

# Problem Set #3

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## 1 Question 1

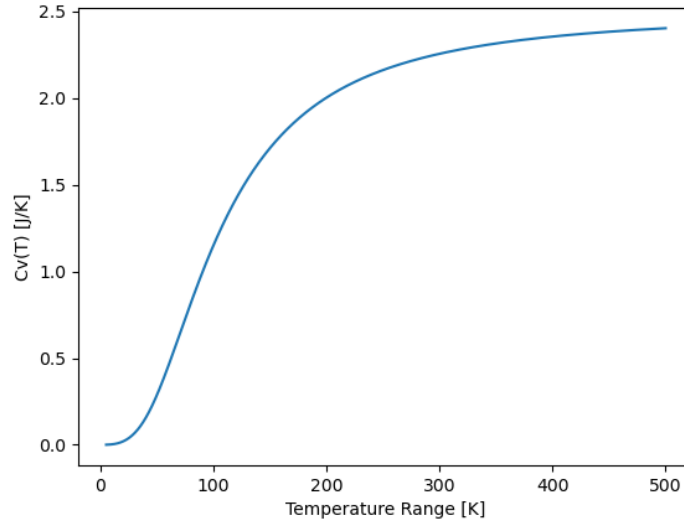


Figure 1: Part a-b: Graph of specific heat capacity  $C_v$  with respect to T range 5-500K

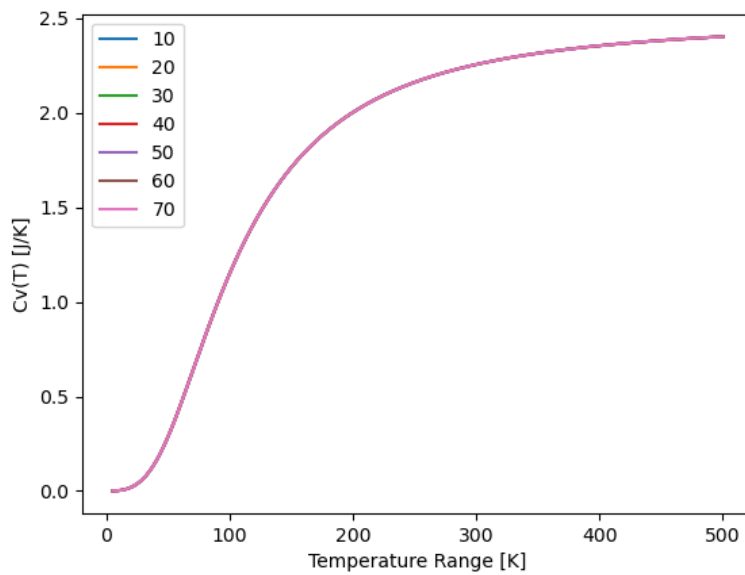


Figure 2: Part c: Graph of specific heat capacity  $C_v$  with respect to T range 5-500K, iterated through different  $N$ 's from 10-70. We see that there is very minimal change to the plot when we change  $N$  which shows how the gaussian quadrature integral evaluation is very accurate regardless of the number of sample points.

## 2 Question 2

$$\begin{aligned}
 E &= \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 + V(x) \\
 \frac{2}{m}[E - V(x)] &= \left(\frac{dx}{dt}\right)^2 \\
 \sqrt{\frac{2}{m}[E - V(x)]} &= \frac{dx}{dt} \\
 \frac{\sqrt{\frac{2}{m}[E - V(x)]}}{dx} &= \frac{1}{dt} \\
 E = V(a) \quad \int_0^a \frac{dx}{\sqrt{\frac{2}{m}[V(a) - V(x)]}} &= \int_0^{1/4 T} dt \\
 \frac{1}{\sqrt{2/m}} \int_0^a \frac{dx}{\sqrt{V(a) - V(x)}} &= 1/4 T \\
 \frac{1}{\sqrt{2/m}} \int_0^a \frac{dx}{\sqrt{V(a) - V(x)}} &= 1/4 T
 \end{aligned}$$

$\frac{4 \cdot \sqrt{m}}{\sqrt{2}} = \frac{4}{\sqrt{2}} = \frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{4\sqrt{2}}{2} = 2\sqrt{2}$   
 $4 = \sqrt{2} \cdot \sqrt{2} \cdot \sqrt{2} \cdot \sqrt{2}$   
 $= \sqrt{2 \cdot 2 \cdot 2 \cdot 2}$   
 $= \sqrt{8}$

Figure 3: Part a: Rearranging the integral

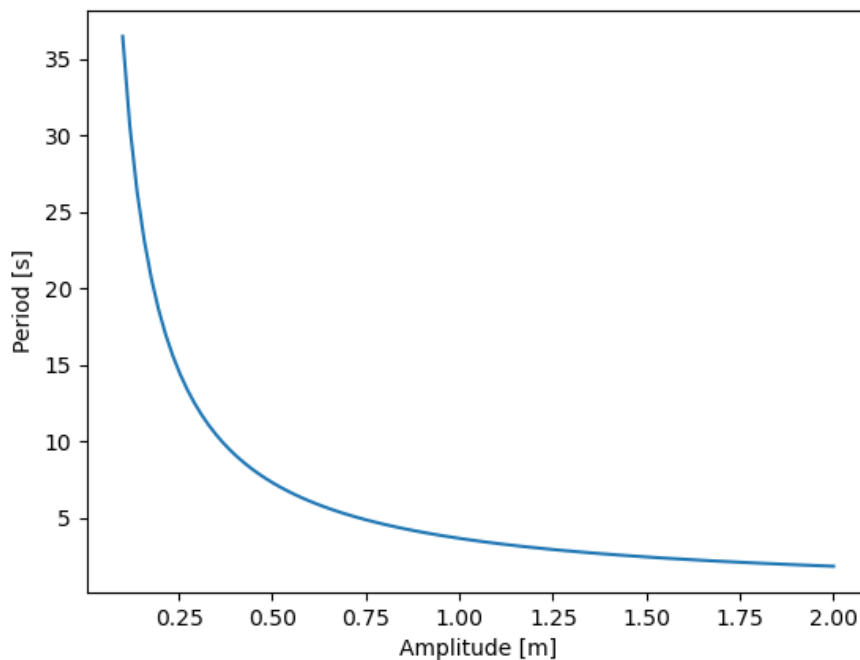


Figure 4: Part b-c: Plot of period vs amplitude. Since energy is constant through time, as the particle travels to larger amplitudes the period has to get smaller, as per the equation derived in part a. As the amplitude goes to zero then the period has to get very large so it diverges.

### 3 Question 3

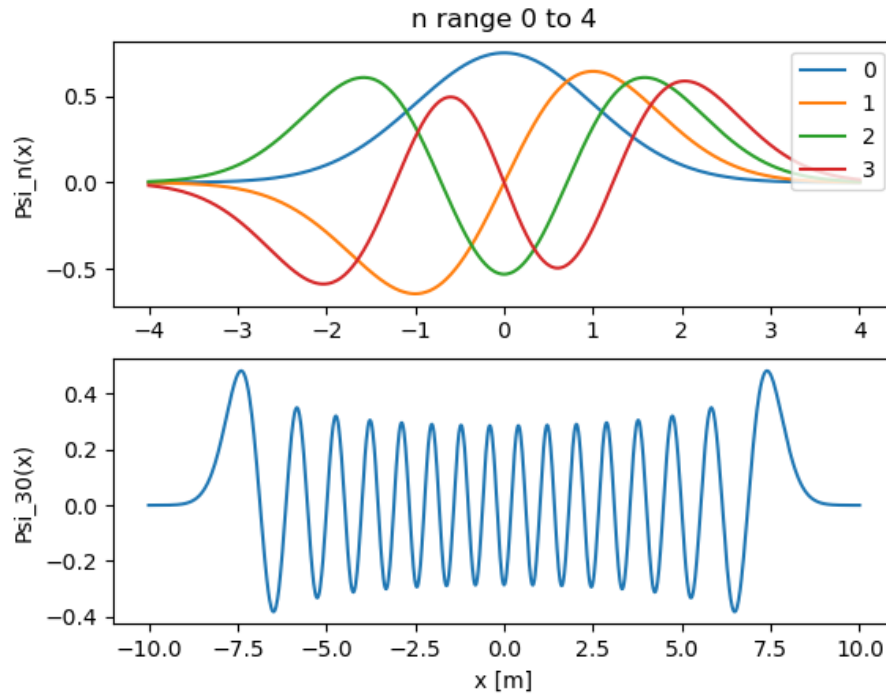


Figure 5: Part a-b: Plots of the wave function with differing values of  $n$  versus  $x$

Part c: I get 2.345207873785818 as the uncertainty for  $n = 5$ .

Part d: I get 2.3452078799117144 as the uncertainty for  $n = 5$ . Yes you can evaluate this integral exactly. The condition for exact evaluation is the degree of the polynomial must be  $2N - 1$  where  $N$  is the number of points. In our case the degree of the polynomial was  $n = 5$  and the number of points was  $N = 100$ , so the calculation was exact.