ACM 20030 19337426 Pg 1

1 a True

b

i	1	2	3	4	5
x	-2	-1	1	2	3
y	-0.9	7.2	0.8	-3.1	0.1
FD1	8.1	-3.2	-3.9	3.2	
FD2	-3.7667	-0.2333	3.55		
FD3	0.8834	0.9458			

$FD1_i = \frac{y_{i+1} - y_i}{x_{i+1} - x_i}$ $FD2 = \frac{FD1_{i+1} - FD1_i}{x_{i+2} - x_i}$

$$y(x) = f(x_1) + (x - x_1)FD1 + (x - x_1)(x - x_2)FD2 + (x - x_1)(x - x_2)(x - x)_3 FD3$$

$$= -0.9 + (x+2)8.1 + (x+2)(x+1)(-3.7667) + (x+2)(x+1)(x-1)(0.8834)$$

$$y(0.5) = -0.9 + (0.5+2)8.1 + (0.5+2)(0.5+1)(-3.7667) + (0.5+2)(0.5+1)(0.5-1)(0.8834)$$

$y(0.5) = \boxed{3.5685}$ using 3rd order

2nd order $y(0.5) = -0.9 + (0.5+2)8.1 + (0.5+2)(0.5+1)(-3.7667)$

$= \boxed{5.2249}$ Second order

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2 a Eg: $f(x) = x$

$x_0 = 1 \qquad x_1 = 2$

as no root is bracketed the method will not converge

b $$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

~~One dimensio~~

The newton Raphson method can be seen as an autonomous 1 D map as $x_{i+1} = F(x_i)$

A period two orbit occurs where

$$x = F(F(x)) \qquad F(x) = x - \frac{f(x)}{f'(x)}$$

d $$x_{i+1} = x_i - f(x_i)\frac{x_i - x_{i-1}}{f(x_i) - f(x_{i-1})}$$

$$x_2 = x_1 - f(x_1)\frac{x_1 - x_0}{f(x_1) - f(x_0)}$$

$$= 3 - (-0.12683)\frac{(-0.1)}{0.07091}$$

$$x_2 = \cancel{3.17886}\ 2.82114$$

$f(x)$	x
-0.19774	3.1
-0.12683	3.0
~~-0.25432~~	~~3.17886~~
-0.00539	2.82114

$$\cancel{x_3 = 3.17886 - (-0.25432)(.17886)}$$

$$x_3 = 2.82114 - \frac{(-0.00539)(2.82114 - 3)}{(-0.00539 + 0.12683}$$

$$\boxed{x_3 = 2.8132}$$

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2 c

$F(x) = x - \frac{f(x)}{f'(x)}$

$f'x = \frac{d}{dx} \frac{1-2x}{x}$

$= \frac{x(-2) - (1-2x)}{x^2}$

$= \frac{-2x-1+2x}{x^2} = \frac{-1}{x^2}$

$F(x) = x - \frac{\frac{1-2x}{x}}{-\frac{1}{x^2}}$

$= x + \frac{x^2(1-2x)}{x}$

$= x + x - 2x^2$

$= 2x - 2x^2$

$= -2x^2 + 2x$

not continuous at $x = 0$

$F'(x) = -4x + 2 = 0$

$-4x = -2$

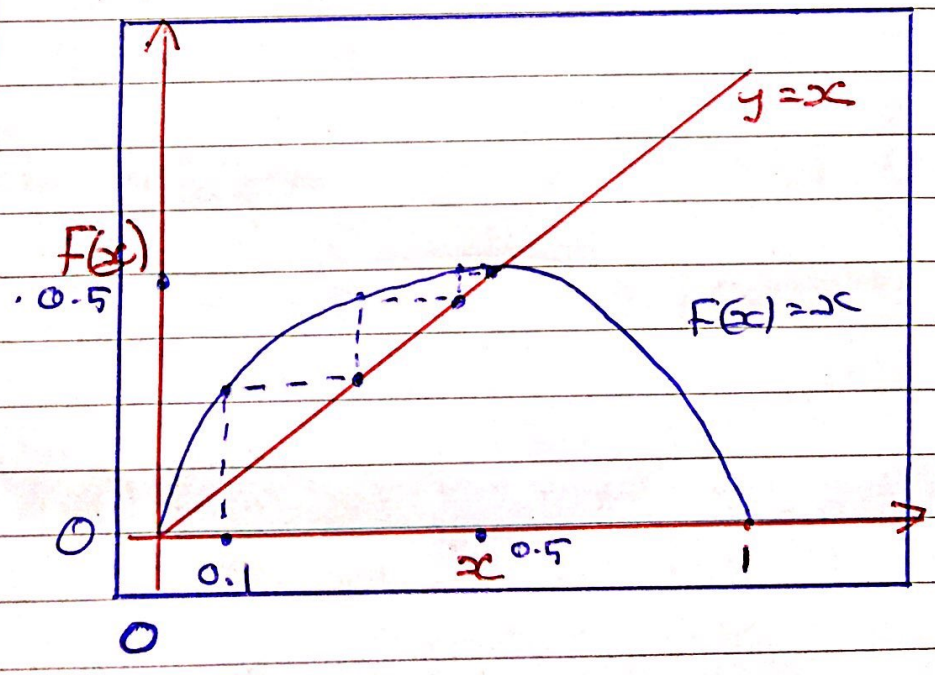
$x = 0.5$

$f(x) = 0 = -2x^2 + 2x$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 1$



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3 a In an initial value problem, values for the data (y and its derivatives) are given at a single point,

In a boundary value problem values for the data are given at multiple points

b $x_{i+1} = x_i + \Delta x$

$y_{i+1} = y_i + \Delta x \, f(x_i, y_i)$

$\Delta x = 0.1$

$x_0 = 0$

$y_0 = -1$

$x_1 = 0.1$

$y_1 = -1 + 0.1 \left(\sin(0.1)(-1)^2 + 0 \right)$

$= -1 + 0.1(0)$

$y_1 = -1$

$x_2 = 0.2$

$y_2 = -1 + 0.1 \left(\sin(0.1)(-1)^2 + 0.1 \right)$

$y_2 = -0.980017$

$y_3 = \overset{-0.980017}{\cancel{-0.9800}} + 0.1 \left(\sin(0.2)(\overset{-0.980017}{\cancel{-0.98}})^2 + 0.2 \right)$

$\boxed{\cancel{y_3 = -0.9409}}$ $y_3 = -0.94390$

$y(0.3) = \cancel{-0.9409}\ -0.94390$

$y_0 = -0.94390$ $x_1 = 0.2$ $\Delta x = -0.1$

$x_0 = 0.3$

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$y(0.1) = y_2 = -1.00023 - 0.1\,(\sin(0.2)(-1.00023)^2 + 0.2)$

$y_2 = -1.04011$

$x_2 = 0.2 - 0.1 = 0.1$

$y(0) = y_3 = -1.04011 - 0.1\,(\sin(0.1)(-1.04011)^2 + 0.1)$

$y_3 = -1.06091$

$|-1| - |-1.06091|$

$= 0.6091$ error

C $\Delta x = 0.1$

$x_0 = 0 \qquad y_0 = 1$

$x_1 = 0 + .1 = 0.1$

$K_1 = (0 - 1^2)$

$= -1$

$K_2 = f(0 + \frac{0.1}{2}, 1 + \frac{0.1}{2}(-1))$

$= f(0.05, 0.95)$

$= 0.05 - (0.95)^2$

$= -0.8525$

$K_3 = f(0 + .5, 1 + 0.1(2(-0.8525) + 1))$

$= -0.7640$

$y_1 = 1 + \frac{0.1}{6}(-1 + 4(-0.8525) + -0.764)$

$= 0.91377$

$K_1 = 0.1 - .91377^2 = -0.73500$

$K_2 = \cancel{0.15}$

$0.15 - .87702^2 = -0.61916$

$K_3 = f(0.2, 0.863438)$

$= -0.54553$

$y_2 = 0.91377 + \frac{0.1}{6}(-0.73500 + 4(-0.61916) - 0.54553)$

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Q 3 c
Cont

$y_2 = 0.85115 = y(0.2)$

$x_2 = 0.2$

$K_1 = -0.52446$

$K_2 = f(0.25, 0.82493)$
$= -0.43051$

$K_3 = f(0.3, 0.81749)$
$= -0.36829$

$y_3 = 0.85115 + \frac{0.1}{6}(K_1 + 4K_2 + K_3)$

$= 0.80757$

$y_4 = y(0.4)$

$x_3 = 0.3$

$K_1 = -0.35217$

$K_2 = f(0.35, 0.78996$
$= -0.27404$

$K_3 = f(0.4, 0.78798)$
$= -0.22091$

$y_4 = y_3 + \frac{0.1}{6}(K_1 + 4K_2 + K_3)$

$\boxed{y(0.4) = 0.74642}$

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4 a ~~a poly~~ a lagrange polynomial can be formed given two points and their mid point, the integral of this polynomial is given by $\frac{b-a}{6}\left[f(a)+4\left(\frac{a+b}{2}\right)+f(b)\right]$ where we are integrating from a to b and f is the original function.

By ~~Re~~ dividing the function into strips, the integral is calculated as the sum of these strips, Simpsons method

b $\Delta x = \frac{0.5+0.1}{6} = 0.1$

$$\cancel{I_{i+1} = \frac{0.1}{6}\left(f(x_i)+4f(x_{i+1})+f(x_{i+2})\right)}$$

$$\cancel{= \frac{0.1}{6}\left(f(x_i)+5f(x_{i+1})+6f(x_{i+2})+6f(x_{i+3})\right)}$$

$$I = \frac{\Delta x}{3}\left(y_0 + 4\sum_{j:odd} y_j + 2\sum_{j:even} y_j + y_N\right)$$

$y_0 = \sin(2(-0.1)) = -0.19867$

$y_1 = \quad = 0$

$y_2 = \quad = 0.19867$

$y_3 \quad = 0.38942$

$y_4 \quad = 0.56464$

$y_5 \quad = 0.71736$

$y_6 \quad = 0.84147$

$$I = \frac{0.1}{3}\left(-0.19867 + 4(0+0.38942+0.71736) + 2(0.19867+0.56464) + 0.84147\right)$$

~~I = 0.2220~~

$$\boxed{I = 0.21988467}$$

$$I = -\frac{\cos(2x)}{2}\Big|_{-0.1}^{0.5}$$

$$= 0.21988214$$

error $= 0.21988467 - 0.21988214 = \boxed{2.53\times10^{-6}}$

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c

$$\text{Area} = f(x_i)\cdot\left(\frac{\Delta x}{2}\right) + f(x_i+\Delta x)\cdot\left(\frac{\Delta x}{2}\right)$$

$$I_i = \Delta x \; f\left(x_i + \frac{\Delta x}{2}\right)$$

$$\boxed{I = \sum_{i=0}^{N-1} \Delta x \cdot f\left(x_i + \frac{\Delta x}{2}\right)}$$

5 a $Ax = b$ is ill conditioned if a small change in b results in a very large change in x

b

$$K(a) = \|A\|_\infty \, \|A^{-1}\|_\infty$$

$$\det A = ad - bc$$

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$$A^{-1} = \frac{25000}{31}\begin{pmatrix} 4.936 & 3.29 \\ 1.86 & 1.24 \end{pmatrix}$$

$$\text{inv } A = \frac{1}{\det}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$\|A\|_\infty = \max_i \sum_{j=1}^{n} |a_{ij}|$$

$$= \max\begin{pmatrix} 4.53 \\ 6.796 \end{pmatrix} = 6.796$$

$$\|A^{-1}\|_\infty = \frac{25000}{31}\max\begin{pmatrix} 8.226 \\ 3.1 \end{pmatrix} = \frac{25000}{31}(8.226)$$

$$K(a) = 6.796 \cdot \frac{25000}{31}(8.226)$$

$$= 45083.79$$

As this is very large A is ill conditioned

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c

$$\begin{array}{cc|cc|c} 3 & 12 & 1 & 0 & 2 \\ 6 & 25 & 0 & 1 & 3 \end{array}$$

- $L_2 - 2L_1$

$$\begin{array}{cc|cc|c} 3 & 12 & 1 & 0 & 2 \\ 0 & 1 & -2 & 1 & -1 \end{array}$$

Could solve system from here using back substitution but contine to find inverse

- $L_1 - 12L_2'$

$$\begin{array}{cc|cc|c} 3 & 0 & 25 & -12 & 14 \\ 0 & 1 & -2 & 1 & -1 \end{array}$$

$L_1' \to \frac{L_1}{3}$

$$\left|\begin{array}{cc|cc|c} 1 & 0 & \frac{25}{3} & -4 & \frac{14}{3} \\ 0 & 1 & -2 & 1 & -1 \end{array}\right.$$

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{14}{3} \\ -1 \end{pmatrix}$ inverse $= \begin{pmatrix} \frac{25}{3} & -4 \\ -2 & 1 \end{pmatrix}$

using back sub

$y = -1$

$3x + 12y = 2$

$3x = 2 - 12y$

$= 2 + 12$

$x = \frac{14}{3}$

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Q4C

$$I = \sum_{i=0}^{n-1} \Delta x \, f\left(x_i + \frac{\Delta x}{2}\right)$$

$\Delta x = 0.1$

$f(x) = \sqrt{x}$

$x_0 = 0$

$x_1 = 0.1$

$x_2 = 0.2$

$x_3 = 0.3$

$\frac{\Delta x}{2} = 0.05$

$$I = 0.1\left(\sqrt{0+0.05}\right) + 0.1\left(\sqrt{0.15}\right) + 0.1\left(\sqrt{0.25}\right) + 0.1\left(\sqrt{0.35}\right)$$

$$\boxed{= 0.17025}$$

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