

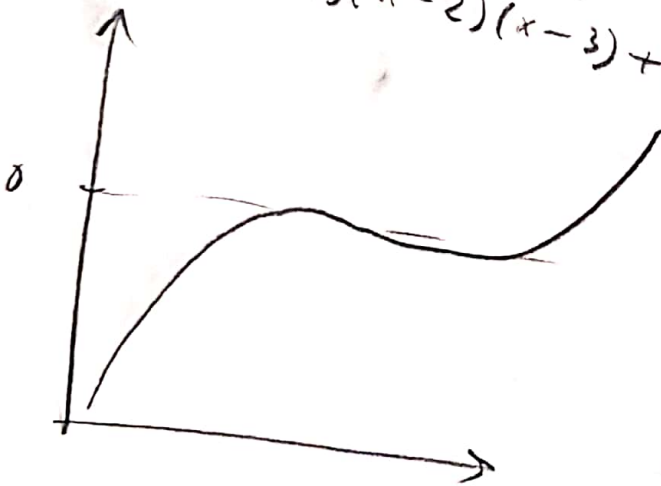
a) See attached code.

b) Rewrites as $f(x) = \begin{bmatrix} xy - 2 \\ \frac{x^4}{4} + \frac{y^3}{3} - 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

See attached code.

The first iteration overshoots correct answer, but quickly converges after this.

c) $f(x) = (x-1)(x-2)(x-3)+1$



looks like this:

Thus, $f'(x) \approx 0$ where $f(x) = 0$. When Newton's approaches 0, the linearization it does will start to overshoot. This can be fixed with a variable step size.

- d) The Jacobian J is singular at the root $x^* = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.
- Newton converges to the closest root.

e) ~~When the Newton method gets close to the solution, the gradient approaches 0, which makes each step smaller and smaller, which breaks the convergence.~~

The convergence is quadratic. ~~The~~ Newton's has the benefit over other methods of finding mins, such as gradient descent, that it uses information in the Hessian to not stall in a zig-zag pattern when it gets close to a solution.

(2) a) See attached code.

b) The results are pretty good.