FIN30200: Econometrics of Financial Markets

Group Project - Modeling Exchange Rate Volatility: Application of the GARCH and EGARCH Models

Ciaran Maxwell: 19327416

Conor Gallagher: 19415714

Liam Holland: 19457642

Mark Mullins: 19341043

Odhran Murphy: 19469442

Import Libraries

In [1]:
```
# Import the standard data analysis libraries
import pandas as pd
import numpy as np
import statsmodels.api as sm
from scipy import stats

# Import libraries for web scraping
import requests
import numpy as np
from tqdm import tqdm
import pickle, os
import bs4 as bs
from time import sleep

# Time Series Analysis Library: AR,VAR,ARMA; ACF&PACF
import statsmodels.tsa as tsa

# Regression and Regression Test Libraries
import statsmodels.formula.api as smf
import statsmodels.stats.api as sms

# Import acf & pacf plotting functions
from statsmodels.graphics.tsaplots import plot_acf
from statsmodels.graphics.tsaplots import plot_pacf

# Plotting library
import matplotlib
import matplotlib.pyplot as plt
plt.style.use("seaborn-darkgrid")
import seaborn as sns

# Calling Python magic command
%matplotlib inline
# Adjust the figure size
matplotlib.rcParams['figure.figsize']=[10,6]

# Module to use
import os

# Import ARCH
from arch import arch_model

# Import ADF and PP unit root tests
from arch.unitroot import ADF
from arch.unitroot import PhillipsPerron as PP

# Import Shapiro-Wilk test
from scipy.stats import shapiro as sw

# Ljung-Box test function
from statsmodels.stats.diagnostic import acorr_ljungbox

# Import datetime library
import datetime as dt

# Import markdown display module
from IPython.display import Markdown

# Forecast accuracy measures: 'sklearn' library: sklearn.metrics
from sklearn.metrics import mean_squared_error
from sklearn.metrics import mean_absolute_error
```

Current Directory

In [2]:
```
print(os.getcwd())
```

```
C:\Users\Odhran Murphy\Documents\College\4. Fourth Year\Semester 1\FIN30200 - Econometrics of Financial Markets\Grou
p Project
```

Set GitHub Repository

In [3]:
```
base_url = 'https://raw.githubusercontent.com/odhran-murphy/EFM_Group_Project/main'
```

Common Plotting Functions

`plot_series`

In [4]:
```
def plot_series(series, title, ylabel):
    """Plot the series against the time index of dataframe."""

    plt.plot(series)
    plt.xlabel('Date')
    plt.ylabel(ylabel)
    plt.title(title)
    plt.show()
```

`plot_acf_pacf`

In [5]:
```
def plot_acf_pacf(series, title):
    """Display both acf and pacf using a shared x-axis."""

    fig, axes = plt.subplots(nrows=2, figsize=(10, 6))

    plot_acf(series, lags=40,ax=axes[0])
    plot_pacf(series, lags=40,ax=axes[1], method='ywm')
    plt.suptitle(title)
    plt.show()
```

`plot_series_v_normal`

In [6]:
```
def plot_series_v_normal(series, series_label, title):
    """Plot series histogram v normal distribution"""

    # Applying the seaborn distplot function to get histogram & density curve of series
    sns.histplot(series, kde=True, color='darkblue', stat='density', label = series_label)

    # Normal distribution with the same mean & variance
    np.random.seed(0) # Fixed seed to use

    # Random variable with normal distribution
    normal_sample = np.random.normal(np.mean(series), np.std(series), 249)

    # Applying the Gaussian kernel density estimate to get the density curve
    sns.kdeplot(normal_sample, color='red', shade=True, label = "Normal")

    plt.title(title)
    plt.legend()
    plt.show()
```

Data Download and Data Processing

This paper uses both GARCH (1,1) and EGARCH (1,1) to model the volatility of TZS/USD exchange rate for the January 4, 2009 - July 27, 2015 period.

A number of sources were accessed to obtained the data needed to reproduce the work described in REF.

Initially, obtaining the data directly from the Bank of Tanzania was attempted, but the web interface only reported historical exchange rates on a daily basis. So other options were explored. Some services such as Exchangr, had limited historical data while others on the free tier, such as, FXMarketAPI, do not support TZS.

Investing.com does support TZS over the required time interval. However, the Investing.com data contained more data points (1711 vs 1593) and is both more noisy and contains a number of outlier observations. So while the global structure of

Investing.com data is comparable to the data used in the paper, it is not suitable for any analysis focusing on the differences between continuous data points.

Obtaining the data directly from the Bank of Tanzania required a web scraper. Looping over each day in the interval 2009-01-04 to 2015-07-27. This represents 2396 days, or on skipping weekends, 1711 days. Web scraping over each of the 1711 weekdays resulted in 1613 data points (most probably due to bank holidays, etc.). Due to speed of remote server, run time is about 1 hour.

The Bank of Tanzania data set contained *Buying*, *Selling*, and *Mean* variables. On comparison with the paper, it appears that *Selling* or *Mean* was used.

Using a plot digitizer (https://automeris.io/WebPlotDigitizer/), we can scan figure 1 and obtain data. We then compare this to the data set obtained from investing.com and directly from the

www.investing.com

- https://www.investing.com/currencies/usd-tzs-historical-data -> USD_TZS_Historical_Data.csv

In [7]:
```
filename = 'USD_TZS_Historical_Data.csv'
df_1 = pd.read_csv(f'{base_url}/{filename}') # TZS to 1 USD rate

df_1.Date = pd.to_datetime(df_1['Date'], format='%m/%d/%Y')
df_1.Price = df_1.Price.apply(lambda x: float(x.split()[0].replace(',', '')))

start_date = dt.datetime(2009,1,4)
end_date = dt.datetime(2015,7,27)
print("Number of days in interval of interest:", end_date-start_date)
paper_interval = (df_1.Date>=start_date) & (df_1.Date<=end_date)

df_1 = df_1[paper_interval]

df_1 = df_1[['Date', 'Price']]
df_1.set_index('Date', inplace=True)
df_1.columns.values[0] = 'TZS_USD'
df_1 = df_1.iloc[::-1]

print(df_1.shape)
df_1.head()
```

```
Number of days in interval of interest: 2395 days, 0:00:00
(1711, 1)
```

Out[7]:

Date	TZS_USD
2009-01-05	1325.0
2009-01-06	1340.0
2009-01-07	1339.0
2009-01-08	1343.0
2009-01-09	1357.0

Log TZS/USD exchange rate Transformation

$$P_t = \log_{10}(E_t)$$

$$r_t = \log_{10}(\frac{E_t}{E_{t-1}}) = \log_{10}(E_t) - \log_{10}(E_{t-1})$$

where

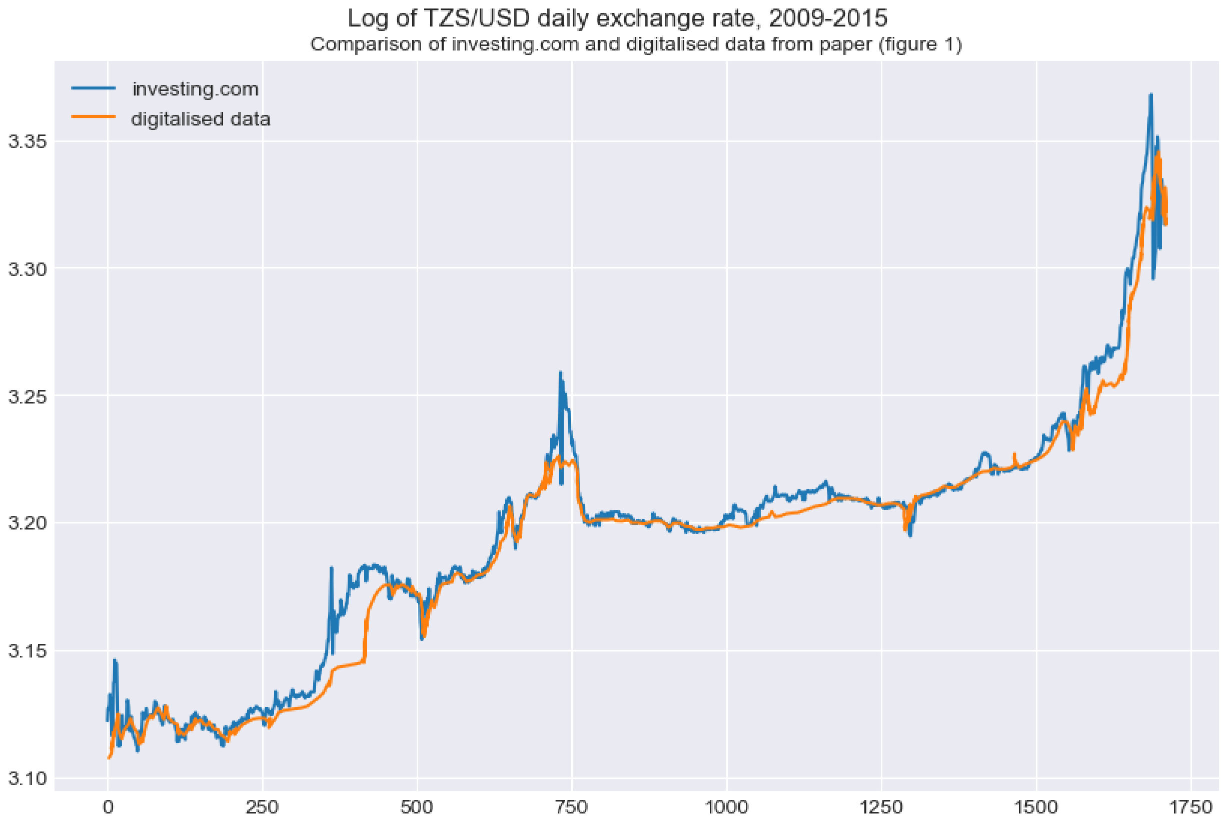
P_t is the log TZS/USD exchange rate at time t

r_t is the log TZS/USD exchange rate relative at time t

E_t is the TZS/USD exchange rate at time t

In [8]:
```
plt.plot(np.log10(df_1.TZS_USD.values), label="investing.com")

# digitalised data from paper fig 1
filename = 'scanned_fig_1_data.csv'
dfp = pd.read_csv(f'{base_url}/{filename}', names=['x','y'])

# dfp = pd.read_csv('scanned_fig_1_data.csv', names=['x','y'])
plt.plot(dfp.x * df_1.TZS_USD.shape / dfp.x.max(), dfp.y, label='digitalised data')

plt.legend()
plt.title('Comparison of investing.com and digitalised data from paper (figure 1)', fontsize=10)
```

```
plt.suptitle('Log of TZS/USD daily exchange rate, 2009-2015')
plt.subplots_adjust(top=0.92) #Adjusting title position
plt.show()
```



https://www.bot.go.tz/

- https://www.bot.go.tz/ExchangeRate/previous_rates -> bank_of_tanzania_USD.csv

In [9]:
```
URL = "https://www.bot.go.tz/ExchangeRate/previous_rates?__RequestVerificationToken=nPOnxVhWt80SQ-DZ346Pm1IAkEGvpI9S

# test pattern
URL.format(month=1, day=11, year=2009)
```

Out[9]:
```
'https://www.bot.go.tz/ExchangeRate/previous_rates?__RequestVerificationToken=nPOnxVhWt80SQ-DZ346Pm1IAkEGvpI9SkfSc8E
LmPiCRKBSu9THfDsnBg5IHVtaVaBMem5M9ftQGLiITCkbtiN9BfwxPGnss8tRgedc3rA81&exchangeDate=01%2F11%2F2009'
```

In [10]:
```
def daterange(start_date, end_date):
    for n in range(int((end_date - start_date).days)):
        yield start_date + dt.timedelta(n)

start_date = dt.datetime(2009,1,4)
end_date = dt.datetime(2015,7,27) + dt.timedelta(days=1)
end_date = start_date + dt.timedelta(days=10)

print("Number of days in interval of interest:", end_date-start_date)

USE_PRE_DOWNLOADED = True

if USE_PRE_DOWNLOADED:
    print("Using pre-downloaded data ...")
    if os.path.isfile("dfs.bin"):
        with open('dfs.bin','rb') as f:
            dfs = pickle.load(f)
    else:
        print("Set USE_PRE_DOWNLOADED to scrape website.")

else:
    # perform web scrapping - takes about an hour
    print("Downloading data ...")

    weekday_count = 0
    dfs = []
    for date in tqdm(daterange(start_date, end_date), total=(end_date-start_date).days):
        url = URL.format(month=date.month, day=date.day, year=date.year)
        #print(date.strftime("%Y-%m-%d"), "WEEKEND" if date.weekday()>4 else date.strftime("%A"))

        if date.weekday()>4: continue
        weekday_count += 1
```

```
        r = requests.get(url)

        soup = bs.BeautifulSoup(r.content, 'html5lib')
        table = soup.find("table",{"id":"table1"})
        if table is None: continue

        headers = [tr.text.split('\n') for tr in table.find_all('thead')][0]
        columns = [h.strip() for h in headers if h.strip()]

        table = soup.find("table",{"id":"table1"})

        data =  []
        for tr in table.find_all('tr'):
            td = tr.find_all('td')
            row = [tr.text for tr in td]
            data.append(row)

        df = pd.DataFrame(data, columns=columns)

        dfs.append(df)

    print("Number of days in interval:", (end_date-start_date).days)
    print("Number of weekdays:", weekday_count)
    print("Number of success data points:", len(dfs))

    with open('dfs.bin','wb') as f:
        dfs = pickle.dump(dfs, f)
```

```
Number of days in interval of interest: 10 days, 0:00:00
Using pre-downloaded data ...
```

In [11]:

```
# check - all df have same headers
for df in dfs:
    if (df.columns.values!=dfs[0].columns.values).any():
        print(df.head)
        break
```

In [12]:

```
df = pd.concat(dfs)
```

In [13]:

```
# Check for missing values
df.isna().sum(axis=1).value_counts()
```

Out[13]:

```
0    52018
6     3226
dtype: int64
```

In [14]:

```
# Missing values are in blank rows (top and bottom of tables) - so just drop them
df.dropna(inplace=True)
```

In [15]:

```
print(df.info())
df.head()
```

```
<class 'pandas.core.frame.DataFrame'>
Int64Index: 52018 entries, 1 to 34
Data columns (total 6 columns):
 #   Column            Non-Null Count  Dtype 
---  ------            --------------  ----- 
 0   S/NO              52018 non-null  object
 1   Currency          52018 non-null  object
 2   Buying            52018 non-null  object
 3   Selling           52018 non-null  object
 4   Mean              52018 non-null  object
 5   Transaction Date  52018 non-null  object
dtypes: object(6)
memory usage: 2.8+ MB
None
```

Out[15]:

	S/NO	Currency	Buying	Selling	Mean	Transaction Date
1	1	FRF	170.6216	173.8696	172.2456	05-Jan-09
2	2	JPY	13.7771	14.0494	13.9133	05-Jan-09
3	3	DKK	236.3939	240.8821	238.638	05-Jan-09
4	4	KES	18.368	18.703	18.5355	05-Jan-09
5	5	ZAR	135.8518	137.6374	136.7446	05-Jan-09

In [16]:

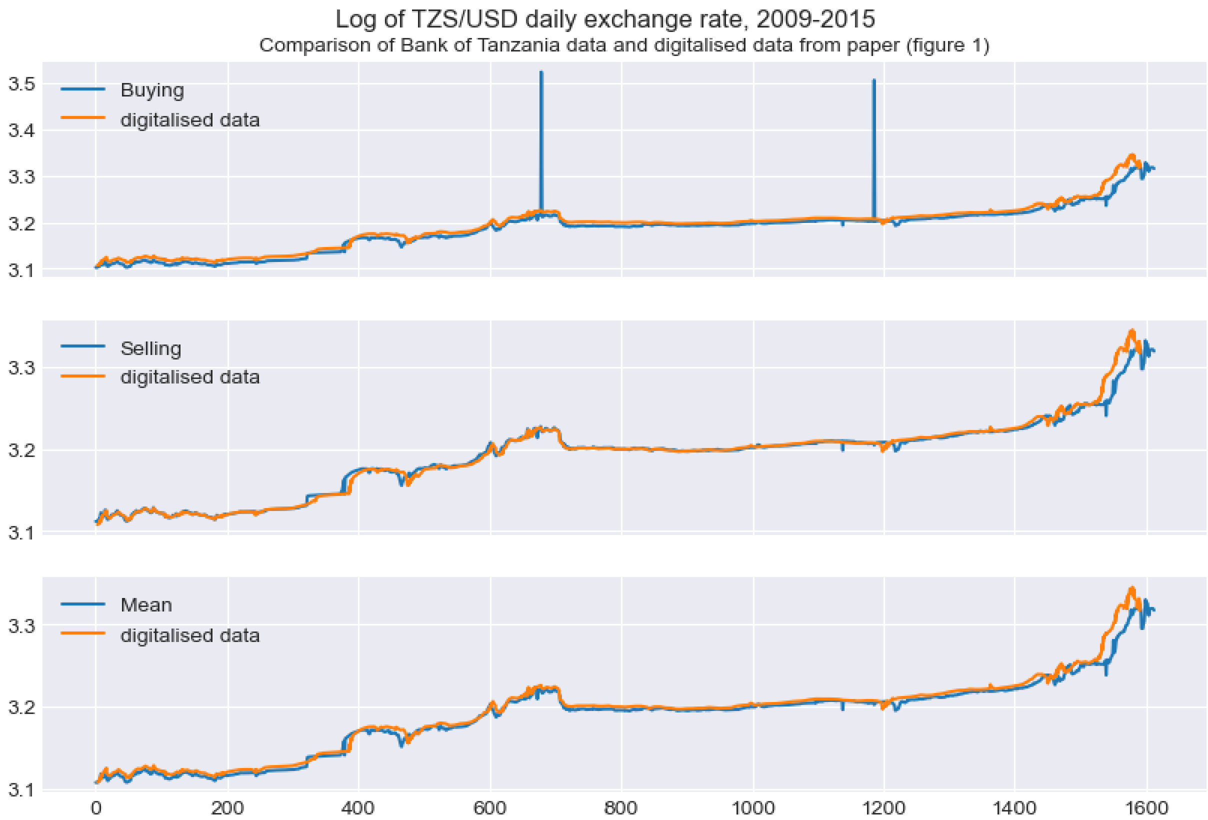
```
# Convert numerical columns from object to float

for c in ['Buying', 'Selling', 'Mean']: df[c] = df[c].astype(float)
df['Date'] = pd.to_datetime(df['Transaction Date'])
df = df[['Date', 'Currency', 'Buying', 'Selling', 'Mean']]
df.head()
```

Out[16]:

	Date	Currency	Buying	Selling	Mean
1	2009-01-05	FRF	170.6216	173.8696	172.2456
2	2009-01-05	JPY	13.7771	14.0494	13.9133
3	2009-01-05	DKK	236.3939	240.8821	238.6380
4	2009-01-05	KES	18.3680	18.7030	18.5355
5	2009-01-05	ZAR	135.8518	137.6374	136.7446

In [17]:

```
# Comparing BoT data against digitalised data from figure 1 in report

fig,axs = plt.subplots(3,1, sharex=True)
for k, c in enumerate(['Buying', 'Selling', 'Mean']):
    axs[k].plot(np.log10(df.query("Currency=='USD'")[c].values), label=c)
    axs[k].plot(dfp.x, dfp.y, label='digitalised data')
    axs[k].legend(loc ="upper left")
plt.suptitle('Log of TZS/USD daily exchange rate, 2009-2015')
axs[0].set_title('Comparison of Bank of Tanzania data and digitalised data from paper (figure 1)', fontsize=10)
plt.subplots_adjust(top=0.92) #Adjusting title position
plt.show()
display(Markdown("Paper appears to be using Selling as the USD/TZS Exchange Rate"))
```



Paper appears to be using Selling as the USD/TZS Exchange Rate

In [18]:

```
# Isolate USD Currency and Selling column

df.to_csv("bank_of_tanzania_all.csv", index=False)
df.query("Currency=='USD'").to_csv("bank_of_tanzania_USD.csv", index=False)

filename = 'bank_of_tanzania_USD.csv'
df = pd.read_csv(f'{base_url}/{filename}')

# df = pd.read_csv("bank_of_tanzania_USD.csv")
df = df[['Date', 'Selling']]
df.columns.values[1] = 'TZS_USD'
df.Date = pd.to_datetime(df.Date, format='%Y-%m-%d')
df.set_index('Date', inplace=True)
df.head()
```

Out[18]:

	TZS_USD
Date	
2009-01-05	1293.103
2009-01-06	1293.103
2009-01-07	1293.103
2009-01-08	1293.103
2009-01-09	1297.850

In [19]:

```
# Check number of missing values
df['TZS_USD'].isna().sum()
```

Out[19]:

```
0
```

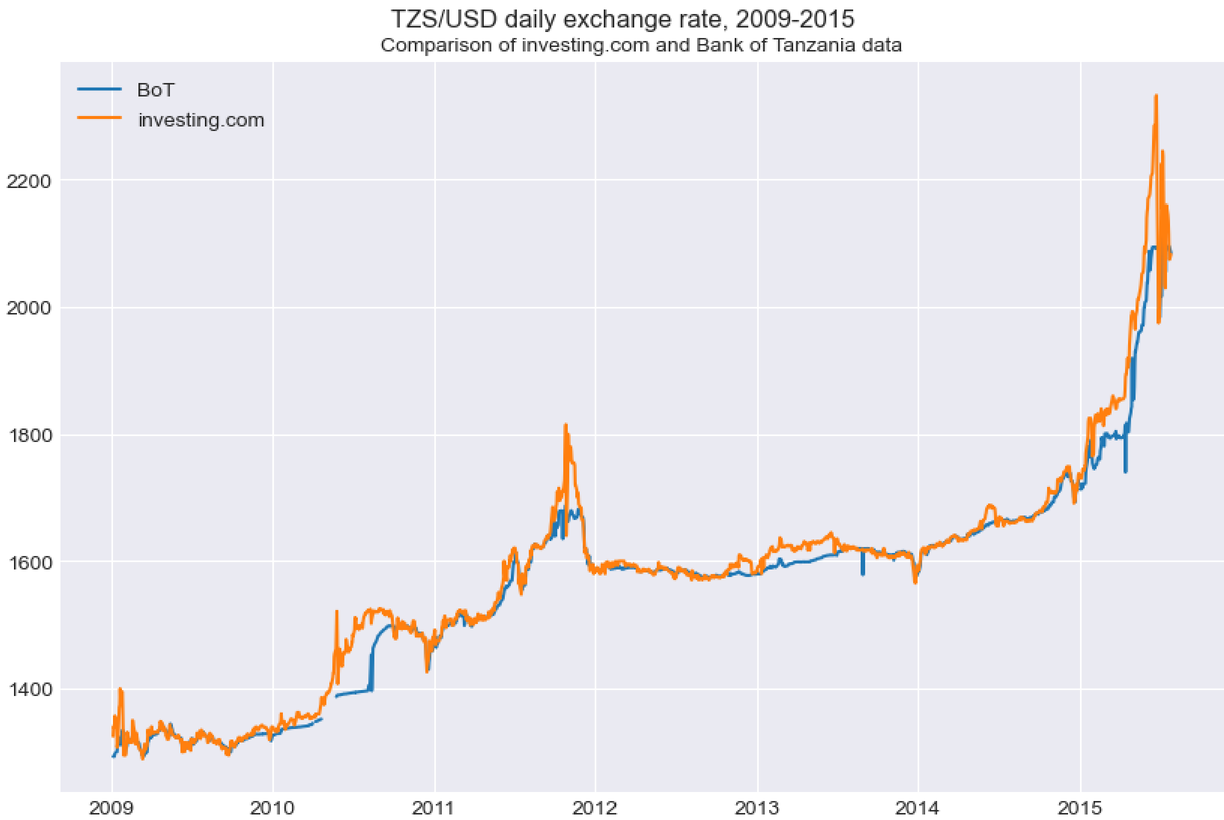
Comparison between datasets

In [20]:

```
df_comparison = pd.concat([df, df_1], axis=1)
# df_comparison = df_comparison[['TZS_USD', 'Price']]

df_comparison.columns.values[0] = 'TZS_USD_1' # Let TZS_USD_1 denote TZS/USD Exchange Rate from Bank of Tanzania
df_comparison.columns.values[1] = 'TZS_USD_2' # Let TZS_USD_2 denote TZS/USD Exchange Rate from investing.com
df_comparison['diff'] = df_comparison['TZS_USD_1'] - df_comparison['TZS_USD_2']
display(df_comparison)

# Plot
plt.plot(df_comparison['TZS_USD_1'], label="BoT")
plt.plot(df_comparison['TZS_USD_2'], label="investing.com")
plt.legend()
plt.title('Comparison of investing.com and Bank of Tanzania data', fontsize=10)
plt.suptitle('TZS/USD daily exchange rate, 2009-2015')
plt.subplots_adjust(top=0.92) #Adjusting title position
plt.show()

display(Markdown("As we can see, Bank of Tanzania data appears to be less volatile than investing.com data."))
```

	TZS_USD_1	TZS_USD_2	diff
Date			
2009-01-05	1293.103	1325.0	-31.897
2009-01-06	1293.103	1340.0	-46.897
2009-01-07	1293.103	1339.0	-45.897
2009-01-08	1293.103	1343.0	-49.897
2009-01-09	1297.850	1357.0	-59.150
...	...	...	...
2015-07-21	2095.470	2135.0	-39.530
2015-07-22	2095.470	2115.0	-19.530
2015-07-23	2095.680	2075.0	20.680
2015-07-24	2093.240	2075.0	18.240
2015-07-27	2085.210	2083.0	2.210

1711 rows × 3 columns



As we can see, Bank of Tanzania data appears to be less volatile than investing.com data.

Data Selection

- We are using the data downloaded from the Bank of Tanzania as it is the best fit to the data in the paper

Log TZS/USD exchange rate Transformation

$$P_t = \log_{10}(E_t)$$

$$r_t = \log_{10}(\frac{E_t}{E_{t-1}}) = \log_{10}(E_t) - \log_{10}(E_{t-1})$$

where

P_t is the log TZS/USD exchange rate at time t

r_t is the log TZS/USD exchange rate relative at time t

E_t is the TZS/USD exchange rate at time t

Daily TZS/USD exchange rate Percentage Change Transformation

$$r_t = 100 \times \left(\frac{E_t - E_{t-1}}{E_{t-1}}\right)$$

where

E_t is the TZS/USD exchange rate at time t

In [21]:

```
# Apply transformations
df['log_TZS_USD'] = np.log10(df['TZS_USD']) # Log Exchange Rate
df['TZS_USD_pct_change'] = 100*df['TZS_USD'].pct_change() # Percentage Change Exchnage Rate
df['log_TZS_USD_relative'] = df['log_TZS_USD'].diff() # Log Exchange Rate relative

# Drop the missing values created by functions
df.dropna(inplace=True)

display(df)
```

Date	TZS_USD	log_TZS_USD	TZS_USD_pct_change	log_TZS_USD_relative
2009-01-06	1293.103	3.111633	0.000000	0.000000
2009-01-07	1293.103	3.111633	0.000000	0.000000
2009-01-08	1293.103	3.111633	0.000000	0.000000
2009-01-09	1297.850	3.113225	0.367101	0.001591
2009-01-13	1300.375	3.114069	0.194553	0.000844
...	...	...	...	...
2015-07-21	2095.470	3.321281	-0.002863	-0.000012
2015-07-22	2095.470	3.321281	0.000000	0.000000
2015-07-23	2095.680	3.321325	0.010022	0.000044
2015-07-24	2093.240	3.320819	-0.116430	-0.000506
2015-07-27	2085.210	3.319150	-0.383616	-0.001669

1612 rows × 4 columns

Stylized Facts (Exploratory Data Analysis)

Clustering Volatility and Leverage Effects

In [22]:
```python
# Plot Log Exchange Rate (TZS/USD)
plot_series(df['log_TZS_USD'], 'Log of TZS/USD daily exchange rate, 2009-2015', 'Log Exchange Rate (TZS/USD)')
```

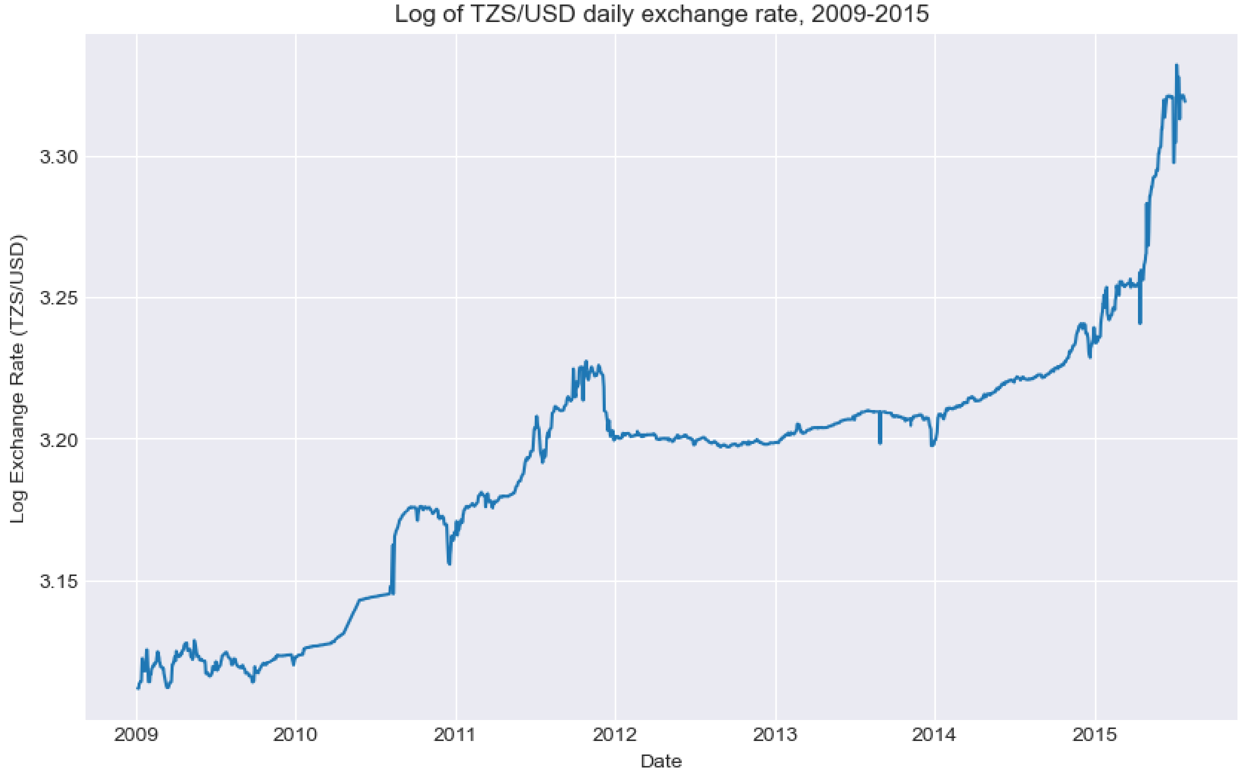
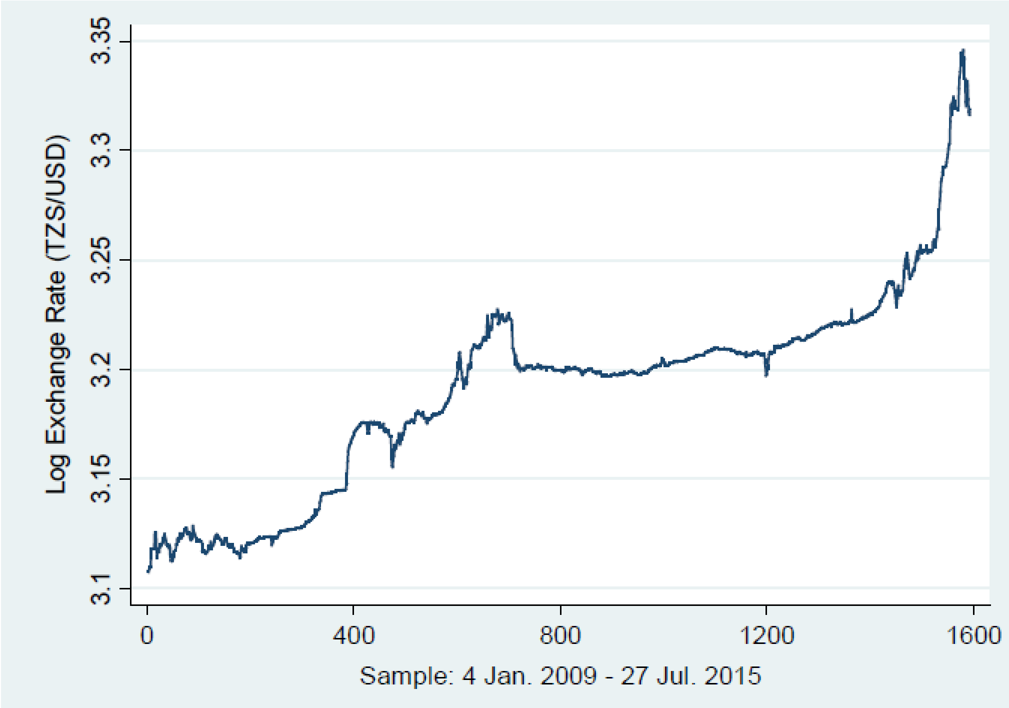


Figure 1 comparison



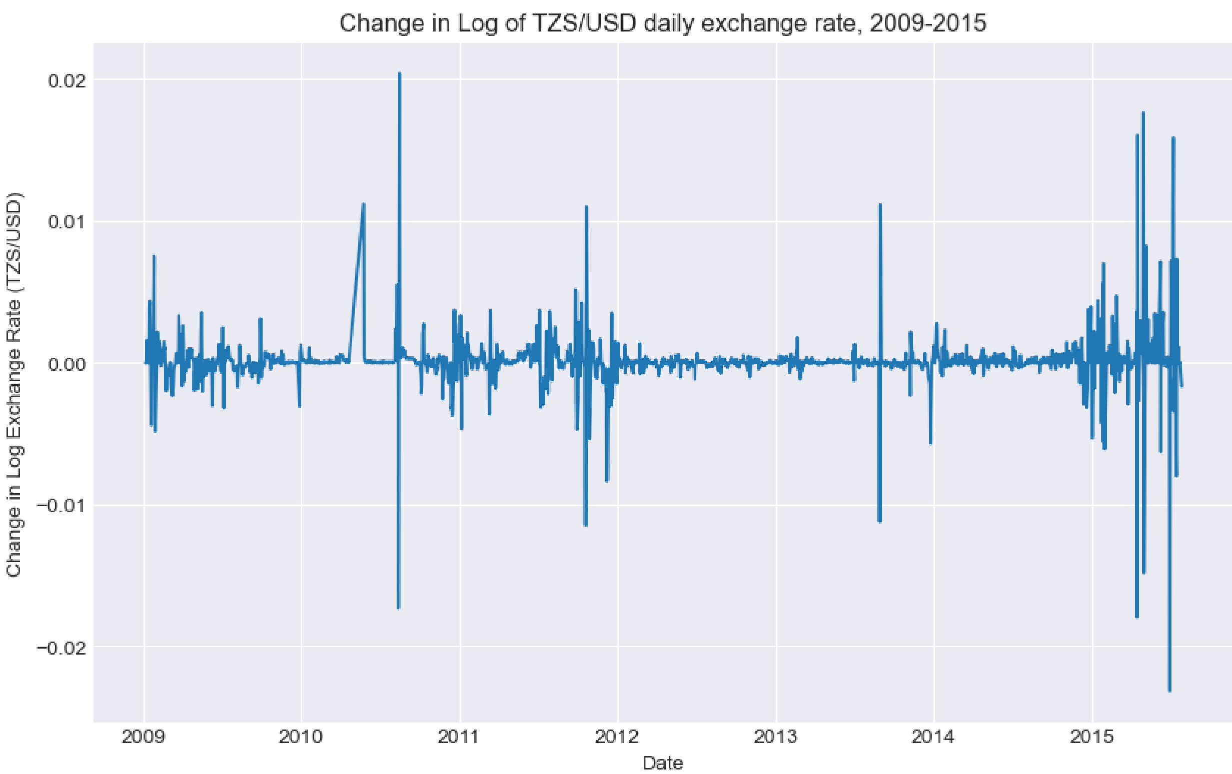
Source: Author's computations using data from Bank of Tanzania [24].

Figure 1. Log of TZS/USD daily exchange rate, 2009-2015.

The generated plot is similar but not identical to figure 1 in the paper. Leaving the issue of different number of observations (1612 vs 1593) aside the data used in figure 1 appears to:

- exclude a number of (outlier) observations, for example 2009-01-21 to 2009-01-25 (≈ day 10 in paper), 2010-05-25 to 2010-05-27 (≈ day 400 in paper), etc.
- be less noisy. Applying moving averages does not remove the inconsistencies.

In [23]:
```
# Plot Change in Log Exchange Rate (TZS/USD)
plot_series(df['log_TZS_USD_relative'], 'Change in Log of TZS/USD daily exchange rate, 2009-2015',
            'Change in Log Exchange Rate (TZS/USD)')
plot_series(df.log_TZS_USD_relative[df.log_TZS_USD_relative.abs()<0.011],
            'Constrained Change in Log of TZS/USD daily exchange rate, 2009-2015',
            'Change in Log Exchange Rate (TZS/USD)')
```



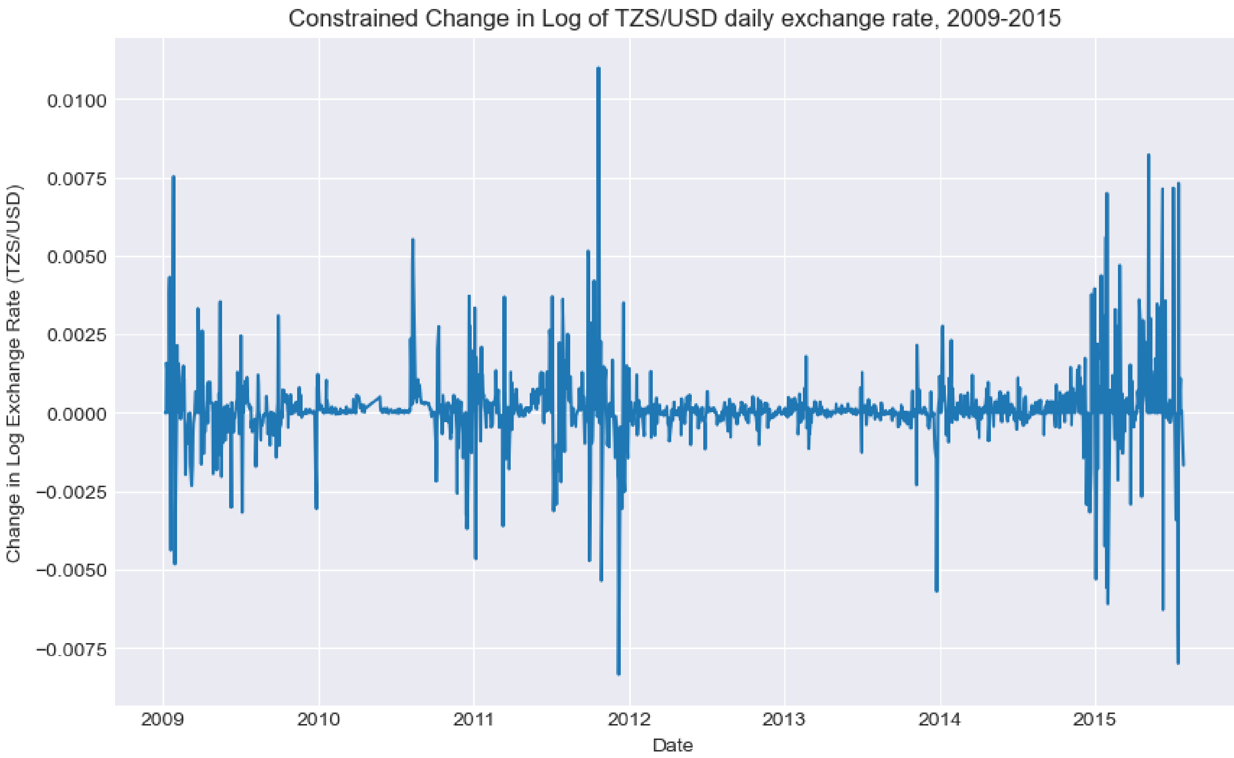
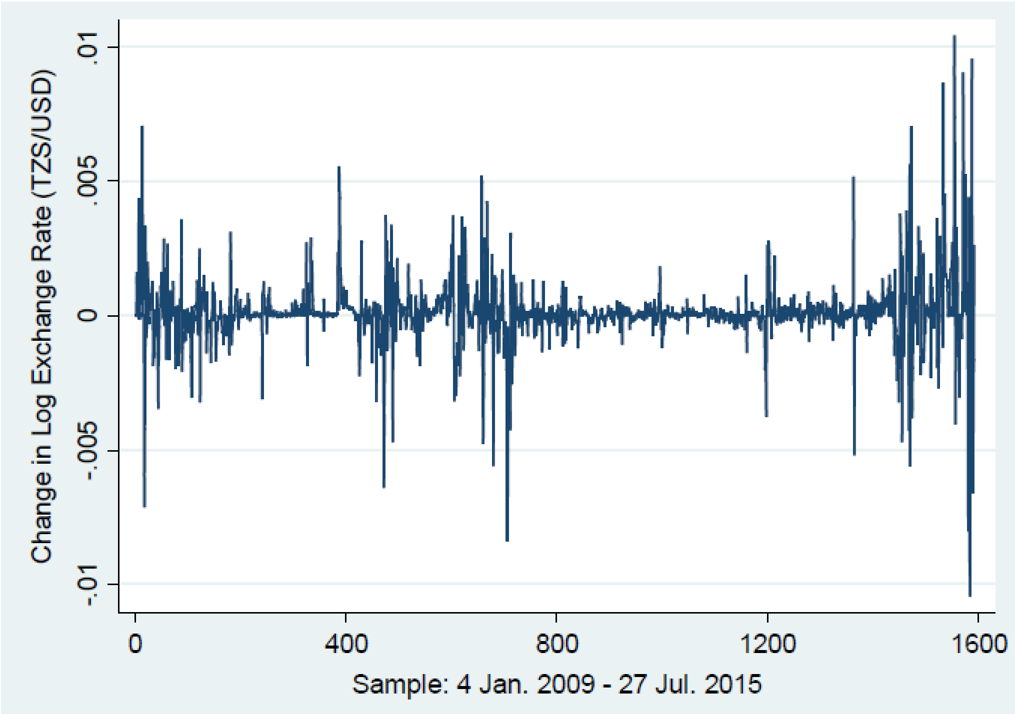


Figure 2 comparison



Source: Author's computations using data from Bank of Tanzania [24].

Figure 2. Change in log of TZS/USD daily exchange rate.

Scale in paper is smaller because that data appears to have excluded a number of observations containing spikes.

Rerunning plot with large changes removed, we get a plot which appears to have some of the structure seen in Figure 2 but is more noisy. (This is consistent with observed discrepancies seen when generating Figure 1)

Furthermore, it is also important to note the spike in mid 2010 in our plot. This may have been a consequence of the web scraping process.

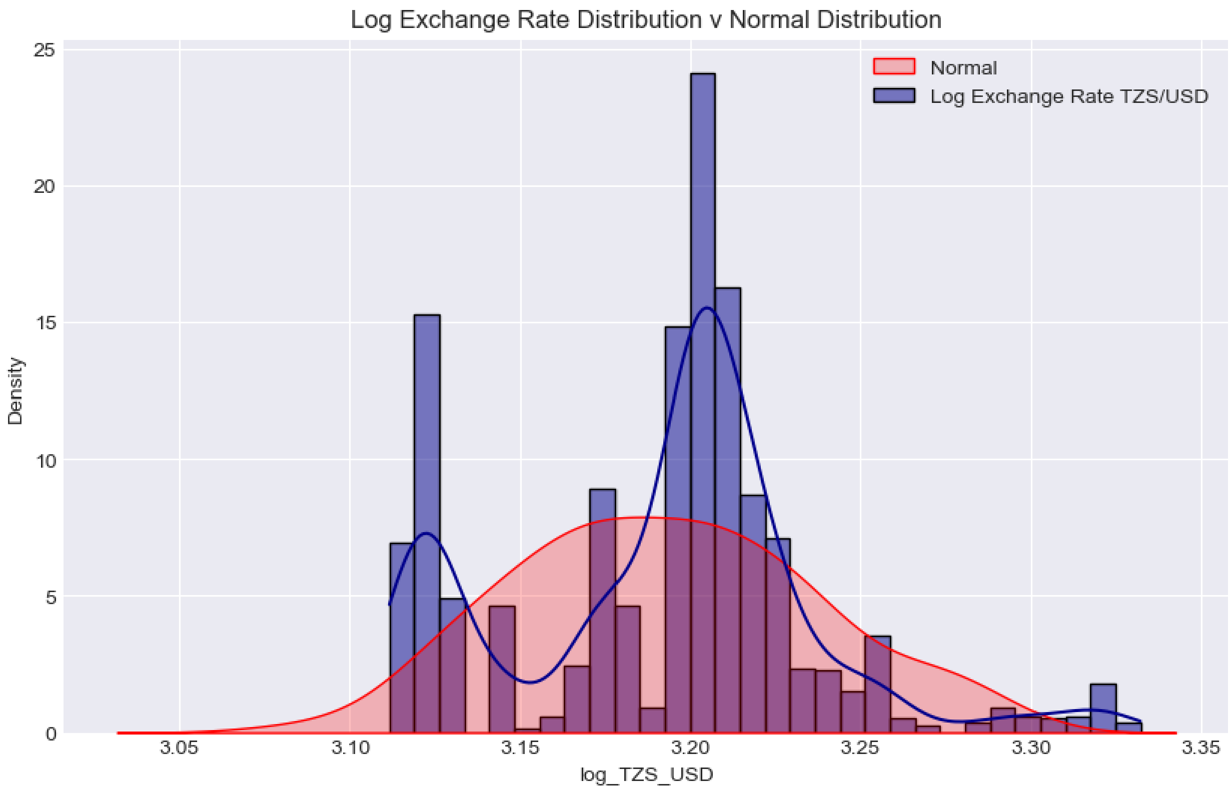
Non-Normality Distribution of Fat Tails

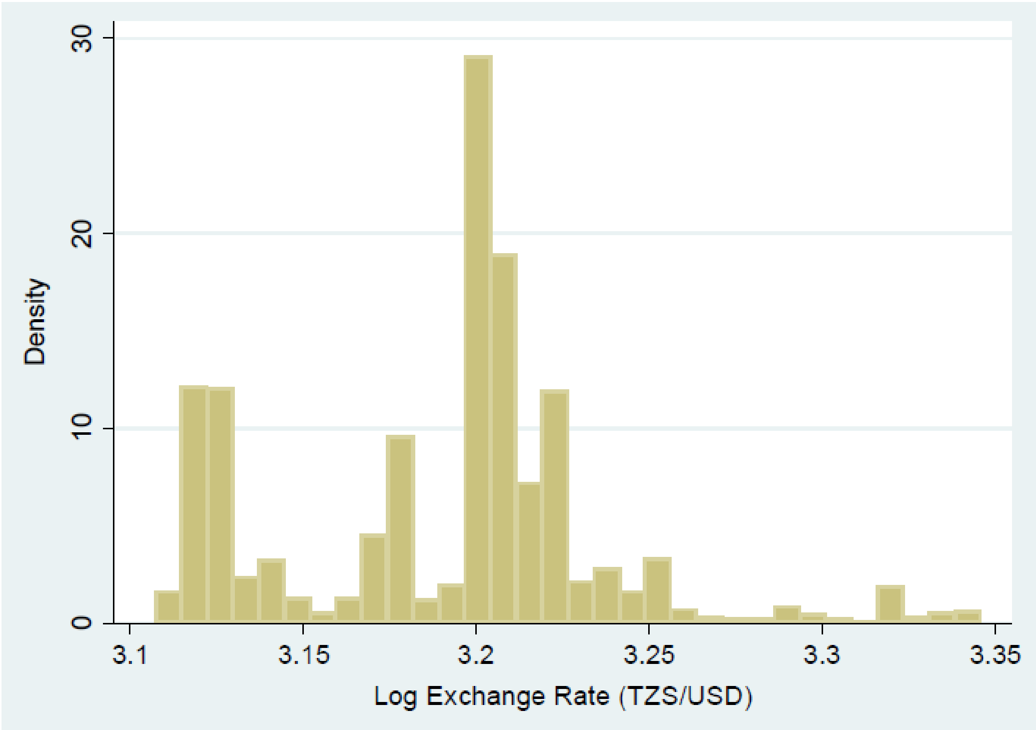
In [24]:
```
# Jarque-Bera (JB) test
```

```python
jb_test_data = sms.jarque_bera(df['log_TZS_USD'])
jb_test_df = pd.DataFrame(jb_test_data)
jb_test_df.columns = ['Jarque-Bera']
jb_test_df = jb_test_df.T
jb_test_df.columns = ['Test-Statistic', 'Chi^2 two-tail prob.', 'Skewness', 'Kurtosis']
display(jb_test_df)
```

	Test-Statistic	Chi^2 two-tail prob.	Skewness	Kurtosis
Jarque-Bera	12.147969	0.002302	0.133224	3.331465

In [25]:
```python
# Log Exchange Rate Distribution v Gaussian Distribution plot
plot_series_v_normal(df['log_TZS_USD'], 'Log Exchange Rate TZS/USD',
                     'Log Exchange Rate Distribution v Normal Distribution')
```

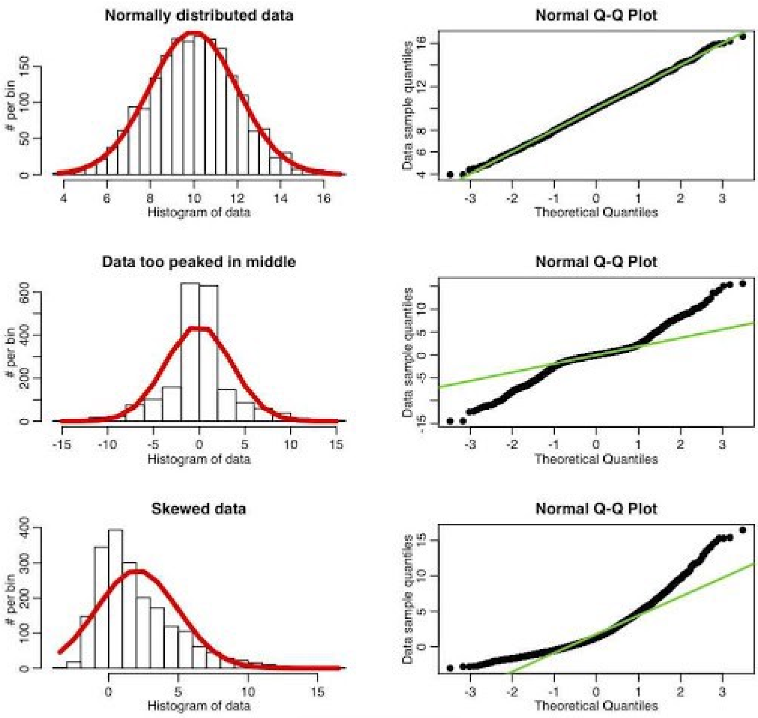




JB = 24.13; Prob =0.00; Skewness = 0.20; Kurtosis = 3.55. Source: Author's computations using data from Bank of Tanzania [24]

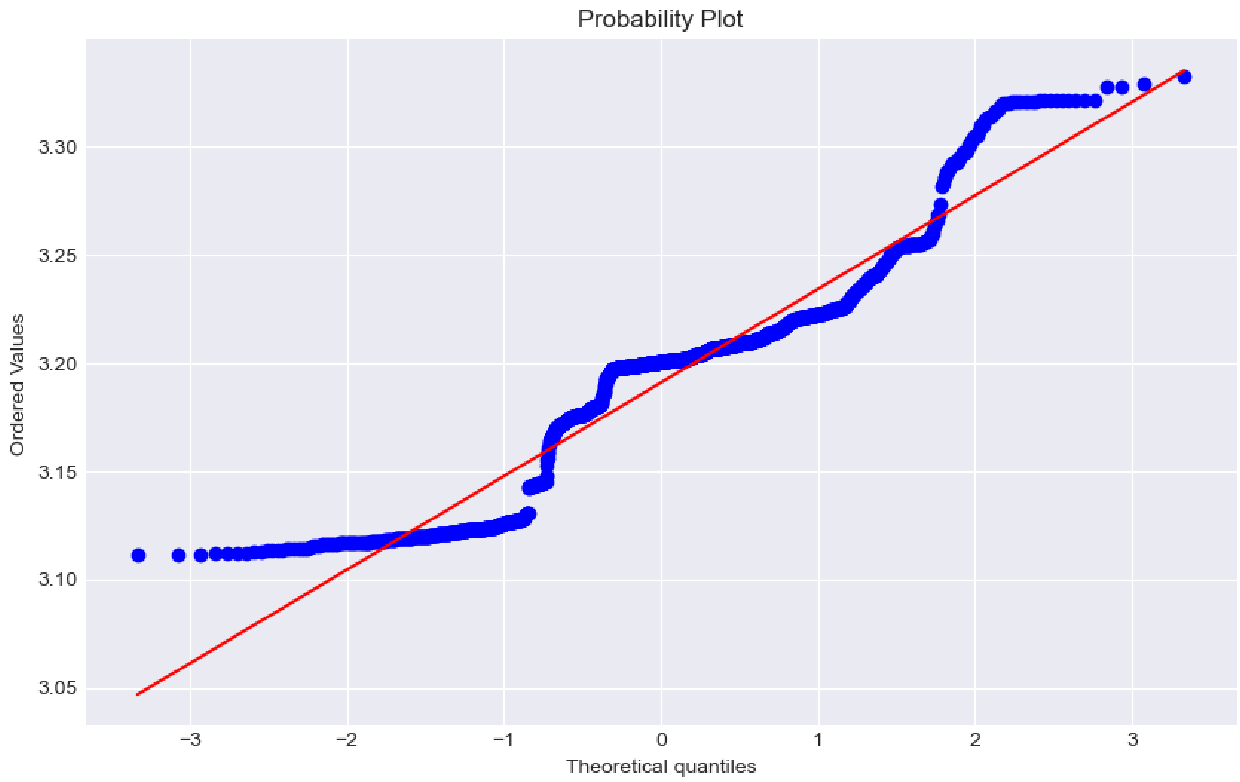
Figure 3. Normality test skewness and kurtosis of the daily TZS/USD.

QQ-Plot: An assessment for data normality A QQ-Plot is used to visually determine how close a sample is to a specified



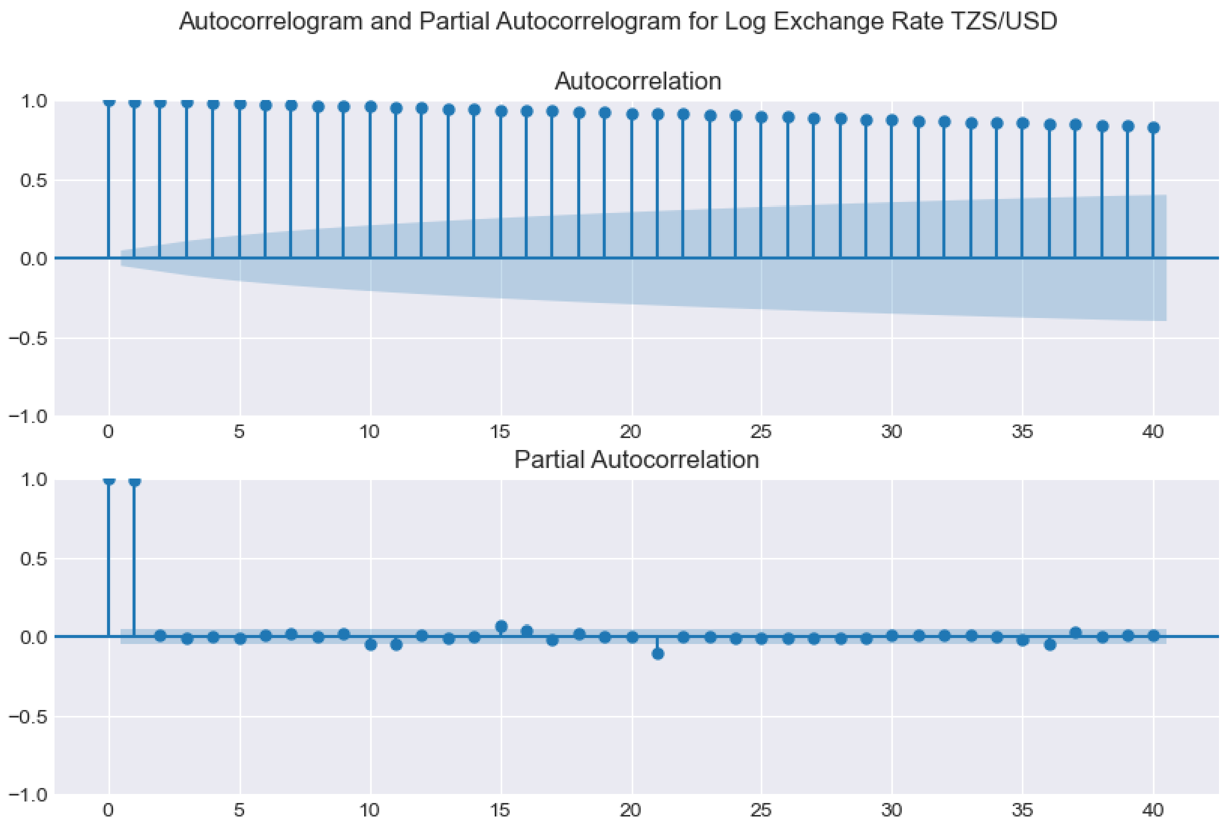
distribution

In [26]:
```
#QQ plot - addition -  nice to have

fig, (ax) = plt.subplots()
res = stats.probplot(df['log_TZS_USD'],dist='norm', plot=ax)
```

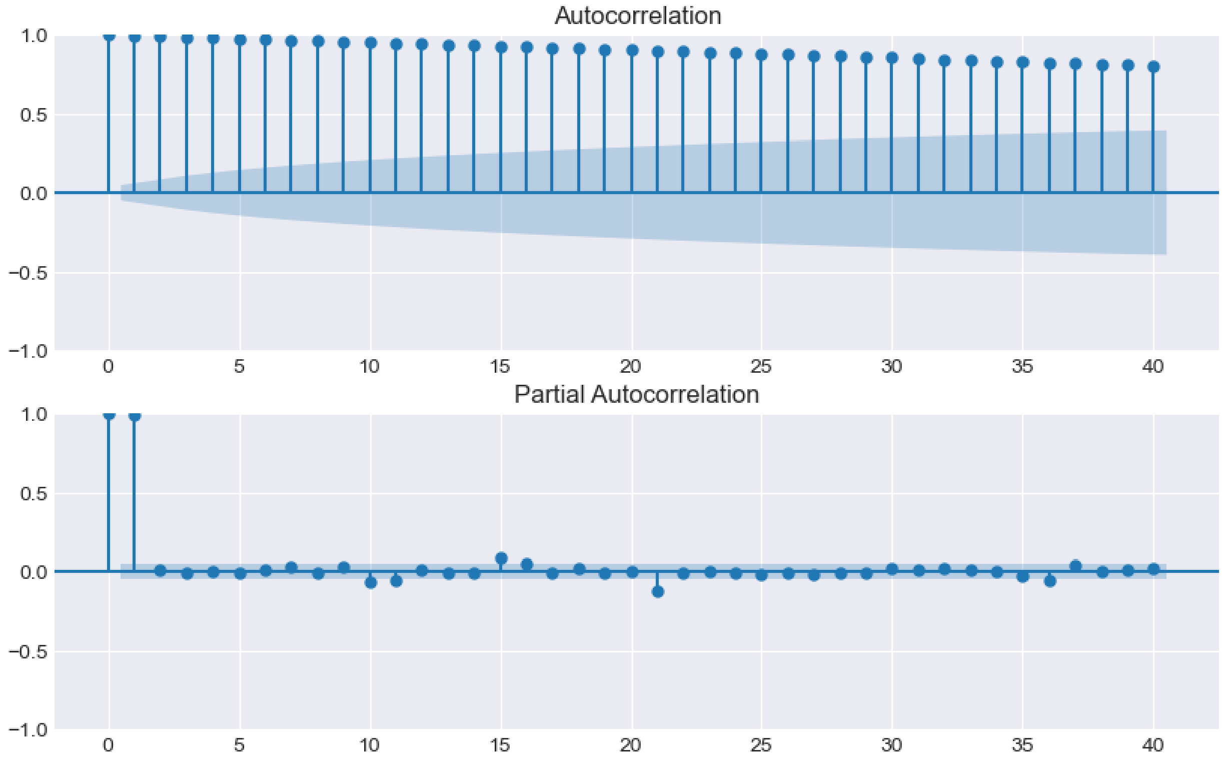


Serial Correlation and Unit Root

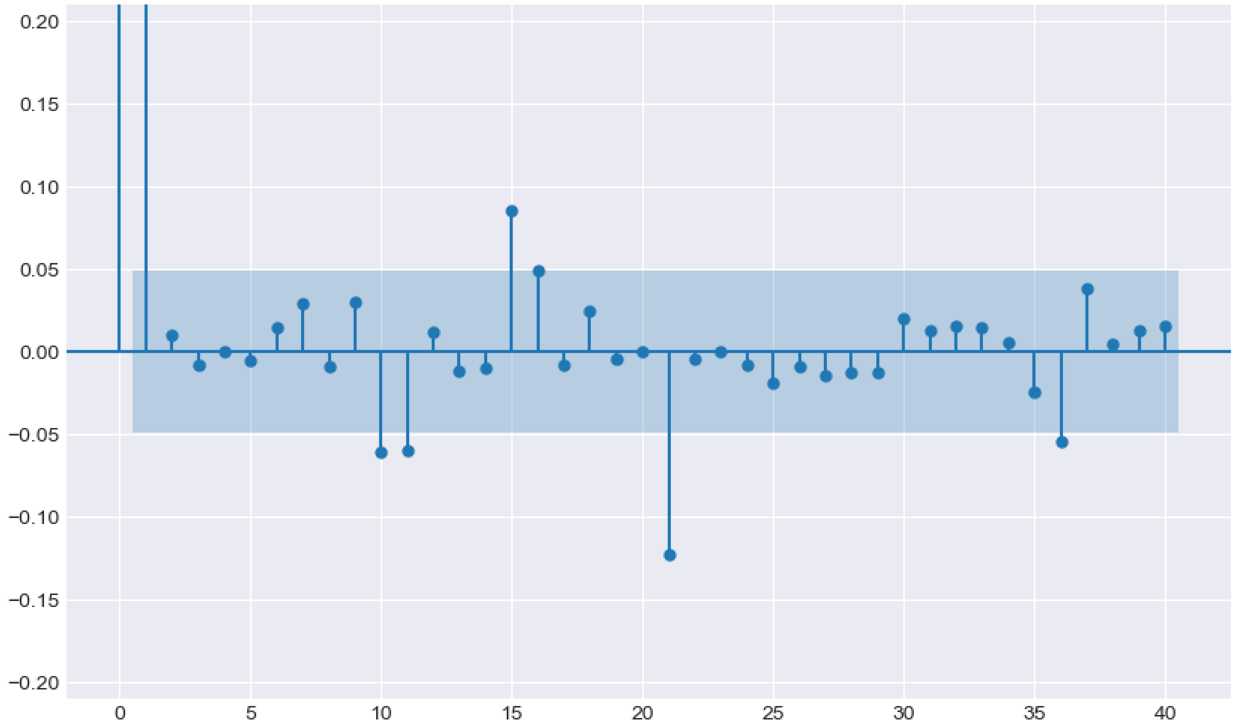
In [27]:
```
# Autocorrelogram and partial autocorrelogram (with 40 lags) plot
plot_acf_pacf(df['log_TZS_USD'], 'Autocorrelogram and Partial Autocorrelogram for Log Exchange Rate TZS/USD')
plot_acf_pacf(df['TZS_USD'], 'Autocorrelogram and Partial Autocorrelogram for Exchange Rate TZS/USD')

# Constraining PACF to match figure 5
plot_pacf(df['TZS_USD'], lags=40, method='ywm')
plt.ylim(-0.21, 0.21)
plt.title('Constrained Partial Autocorrelogram for Exchange Rate TZS/USD')
plt.show()
```

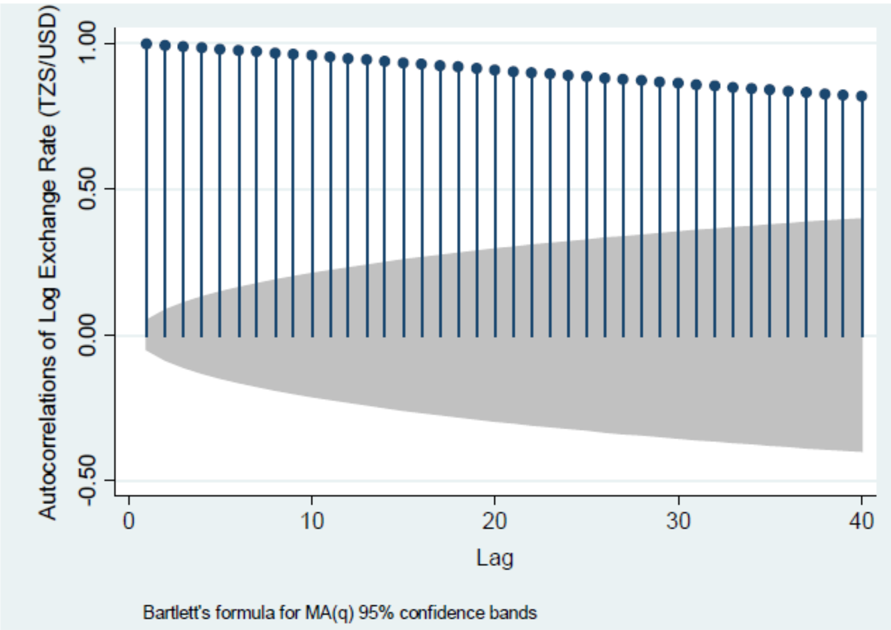


Autocorrelogram and Partial Autocorrelogram for Exchange Rate TZS/USD



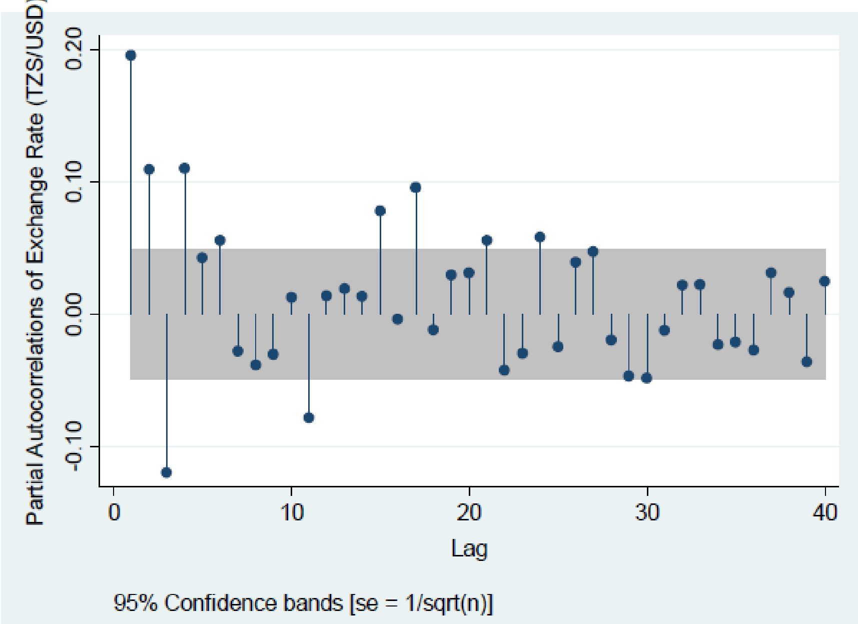
Constrained Partial Autocorrelogram for Exchange Rate TZS/USD





H0: There is no serial correlation in the series; H1: There is serial correlation in the series. Source: Author's computations using data from Bank of Tanzania [24]

Figure 5. Autocorrelation of exchange rate.



H0: There is no serial correlation in the series; H1: There is serial correlation in the series. Source: Author's computations using data from Bank of Tanzania [24]

Figure 4. Partial autocorrelation of exchange rate.

Figure 4 and 5 comparison

Author seems to be plotting Autocorrelogram and Partial Autocorrelogram for both Exchange Rate TZS/USD and Log Exchange Rate TZS/USD.

Our Autocorrelogram matches figure 5 as per the paper. The PACF does not appear to match figure 4, even after constraining it to the limits to match figure 4. This will need to be investigated.

Note: Author is using STATA which does include lag=0 in its plot

In [28]:
```
# ADF and PP unit root tests for stationarity code

adf_level = ADF(df['log_TZS_USD'], lags=40)
print(adf_level.summary())

pp_level = PP(df['log_TZS_USD'], lags=40)
print(pp_level.summary())
```

```
   Augmented Dickey-Fuller Results
=====================================
Test Statistic                  0.884
P-value                         0.993
Lags                               40
-------------------------------------

Trend: Constant
Critical Values: -3.43 (1%), -2.86 (5%), -2.57 (10%)
Null Hypothesis: The process contains a unit root.
Alternative Hypothesis: The process is weakly stationary.
     Phillips-Perron Test (Z-tau)
=====================================
Test Statistic                  0.870
P-value                         0.993
Lags                               40
-------------------------------------

Trend: Constant
Critical Values: -3.43 (1%), -2.86 (5%), -2.57 (10%)
Null Hypothesis: The process contains a unit root.
Alternative Hypothesis: The process is weakly stationary.
```

In [29]:

```
# ADF and PP unit root tests for stationarity in first difference code

adf_diff = ADF(df['log_TZS_USD_relative'], lags=40)
print(adf_diff.summary())

pp_diff = PP(df['log_TZS_USD_relative'], lags=40)
print(pp_diff.summary())
```

```
   Augmented Dickey-Fuller Results
=====================================
Test Statistic                 -6.124
P-value                         0.000
Lags                               40
-------------------------------------

Trend: Constant
Critical Values: -3.43 (1%), -2.86 (5%), -2.57 (10%)
Null Hypothesis: The process contains a unit root.
Alternative Hypothesis: The process is weakly stationary.
     Phillips-Perron Test (Z-tau)
=====================================
Test Statistic                -45.285
P-value                         0.000
Lags                               40
-------------------------------------

Trend: Constant
Critical Values: -3.43 (1%), -2.86 (5%), -2.57 (10%)
Null Hypothesis: The process contains a unit root.
Alternative Hypothesis: The process is weakly stationary.
```

Parametric Volatility Models & Empirical Results

In this paper both GARCH (1,1) and EGARCH (1,1) are used to model the volatility of TZS/USD exchange rate for the January 4, 2009 - July 27, 2015 period.

Before estimating the ARCH and GARCH models, the paper investigates the exchange rate series in order to identify its statistical properties and to see if it meets the pre-conditions for the ARCH and GARCH models, that is, clustering volatility and ARCH effect in the residuals.

The ARCH Effect

Test for the presence of ARCH effect is performed by first applying the least squares (LS) method in order to generate regression residuals. Then the ARCH heteroskedasticity test is applied to the residuals to ascertain whether time varying volatility clustering does exist.

In [30]:

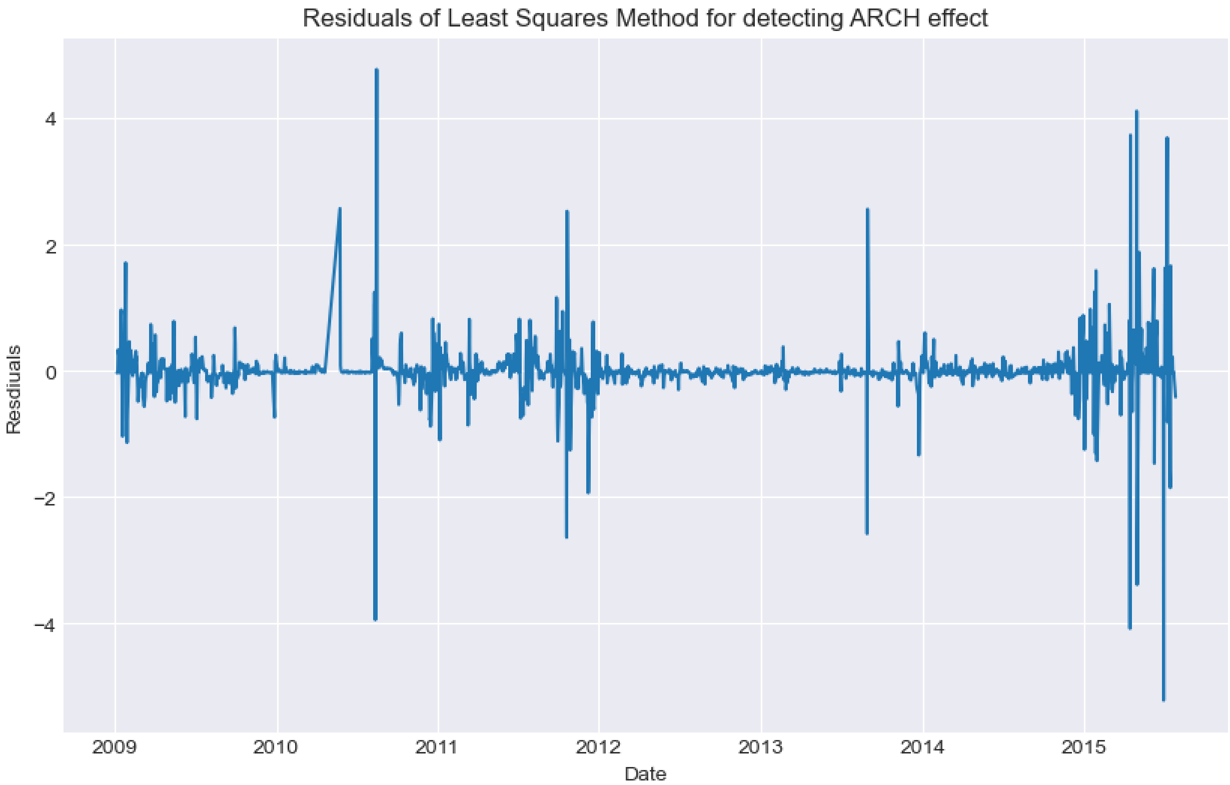
```
# ARCH effect test using Least Square method

ex_rate_pct_change = df['TZS_USD_pct_change']

# Fit a model (From Tutorial 9)
arch_effect_ols = smf.ols(formula='ex_rate_pct_change ~ 1',data=df).fit()
print(arch_effect_ols.summary())
dw_test = sms.durbin_watson(arch_effect_ols.resid)
print('\n','Durbin-Watson Statistic: ', dw_test)
```

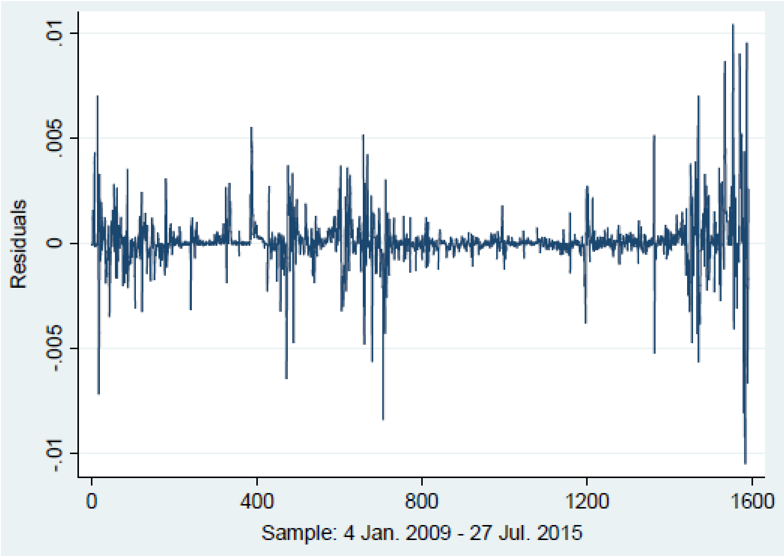
```
                            OLS Regression Results
==============================================================================
Dep. Variable:     ex_rate_pct_change   R-squared:                       0.000
Model:                            OLS   Adj. R-squared:                  0.000
Method:                 Least Squares   F-statistic:                       nan
Date:                Fri, 02 Dec 2022   Prob (F-statistic):                nan
Time:                        18:15:50   Log-Likelihood:                -917.77
No. Observations:                1612   AIC:                             1838.
Df Residuals:                    1611   BIC:                             1843.
Df Model:                           0
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
Intercept      0.0306      0.011      2.869      0.004       0.010       0.051
==============================================================================
Omnibus:                      568.682   Durbin-Watson:                   2.252
Prob(Omnibus):                  0.000   Jarque-Bera (JB):           185284.819
Skew:                           0.112   Prob(JB):                         0.00
Kurtosis:                      55.522   Cond. No.                         1.00
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.

 Durbin-Watson Statistic:  2.2519899851275613
```

Furthermore, to capture the volatility in the exchange rate illustrated in Figure 6, the paper considers a very simple model (Gujarati & Porter [1]), $r_t = \mu + u_t$ where $r_t =$ percentage change in the exchange rate and u_t = random error term. Using the daily TZS/USD exchange rate spanning from 2009 to 2015, the paper obtains the following OLS regression

$$\hat{r}_t = 0.00131$$
$$t = (4.23) \tag{20}$$
$$\text{Durbin} - \text{Watson stat}\ (DW) = 1.6182$$

In [31]:
```
plot_series(arch_effect_ols.resid, 'Residuals of Least Squares Method for detecting ARCH effect', ylabel='Resdiuals'
```

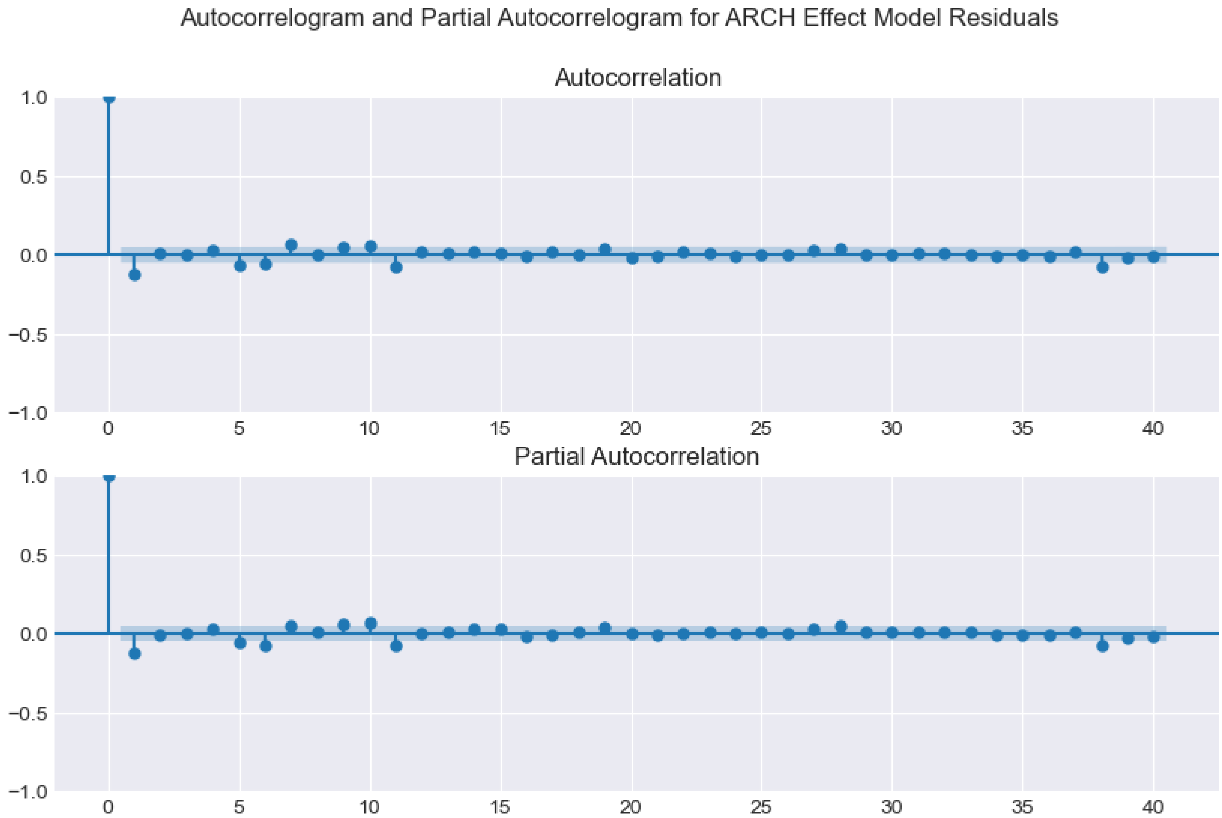


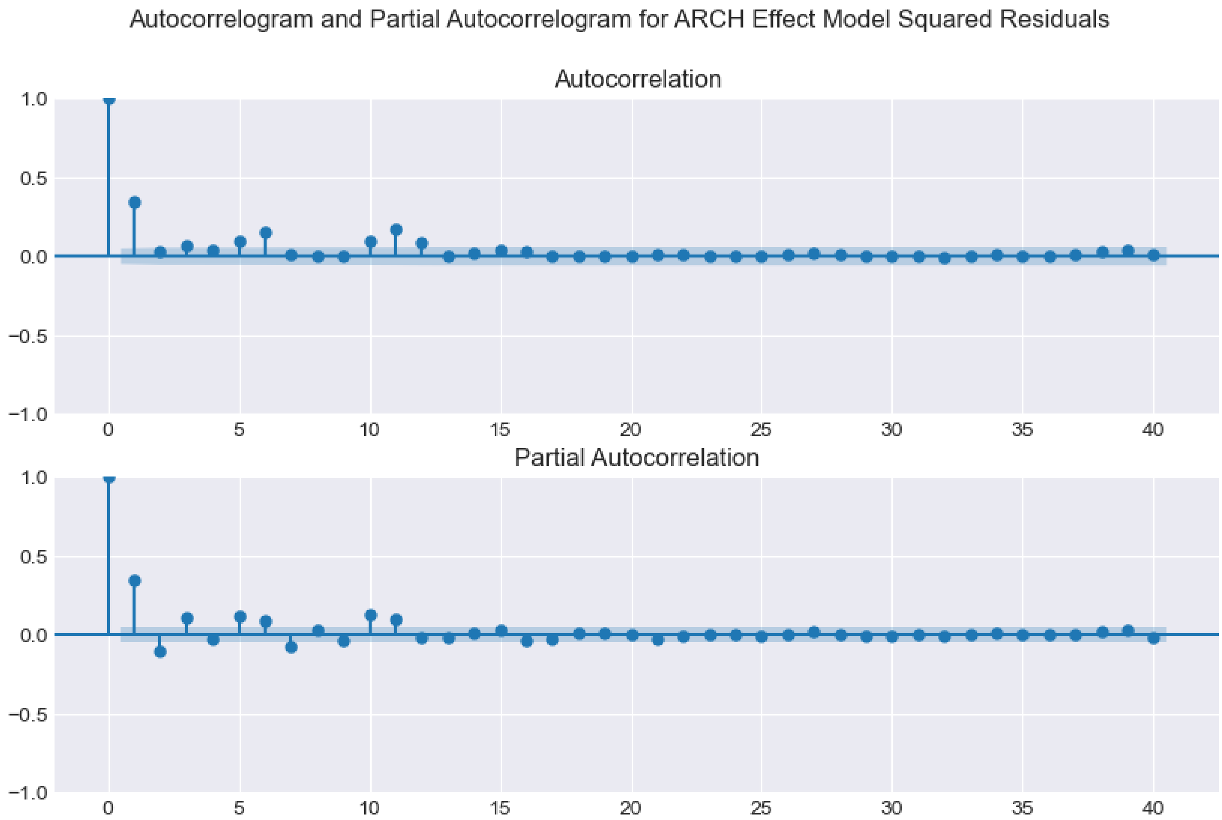


GARCH (1,1). Source: Author's computations using data from Bank of Tanzania [24].

Figure 6. Volatility clustering test: Residuals.

```
In [32]: # Optional (from tutorial)
# Residuals correlation: No serial correlation for residuals from the model (assumption)
# Positive correlations found for squared residuals: one way to detect ARCH effect

plot_acf_pacf(arch_effect_ols.resid, 'Autocorrelogram and Partial Autocorrelogram for ARCH Effect Model Residuals')
plot_acf_pacf(np.square(arch_effect_ols.resid), 'Autocorrelogram and Partial Autocorrelogram for ARCH Effect Model S
```





Testing for ARCH Effect: Engle's Test for Autoregressive Conditional Heteroscedasticity

H_0: There is no conditional heteroscedasticity/ No ARCH effect is present

H_1: There is conditional heteroscedasticity/ ARCH effect is present

In [33]:
```
# ARCH LM test (Engle's Test) for autoregressive conditional heteroskedasticity (ARCH)

engle_test_data = sm.stats.diagnostic.het_arch(arch_effect_ols.resid)
engle_test_df = pd.DataFrame(engle_test_data)
engle_test_df.columns = ['Engle Test']
engle_test_df = engle_test_df.T
engle_test_df.columns = ['Lagrange Multiplier Test Statistic', 'LM p-value', 'F-statistic', 'F p-value']
display(engle_test_df)
```

	Lagrange Multiplier Test Statistic	LM p-value	F-statistic	F p-value
Engle Test	284.122914	3.511567e-55	34.300585	6.092688e-61

Table 3. LM test for autoregressive conditional heteroskedasticity (ARCH).

Lags (P)	χ^2	Df	Prob > χ^2
1	175.353	1	0.000

H0: no ARCH effects vs. H1: ARCH (p) disturbance. Source: Authors computations using data from Bank of Tanzania [24].

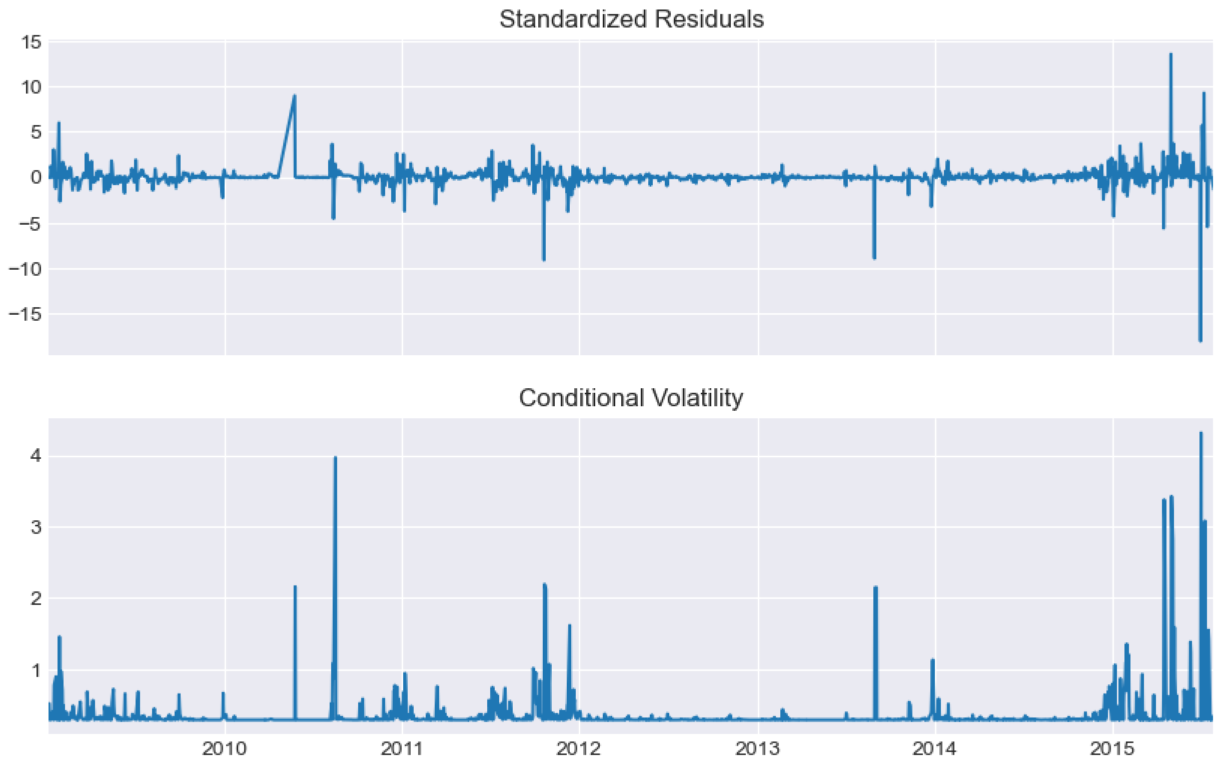
In [34]:
```
# From tutorial - ARCH(1)
arch_effect = arch_model(df['TZS_USD_pct_change'], vol='ARCH', p=1, q=0, rescale=False).fit()
print(arch_effect.summary())
dw_test = sms.durbin_watson(arch_effect.resid)
print('\n','Durbin-Watson Statistic: ', dw_test)
```

```
Iteration:      1,   Func. Count:      5,   Neg. LLF: 18634.843418051947
Iteration:      2,   Func. Count:     14,   Neg. LLF: 2073.7121098661282
Iteration:      3,   Func. Count:     21,   Neg. LLF: 495.93846597236677
Iteration:      4,   Func. Count:     26,   Neg. LLF: 490.7282989072106
Iteration:      5,   Func. Count:     31,   Neg. LLF: 490.5375165153817
Iteration:      6,   Func. Count:     35,   Neg. LLF: 490.53614887474953
Iteration:      7,   Func. Count:     39,   Neg. LLF: 490.53614574637714
Iteration:      8,   Func. Count:     42,   Neg. LLF: 490.53614574637095
Optimization terminated successfully    (Exit mode 0)
            Current function value: 490.53614574637714
            Iterations: 8
            Function evaluations: 42
            Gradient evaluations: 8
                      Constant Mean - ARCH Model Results
==============================================================================
Dep. Variable:     TZS_USD_pct_change   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                       ARCH   Log-Likelihood:               -490.536
Distribution:                  Normal   AIC:                           987.072
Method:            Maximum Likelihood   BIC:                           1003.23
                                        No. Observations:                 1612
Date:                Fri, Dec 02 2022   Df Residuals:                     1611
Time:                        18:15:51   Df Model:                            1
                                 Mean Model
============================================================================
                 coef    std err          t      P>|t|      95.0% Conf. Int.
----------------------------------------------------------------------------
mu             0.0203  1.021e-02      1.985  4.712e-02 [2.581e-04,4.028e-02]
                               Volatility Model
==========================================================================
                 coef    std err          t      P>|t|    95.0% Conf. Int.
--------------------------------------------------------------------------
omega          0.0834  2.573e-02      3.243  1.183e-03 [3.300e-02,  0.134]
alpha[1]       0.6841      0.189      3.619  2.962e-04   [  0.314,  1.055]
==========================================================================

Covariance estimator: robust

 Durbin-Watson Statistic:  2.2506862532653678
```

siduals are obtained from the preceding regression and estimate the ARCH (1) model that give the following results.

$$\begin{aligned} \hat{u}_t^2 &= 0.00051 + 0.17144\,\hat{u}_{t-1}^2 \\ t &= (0.000) \qquad (14.93) \\ R^2 &= 0.0024 \qquad DW = 1.621 \end{aligned} \tag{21}$$

In [35]:
```python
# From tutorial
std_resid_con_vol_plot = arch_effect.plot()
```



The GARCH Model

Results

In [36]:
```
# GARCH(1,1)
garch_model_11 = arch_model(df['TZS_USD_pct_change'], vol='GARCH', p=1, q=1, rescale=False).fit()
print(garch_model_11.summary())
dw_test = sms.durbin_watson(garch_model_11.resid)
print('\n','Durbin-Watson Statistic: ', dw_test)
```

```
Iteration:      1,   Func. Count:      6,   Neg. LLF: 1096588.622812618
Iteration:      2,   Func. Count:     18,   Neg. LLF: 33765.74062626453
Iteration:      3,   Func. Count:     25,   Neg. LLF: 676.1954087330912
Iteration:      4,   Func. Count:     33,   Neg. LLF: 2567.4046552259497
Iteration:      5,   Func. Count:     40,   Neg. LLF: 176.10488303449847
Iteration:      6,   Func. Count:     45,   Neg. LLF: 174.1489575719456
Iteration:      7,   Func. Count:     50,   Neg. LLF: 173.30466491282678
Iteration:      8,   Func. Count:     55,   Neg. LLF: 173.30453492401776
Iteration:      9,   Func. Count:     60,   Neg. LLF: 173.30453311231534
Iteration:     10,   Func. Count:     65,   Neg. LLF: 173.30453476219193
Optimization terminated successfully    (Exit mode 0)
            Current function value: 173.30453311466707
            Iterations: 10
            Function evaluations: 75
            Gradient evaluations: 10
                     Constant Mean - GARCH Model Results
==============================================================================
Dep. Variable:     TZS_USD_pct_change   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                      GARCH   Log-Likelihood:               -173.305
Distribution:                  Normal   AIC:                           354.609
Method:            Maximum Likelihood   BIC:                           376.150
                                        No. Observations:                 1612
Date:                Fri, Dec 02 2022   Df Residuals:                     1611
Time:                        18:15:51   Df Model:                            1
                                 Mean Model
=============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
-----------------------------------------------------------------------------
mu         7.2673e-03  6.767e-03      1.074      0.283 [-5.995e-03,2.053e-02]
                              Volatility Model
=============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
-----------------------------------------------------------------------------
omega      4.2198e-03  2.873e-03      1.469      0.142 [-1.411e-03,9.850e-03]
alpha[1]       0.1689  6.016e-02      2.807  4.996e-03   [5.097e-02,  0.287]
beta[1]        0.8311  6.711e-02     12.384  3.171e-35    [  0.700,  0.963]
=============================================================================

Covariance estimator: robust

 Durbin-Watson Statistic:  2.2453268024019994
```

In [37]:

```
# Checking sum of coefficients
alpha = garch_model_11.params[2] # coefficient of lagged squared residual
beta = garch_model_11.params[3] # coefficient of lagged conditional variance
sum_coefficients_garch_model_11 = alpha + beta
sum_coefficients_garch_model_11
```

Out[37]:

```
1.0000000097808792
```

GARCH (1,1) model of the daily percentage change in exchange rate, is estimated using data from January 4, 2009 through July 27, 2015, and the results as reported as follows

$$\begin{aligned} \hat{\sigma}_t^2 &= 0.0005 + 0.151\hat{u}_{t-1}^2 + 0.603\,\sigma_{t-1}^2 \\ t &= (0.059) \quad (5.055) \quad (11.491) \\ R^2 &= 0.001 \quad DW = 1.622 \end{aligned} \tag{22}$$

The coefficients on both the lagged squared residual $\left(u_{t-1}^2\right)$ and lagged conditional variance $\left(\sigma_{t-1}^2\right)$ terms in the conditional variance equation are individually statistically significant at the 1 percent significance level. This suggests that volatility from the previous periods has a power of explaining the current volatility condition. One measure of the persistence of movements in the variance is the sum of the coefficients on u_{t-1}^2 and σ_{t-1}^2 in the GARCH model (Stock & Watson [46]). The sum of 0.75 is large, indicating that changes in the conditional variance are persistence. In other words, a large sum of these coefficients will imply that a large positive or a large negative return will lead future forecasts of the variance to be high for a protracted period. This implication is consistent with the long periods of volatility clustering reported in Figure 6. Likewise, the GARCH (1,1) coefficients are positive confirming the non-negativity condition of the model.

Diagnostic Checking of the GARCH (1,1) Model

Goodness of fit of the ARCH-GARCH model is based on residuals. The residuals are assumed to be independently and identically distributed following a normal or standardized t-distribution. If the model fits the data well the histogram of the residuals should be approximately symmetric. The ACF and the PACF of the standardized residuals are used for checking the adequacy of the conditional variance model.

Normality Test of the Residuals

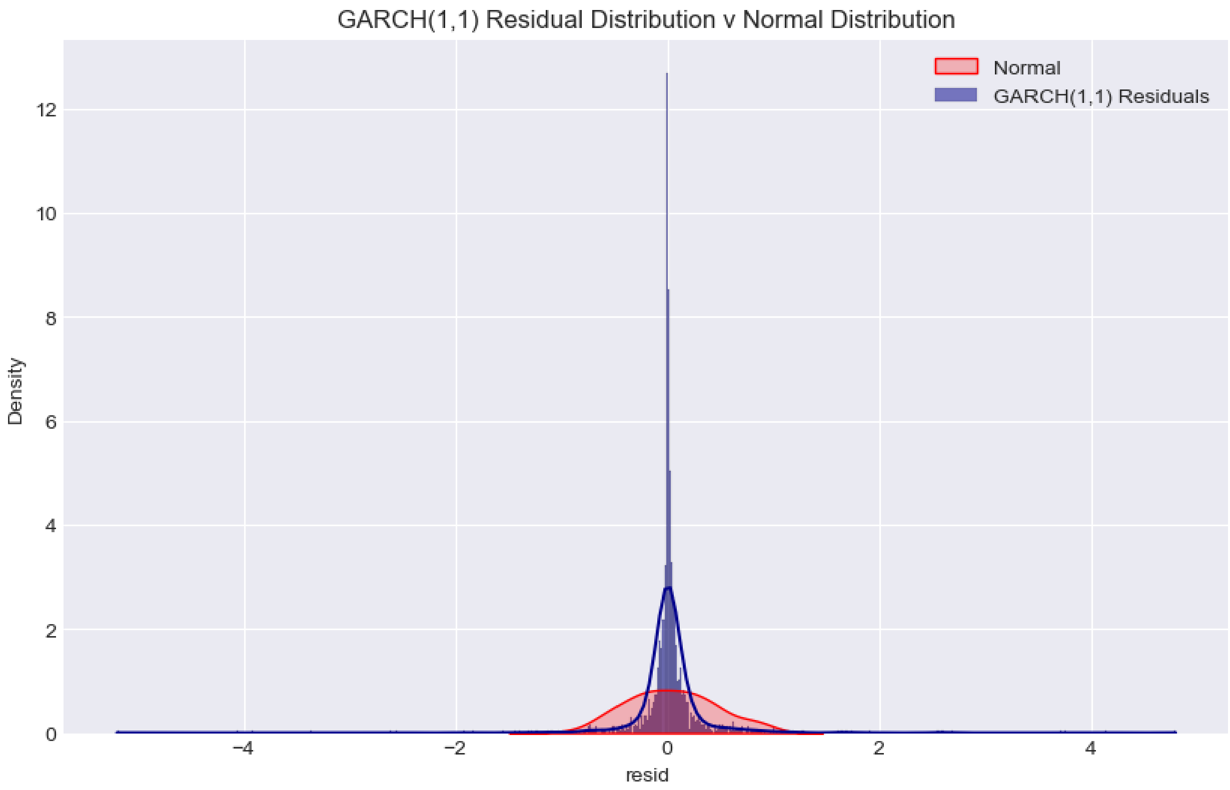
In [38]:
```
# Shapiro-Wilk test
sw_test = sw(garch_model_11.resid)
sw_test
```

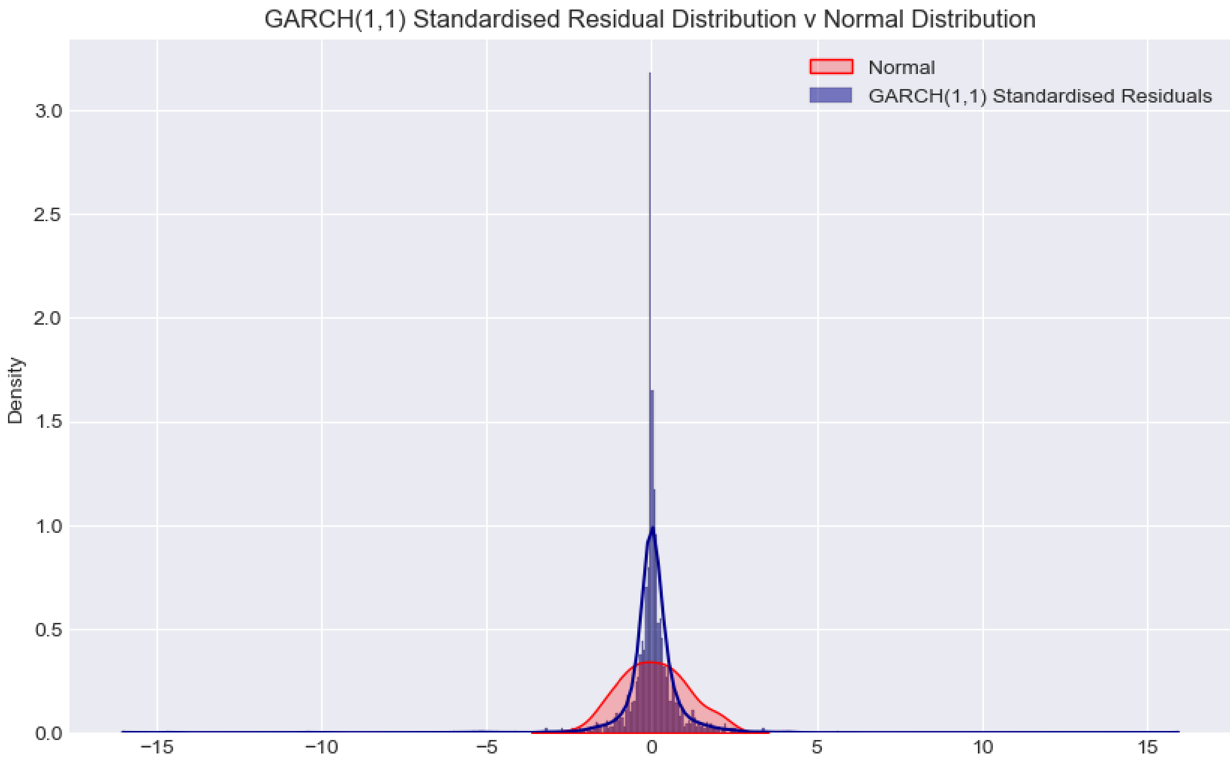
Out[38]:
```
ShapiroResult(statistic=0.497205913066864, pvalue=0.0)
```

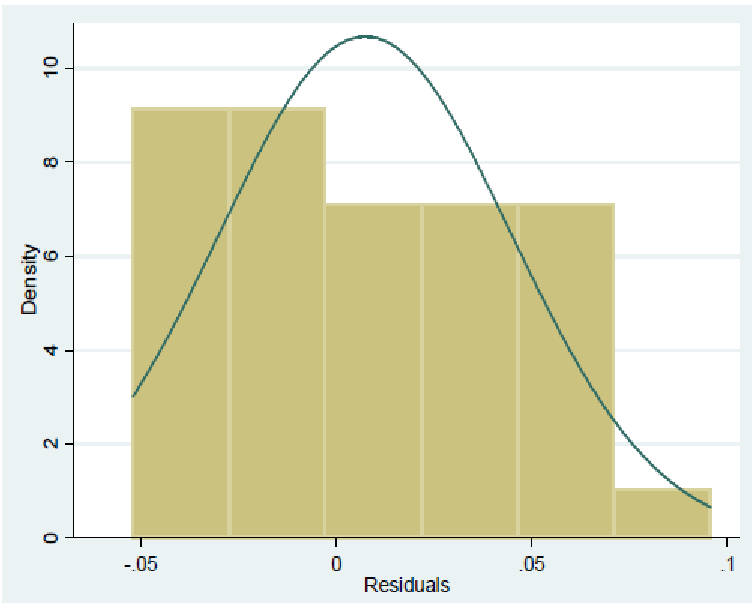
Table 4. Shapiro-wilk W test for normal data.

Variable	Obs	W	V	Z	Prob > z
Residuals	1592	0.95143	1.920	1.373	0.08495

In [39]:
```
# GARCH (1,1) Residual Distribution v Normal Distribution plot
plot_series_v_normal(garch_model_11.resid, 'GARCH(1,1) Residuals',
                     'GARCH(1,1) Residual Distribution v Normal Distribution')

# Tutorial does standardised residuals
garch_model_11_st_resid = np.divide(garch_model_11.resid, garch_model_11.conditional_volatility)

# GARCH (1,1) Standardised Residual Distribution v Normal Distribution plot
plot_series_v_normal(garch_model_11_st_resid, 'GARCH(1,1) Standardised Residuals',
                     'GARCH(1,1) Standardised Residual Distribution v Normal Distribution')
```







JB = 1.02; Prob = 0.60; Skewness = 0.20; Kurtosis = 2.42. Source: Author's computations (2016).

Figue 7. Normality test, skewness and kurtosis of the residuals.

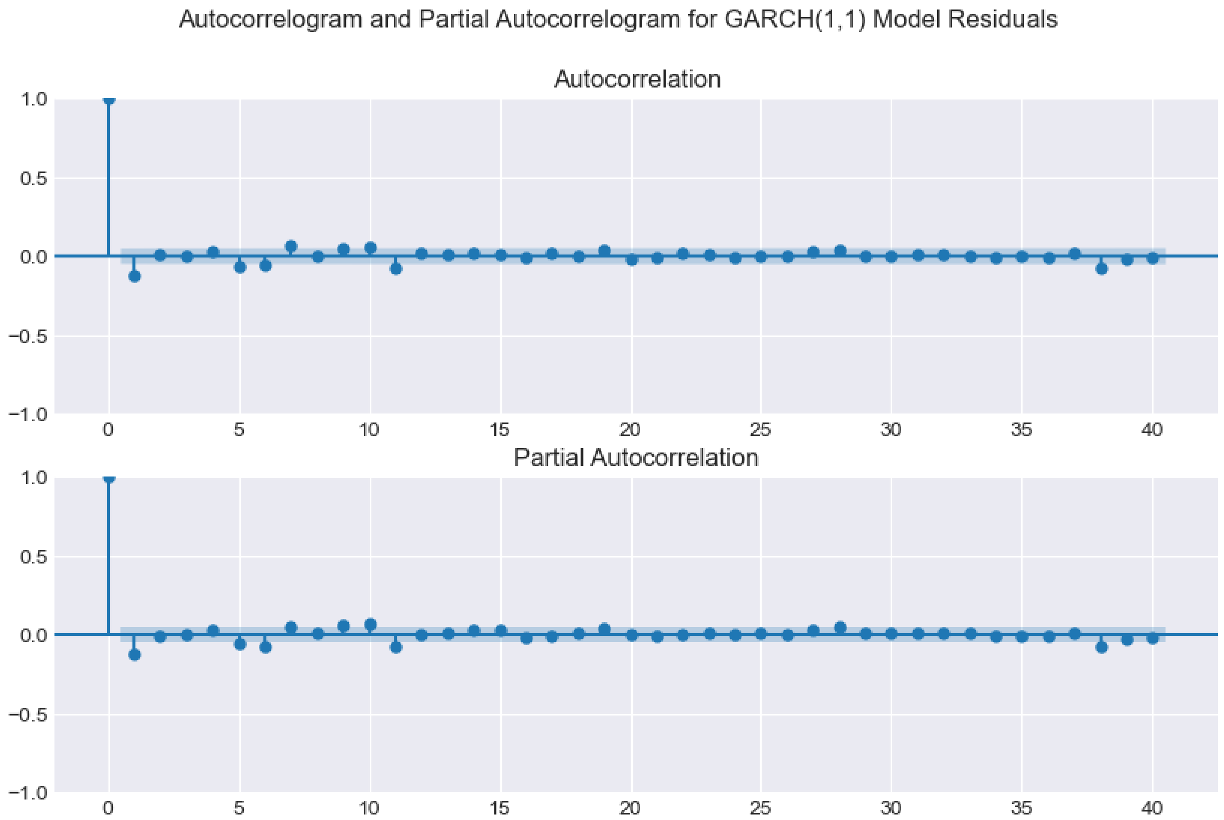
"If the model fits the data well the histogram of the residuals should be approximately symmetric."

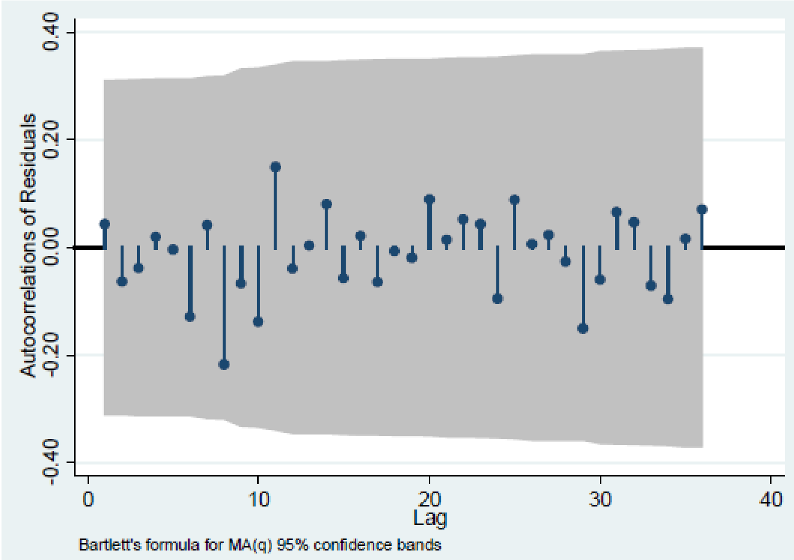
In [40]:
```
# Jarque-Bera (JB) test

jb_test_data = sms.jarque_bera(garch_model_11.resid)
jb_test_df = pd.DataFrame(jb_test_data)
jb_test_df.columns = ['Jarque-Bera']
jb_test_df = jb_test_df.T
jb_test_df.columns = ['Test-Statistic', 'Chi^2 two-tail prob.', 'Skewness', 'Kurtosis']
display(jb_test_df)
```

	Test-Statistic	Chi^2 two-tail prob.	Skewness	Kurtosis
Jarque-Bera	185284.81876	0.0	0.112329	55.521733

Serial Correlation

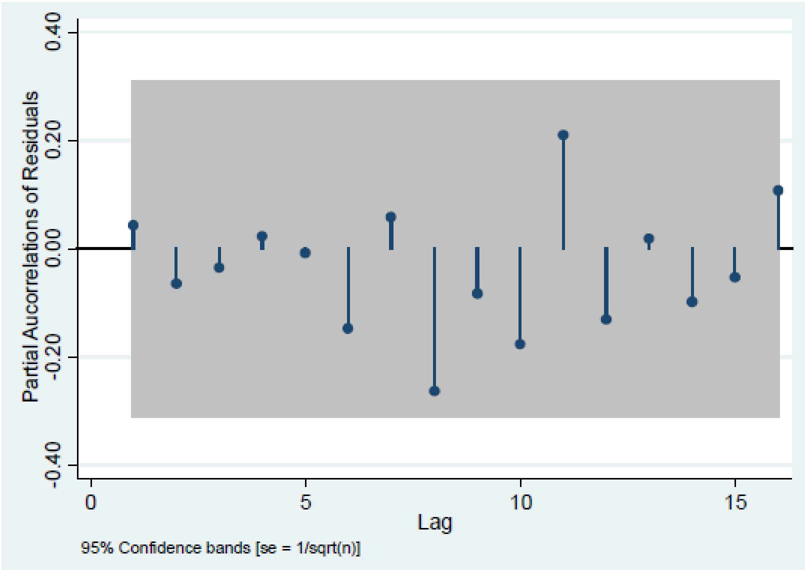
In [41]:
```
# Partial autocorrelogram and autocorrelogram (with 40 lags) of Residuals plot
plot_acf_pacf(garch_model_11.resid, 'Autocorrelogram and Partial Autocorrelogram for GARCH(1,1) Model Residuals')
```


Autocorrelogram and Partial Autocorrelogram for GARCH(1,1) Model Residuals
Autocorrelation
Partial Autocorrelation
1.0
0.5
0.0
−0.5
−1.0
0
5
10
15
20
25
30
35
40



H0: There is no serial correlation in the residuals; H1: There is serial correlation in the residuals. Source: Author's computations using data from Bank of Tanzania [24].

Figure 8. Autocorrelation of residuals.



H0: There is no serial correlation in the residuals; H1: There is serial correlation in the residuals. Source: Author's computations using data from Bank of Tanzania [24].

Figure 9. Partial autocorrelation of residuals.

In [42]:
```
# Serial correlation test - Ljung-Box test
LB_test_data = acorr_ljungbox(garch_model_11.resid, lags=40)
LB_test_df = pd.DataFrame.from_records(LB_test_data)
LB_test_df = LB_test_df.rename_axis('Lags') # Rename index
LB_test_df.index = LB_test_df.index + 1 # Set first row to 1 instead of 0
LB_test_df.rename(columns={'lb_stat': 'Q stat','lb_pvalue':'P-Value'},inplace=True) # Rename columns
LB_test_df
```

Out[42]:

Lags	Q stat	P-Value
1	25.756952	3.872296e-07
2	25.964326	2.301009e-06
3	25.967641	9.687357e-06
4	27.114858	1.884378e-05
5	33.882441	2.512813e-06
6	39.464251	5.803817e-07
7	46.309947	7.608082e-08
8	46.310302	2.074972e-07
9	50.534045	8.547062e-08
10	55.858234	2.182321e-08
11	65.990707	7.013453e-10
12	66.461984	1.461502e-09
13	66.523682	3.480925e-09
14	67.434328	5.605034e-09
15	67.537728	1.221689e-08
16	67.798102	2.417239e-08
17	68.295816	4.229650e-08
18	68.307384	8.719806e-08
19	70.306798	8.178404e-08
20	70.725827	1.385805e-07
21	70.908994	2.510539e-07
22	71.605626	3.688132e-07
23	71.701092	6.623876e-07
24	71.742168	1.187276e-06
25	71.742306	2.113844e-06
26	71.789037	3.629386e-06
27	73.327429	3.713551e-06
28	76.030928	2.564541e-06
29	76.031128	4.344378e-06
30	76.053206	7.180713e-06
31	76.356533	1.067764e-05
32	76.522114	1.636340e-05
33	76.522147	2.601612e-05
34	76.794075	3.755079e-05
35	76.804804	5.784978e-05
36	76.886571	8.608810e-05
37	77.654460	1.041530e-04
38	86.225460	1.308453e-05
39	86.585354	1.818373e-05
40	86.626502	2.745430e-05

Table 5. Serial correlation test.

Lag	AC	PAC	Q-Stat	Prob.
3	−0.065	−0.062	0.3376	0.953
6	−0.101	−0.108	0.8820	0.990
9	−0.043	−0.017	4.2496	0.894
12	−0.049	−0.144	6.4183	0.894
15	−0.111	−0.063	8.0495	0.922
18	−0.042	−0.194	9.0030	0.960

H0: There is no serial correlation in the residuals. Source: Author's computations using data from Bank of Tanzania [24].

The Leverage Effects and Asymmetric GARCH

In [43]:

```
# Exponential GARCH (EGARCH) EGARCH (1,1)
egarch_model_11 = arch_model(df['TZS_USD_pct_change'], vol='EGARCH', p=1, q=1, o=1, rescale=False).fit()
print(egarch_model_11.summary())
dw_test = sms.durbin_watson(egarch_model_11.resid)
print('\n','Durbin-Watson Statistic: ', dw_test)
```

```
Iteration:      1,   Func. Count:      7,   Neg. LLF: 77012796253595.12
Iteration:      2,   Func. Count:     19,   Neg. LLF: 4107987971543.1484
Iteration:      3,   Func. Count:     30,   Neg. LLF: 3094.8053695710128
Iteration:      4,   Func. Count:     40,   Neg. LLF: 446168286696.89905
Iteration:      5,   Func. Count:     49,   Neg. LLF: 1128.6070886910009
Iteration:      6,   Func. Count:     56,   Neg. LLF: 120239383296.81186
Iteration:      7,   Func. Count:     65,   Neg. LLF: 777.4225195334895
Iteration:      8,   Func. Count:     72,   Neg. LLF: 396.6452977622971
Iteration:      9,   Func. Count:     79,   Neg. LLF: 235.1725611672358
Iteration:     10,   Func. Count:     85,   Neg. LLF: 234.94179743224117
Iteration:     11,   Func. Count:     91,   Neg. LLF: 234.92058835721673
Iteration:     12,   Func. Count:     97,   Neg. LLF: 234.92003863943188
Iteration:     13,   Func. Count:    103,   Neg. LLF: 234.92001516038692
Iteration:     14,   Func. Count:    108,   Neg. LLF: 234.92001516079517
Optimization terminated successfully    (Exit mode 0)
            Current function value: 234.92001516038692
            Iterations: 14
            Function evaluations: 108
            Gradient evaluations: 14
                     Constant Mean - EGARCH Model Results
==============================================================================
Dep. Variable:     TZS_USD_pct_change   R-squared:                       0.000
Mean Model:             Constant Mean   Adj. R-squared:                  0.000
Vol Model:                     EGARCH   Log-Likelihood:                -234.920
Distribution:                  Normal   AIC:                             479.840
Method:            Maximum Likelihood   BIC:                             506.766
                                        No. Observations:                 1612
Date:                Fri, Dec 02 2022   Df Residuals:                     1611
Time:                        18:15:55   Df Model:                            1
                                  Mean Model
=============================================================================
                 coef    std err          t      P>|t|       95.0% Conf. Int.
-----------------------------------------------------------------------------
mu             0.0148  2.403e-02      0.617      0.537 [-3.228e-02,6.193e-02]
                              Volatility Model
===========================================================================
                 coef    std err          t      P>|t|     95.0% Conf. Int.
---------------------------------------------------------------------------
omega         -0.0591  9.693e-02     -0.609      0.542    [ -0.249,  0.131]
alpha[1]       0.3204      0.103      3.099  1.944e-03    [  0.118,  0.523]
gamma[1]       0.0821  5.707e-02      1.439      0.150 [-2.971e-02,  0.194]
beta[1]        0.9278  4.604e-02     20.153  2.546e-90    [  0.838,  1.018]
===========================================================================

Covariance estimator: robust

 Durbin-Watson Statistic:   2.2489433003979222
```

EGARCH Coefficents

AS PER PAPER

- Our Omega is constant
- Our Alpha is coefficent on first term
- Our Beta is coefficient on second term
- Our Gamma is coefficient on third term

ing the effect of volatility (Ali [40]). The EGARCH model is specified as follows (Nelson [19], Brooks [10])

$$\ln\left(\sigma_t^2\right) = \omega + \lambda \ln\left(\sigma_{t-1}^2\right) + \theta \frac{u_{t-1}}{\sqrt{\sigma_{t-1}^2}} + \gamma \left[\frac{|u_{t-1}|}{\sigma_{t-1}^2} - \sqrt{\frac{2}{\pi}} \right] \tag{16}$$

$$\begin{aligned} \hat{\sigma}_t^2 &= 2.02E-08 + 0.264\,\hat{u}_{t-1}^2 + 0.717\,\sigma_{t-1}^2 - 0.176\,u_{t-1}^2 I_{t-1} \\ t &= (30.57) \qquad (22.50) \qquad (137.50) \qquad (-6.86) \\ R^2 &= 0.002 \qquad DW = 1.717 \end{aligned} \tag{23}$$

Forecasting Evaluation and Accuracy

To measure the forecasting ability, the paper estimates within sample forecasts. The purpose of forecasting within the sample is to test for the predictability power of the model. If the magnitude of the difference between the actual and forecasted values is small then the model has good forecasting power.

(i.e. they do not split data into training and testing)

Realised Variance

From "Franses, P.H. and Dijk, D.V. (1996) Forecasting Stock Market Volatility Using (Non-Linear) GARCH Models. Journal of Forecasting , 15, 229-235."

To evaluate the ability of (non-linear variants of) GARCH models to adequately forecast the volatility in financial time series, we need a measure of the 'true volatility'. As is standard in related studies on forecasting volatility (see Day and Lewis, 1992; Pagan and Schwert, 1990), we use the measure w_t, defined by

$$w_t = (r_t - \bar{r})^2 \tag{4}$$

In [44]:
```
# Isolate actuals and predict using model

start = dt.datetime(2009,1,5)

predictions = garch_model_11.forecast(horizon=1, start=1, reindex=False)
var_predictions = predictions.variance
var_predictions.columns.values[0] = 'Forecasted Variance'

df_forecast_actuals = pd.concat([df, var_predictions], axis=1)
df_forecast_actuals['Realised Variance'] = np.square(df['TZS_USD_pct_change'] - df['TZS_USD_pct_change'].mean())
df_forecast_actuals.dropna(inplace=True)
df_forecast_actuals = df_forecast_actuals[['Realised Variance','Forecasted Variance']]
df_forecast_actuals
```

Out[44]:

Date	Realised Variance	Forecasted Variance
2009-01-07	0.000934	0.199321
2009-01-08	0.000934	0.169887
2009-01-09	0.113260	0.167283
2009-01-13	0.026893	0.149175
2009-01-14	0.000934	0.128210
...	...	...
2015-07-21	0.001117	1.741809
2015-07-22	0.000934	1.451873
2015-07-23	0.000422	1.210895
2015-07-24	0.021606	1.013197
2015-07-27	0.171542	0.872108

1611 rows × 2 columns

In [45]:
```
actuals = df_forecast_actuals['Realised Variance']
predictions = df_forecast_actuals['Forecasted Variance']
error = actuals - predictions
error = error.dropna()
log_error = np.log10(abs(error))
log_error
```

Out[45]:
```
Date
2009-01-07   -0.702487
2009-01-08   -0.772233
2009-01-09   -1.267421
2009-01-13   -0.912639
2009-01-14   -0.895252
                ...
2015-07-21    0.240722
2015-07-22    0.161649
2015-07-23    0.082955
2015-07-24   -0.003667
2015-07-27   -0.154551
Length: 1611, dtype: float64
```

Mean absolute error (MAE) and Root Mean Squared Error (RMSE)

$$\mathrm{MAE} = \frac{1}{T}\sum_{t=1}^{T}\left|r_t^2 - \hat{\sigma}_t^2\right| \qquad (24)$$

$$\mathrm{RMAE} = \sqrt{\frac{1}{T}\sum_{t=1}^{T}\left(r_t^2 - \hat{\sigma}_t^2\right)^2} \qquad (25)$$

$$U = \left(\frac{\sum_{t=1}^{T-1}\left(\mathrm{FPE}_{t+1} - \mathrm{APE}_{t+1}\right)^2}{\sum_{t=1}^{T-1}\left(\mathrm{APE}_{t+1}\right)^2}\right)^{\frac{1}{2}}$$

Theil's U-statistic

where $\mathrm{FPE}_{t+1} = \frac{\left(\hat{X}_{t+1} - X_t\right)}{X_t}$ is the forecast relative change, and

$\mathrm{APE}_{t+1} = \frac{\left(X_{t+1} - X_t\right)}{X_t}$ is the actual relative change (Diebold & Lopez [51]).

In [46]:
```
# Mean absolute error (MAE); the Root Mean Squared Error (RMSE); and Theil’s U-statistic
rmse = np.sqrt(mean_squared_error(actuals, predictions))
mae = mean_absolute_error(actuals, predictions)

# Theil’s U-statistic - seems to have no library for this test - Calculate manually
def theil_u_statistic(actual, predicted):
```

```
    """Calculate Theil's U-statistic"""

    fpe = (predicted[1:] - actual[:-1]) / actual[:-1]

    ape = actuals.pct_change()

    u = np.sqrt( np.sum(np.square(fpe-ape)) / (np.sum(np.square(ape))) )

    print(f'Theil’s U-statistic: {u}')

print(f'Root Mean Squared Error: {rmse}')
print(f'Mean Absolute Error: {mae}')
theil_u_statistic(actuals, predictions)
```

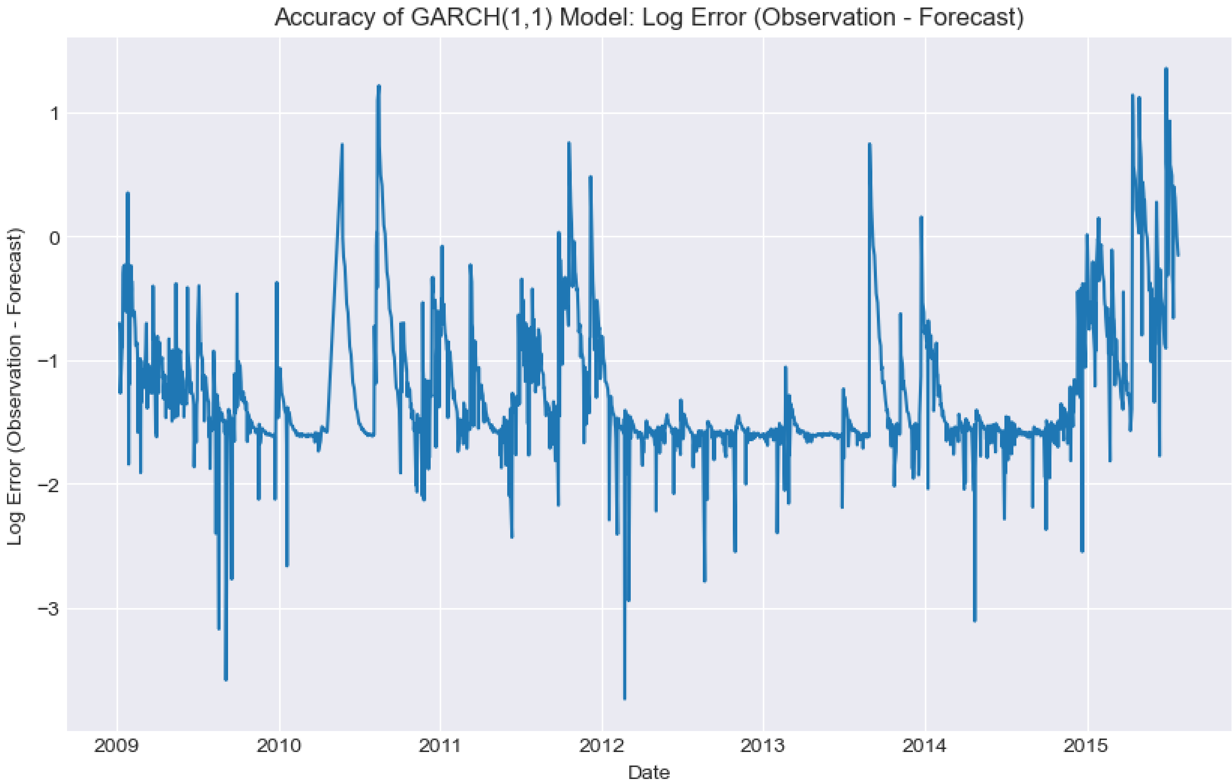
```
Root Mean Squared Error: 1.110774676220311
Mean Absolute Error: 0.23987201344187326
Theil’s U-statistic: 1.4183779162957852
```

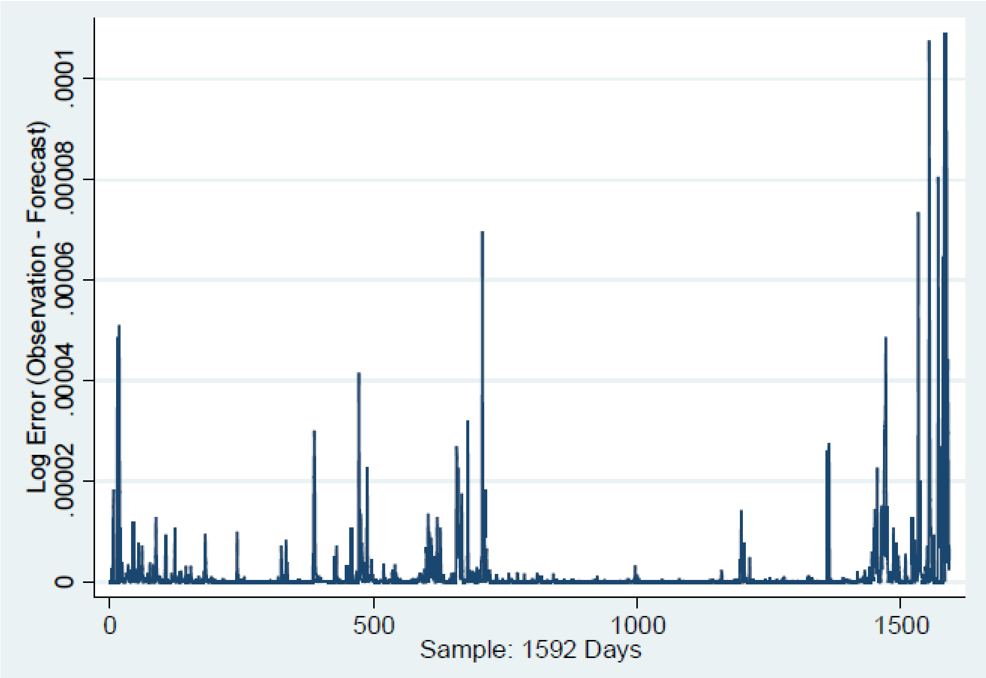
Table 6. Forecast evaluation.

Root Mean Squared Error (*RMSE*)	0.001300
Mean Absolute Error (*MAE*)	0.000640
Theil Inequality Coefficient	0.889055

Obs: 1591. Source: Author’s computations using data from Bank of Tanzania [24].

In [47]:
```
# Difference between actual and forecasted volatility plot
plot_series(log_error, 'Accuracy of GARCH(1,1) Model: Log Error (Observation - Forecast)',
            'Log Error (Observation - Forecast)')
```




Log Error (Observation - Forecast)
0
.00002
.00004
.00006
.00008
.0001
0
500
1000
1500
Sample: 1592 Days