TTK4130 MODELING AND SIMULATION / ASSIGNMENT 5

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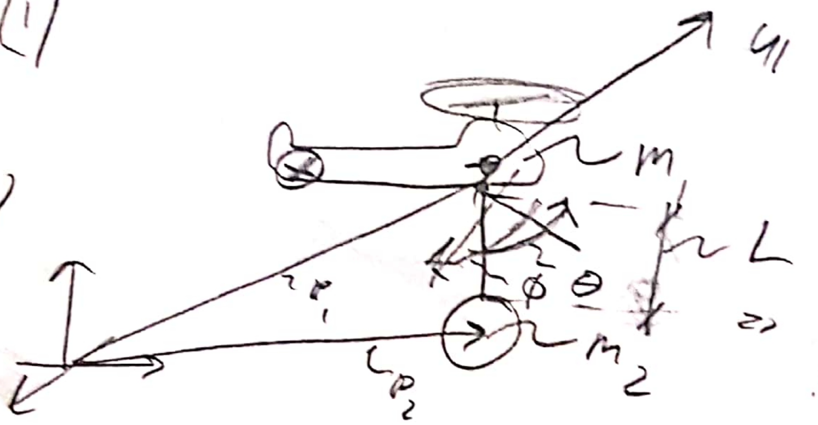
a)



$$\mathbf{q} = \begin{bmatrix} \mathbf{p}_1 \\ \theta \\ \phi \end{bmatrix}$$

$$\Rightarrow \quad \dot{\mathbf{q}} = \mathbf{v}$$

$$\mathbb{M}(\mathbf{q})\dot{\mathbf{v}} = \mathbb{K}(\mathbf{q}, \dot{\mathbf{q}}, u)$$

The position of m_2 is given by

$$\mathbf{p}_2 = \mathbf{p}_1 + \begin{bmatrix} L\sin(\theta)\cos(\phi) \\ L\sin(\theta)\sin(\phi) \\ L\cos\theta \end{bmatrix}$$

The generalized forces are $Q = \overbrace{\begin{bmatrix} u \\ 0 \\ 0 \end{bmatrix}}^{\text{External forces}}$

The kinetic and potential energies are:

$$T = \frac{1}{2} m_1 \dot{\mathbf{p}}_1^T \dot{\mathbf{p}}_1 + \frac{1}{2} m_2 \dot{\mathbf{p}}_2^T \dot{\mathbf{p}}_2$$

$$V = m_1 g \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}^T \mathbf{p}_1 + m_2 g \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}^T \mathbf{p}_2$$

$$\Rightarrow \quad \underline{\mathcal{L} = T - V}$$

b) The kinetic energy can now be written as

$$T = \frac{1}{2} m_1 \dot{\mathbb{P}}_1^T \dot{\mathbb{P}}_2 + \frac{1}{2} m_2 \dot{\mathbb{P}}_2^T \dot{\mathbb{P}}_2$$

$$= \frac{1}{2} \dot{q}^T \underbrace{\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}}_{W} \dot{q}$$

Potential energy: $m_1 g [0 \; 0 \; 1] \mathbb{P}_1 + m_2 g [0 \; 0 \; 1] \mathbb{P}_2$

$$= \underbrace{[0 \; 0 \; m_1 g \; 0 \; 0 \; m_2 g]}_{G^T} q$$

$$\Rightarrow \mathcal{L} = T - V = \frac{1}{2} \dot{q}^T W \dot{q} - G^T q - Z \cdot C \sim \frac{1}{2}(e^T e - \ell^2)$$

$$\Rightarrow \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}} - \frac{\partial \mathcal{L}}{\partial q} - Z \underbrace{\nabla_q C(q)}_{} \; Z \cdot H^T H q$$

$e = \mathbb{P}_1 - \mathbb{P}_2$

$= \underbrace{[\mathbb{I}_3 \; -\mathbb{I}_3]}_{H} q_3$

$$= \frac{d}{dt}(W \dot{q}) - G$$

$$= W \ddot{q} - G = Q \Rightarrow W \ddot{q} = Q + G$$

$\Rightarrow \frac{1}{2}(q^T H^T H q - \ell^2)$

$$\Rightarrow \mathbb{M} = \mathbb{W}, \quad b = Q + G + Z$$

$$\begin{matrix} \mathbb{I}_3 & -\mathbb{I}_3 \\ \mathbb{I}_3 & \mathbb{I}_3 & -\mathbb{I}_3 \\ & -\mathbb{I}_3 & \mathbb{I}_3 \end{matrix}$$

$$\mathbb{M}\ddot{\mathbf{q}} = \mathbb{b}$$

$$\mathbb{M} = \mathbb{W} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}$$

$$\mathbb{b} = Q - \cancel{\frac{\partial}{\partial \mathbf{q}}\left(\mathbb{W}(\mathbf{q})\dot{\mathbf{q}}\right)\dot{\mathbf{q}}}^{\,0} + \cancel{\nabla_{\mathbf{q}} T}^{\,0} - \nabla_{\mathbf{q}} V - Z\nabla_{\mathbf{q}} c$$

$$= Q - \nabla_{\mathbf{q}} V - Z\nabla_{\mathbf{q}} c$$

$$\Rightarrow \mathbb{M}_{impl}\,\ddot{\mathbf{x}} = \mathbb{C}, \Rightarrow \mathbb{M}_{impl} = \begin{bmatrix} W(\mathbf{q}) & \nabla_{\mathbf{q}} c \\ \nabla_{\mathbf{q}} \mathbb{C}^T & 0 \end{bmatrix},$$

$$\mathbf{x} = \begin{bmatrix} \ddot{\mathbf{q}} \\ Z \end{bmatrix}, \quad \mathbb{C} = \begin{bmatrix} Q - \nabla_{\mathbf{q}} \mathcal{L} \\ -\frac{\partial}{\partial \mathbf{q}}\left(\frac{\partial c}{\partial \mathbf{q}}\dot{\mathbf{q}}\right)\dot{\mathbf{q}} \end{bmatrix}.$$

$\boxed{2}$ b)

The $\mathbb{M}$ and $\mathbb{b}$ in 1a, are nasty, but $\mathbb{M}$ and $\mathbb{b}$ in 1 b) are much cleaner.

b)
$$\begin{bmatrix} \mathbb{W}(\mathbf{q}) & \nabla_{\mathbf{q}} \mathbb{C} \\ \nabla_{\mathbf{q}} \mathbb{C}^T & 0 \end{bmatrix}$$ is not too bad, but

$$\begin{bmatrix} W & \nabla_{\mathbf{q}} c \\ \nabla \mathbb{C}_{\mathbf{q}}^T & 0 \end{bmatrix}^{-1}$$ is really nasty-looking

Scanned with CamScanner

$\boxed{2}$ a)

$$\mathfrak{A}(q) = \nabla_q C$$

$$\mathbb{C} = \begin{bmatrix} Q - \nabla_q \mathcal{L} \\ -\frac{\partial}{\partial q}\left(\frac{\partial c}{\partial q}\dot{q}\right)\dot{q} \end{bmatrix}$$

$\boxed{3}$

a) Only using $\alpha_1, \alpha_2, \alpha_3$ would result in two different points for nozzle. Additionally, it is computationally demanding as it includes three pivots and three double rods.

b) $V = mg[0 \quad 0 \quad 1]P$

$T = \frac{1}{2} J \| \dot{\alpha}_i \|^2 + \frac{1}{2} m \| \dot{P} \|^2$

$\mathcal{L} = T - V - z \cdot C(q), \Rightarrow \frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} - \frac{\partial \mathcal{L}}{\partial q} = Q, \quad Q = \begin{bmatrix} u \\ 0 \end{bmatrix}$

$C(q) = 0 \Rightarrow \|P - P_1\|^2 - l^2 = 0$

$\|P - P_2\|^2 - l^2 = 0$

$\|P - P_3\|^2 - l^2 = 0$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_f} = W \ddot{q}_f , \quad T = \frac{1}{2}\dot{q}_f^T W \dot{q}_f \Rightarrow W = \begin{bmatrix} J & & & & & \\ & J & & & 0 & \\ & & J & & & \\ & 0 & & m & & \\ & & & & m & \\ & & & & & m \end{bmatrix}$$

$$\frac{\partial \mathcal{L}}{\partial q_f} = \begin{bmatrix} 0_{5\times 1} \\ -mg \end{bmatrix}$$