120.000000000000003
  gamma(10) = 362879.99999999994

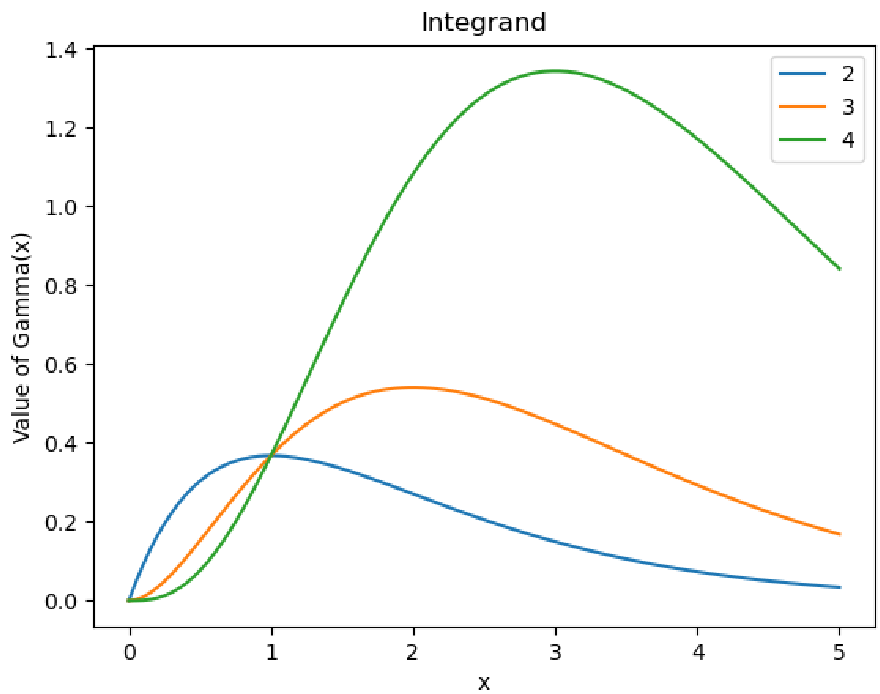

Figure 2: Part a

b. $f(x) = x^{a-1} e^{-x}$

$f'(x) = (a-1)x^{a-2}e^{-x} + x^{a-1}(-1)e^{-x} = 0$

$(a-1)x^{a-2}\cancel{e^{-x}} = x^{a-1}\cancel{e^{-x}}$

$a-1 = \dfrac{x^{a-1}}{x^{a-2}}$

$a-1 = x^{a-1-a+2}$

$a-1 = x$ ← critical point

c. $z = \dfrac{x}{c+x} = \dfrac{1}{2}$ $\quad x = \dfrac{c+x}{2}$ $\quad 2x = c+x$ $\quad x = c$

for $x = a-1$

d. $x^{a-1} = e^{(a-1)\ln x}$

$e^{(a-1)\ln x}e^{-x} \Rightarrow \lim_{x \to 0}$ $\quad \ln x \to -\infty$ so $e^{(a-1)\ln x} \to$ very small

$e^{-x} \to$ ~~very~~ small (1)

$\Rightarrow \lim_{x \to \infty}$ $\quad \ln x \to \infty$ so $e^{(a-1)\ln x} \to$ very large

$e^{-\infty} \to 0$

Gets rid of overflow error and limits underflow

Figure 3: Parts b-d

3 Question 3

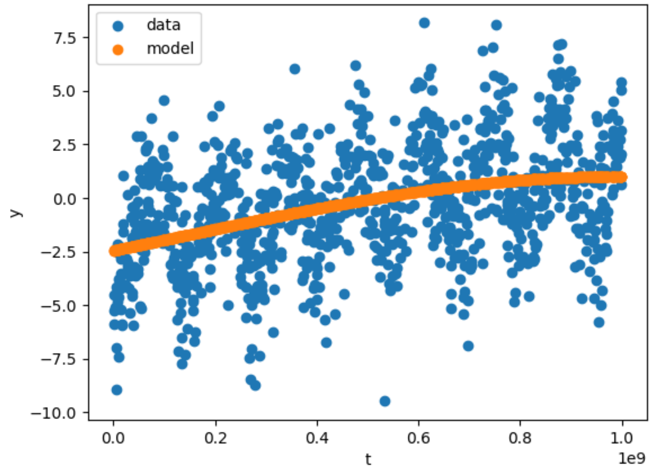

Figure 4: Parts a and b, as we can see, this is not the greatest model. For Part c, the residuals are 2.5262513515109735, this is too far from the expected standard deviation which is 2.0.

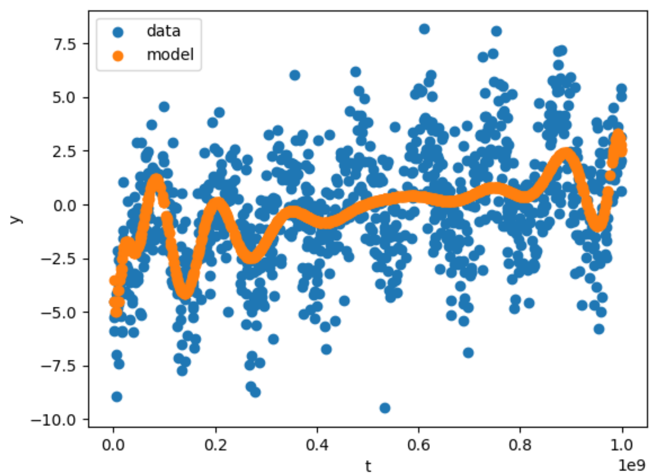

Figure 5: Part d: I used a 40 degree polynomial. The residuals came out to be 2.2734744011188095 which is better than the 3rd degree polynomial but still not 2. There is no reasonable polynomial which will fit the data perfectly because the data is periodic

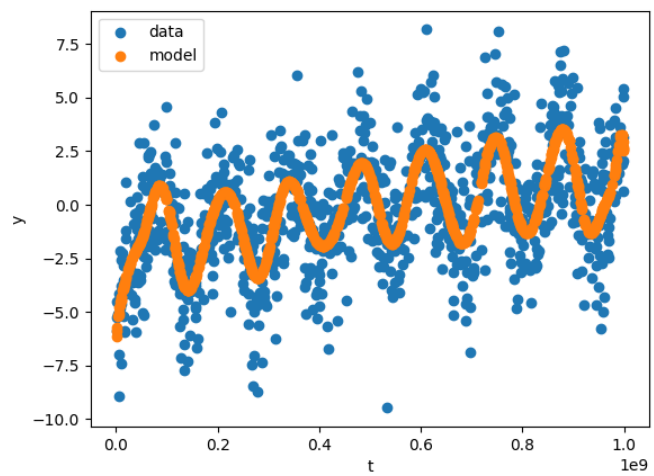

Figure 6: Part e: This model does the best job at approximating the periodicity of the data.