

1) a) See attached code

b) The IRK-method is very close to the exact solution

c) The IRK-method does not become ~~unstable~~ as long as the actual system is stable. This was tested up to  $\lambda = -1460$

2) We have  $\ddot{x} = -g(1 - (\frac{x_d}{x})^k)$ ,  $E = \frac{mg}{k-1} \frac{x_d^k}{x^{k-1}} + mgx + \frac{1}{2} m \dot{x}^2$   
and  $\dot{E} = 0$

$$\Rightarrow \frac{dE}{dt} = \frac{\partial E}{\partial x} \cdot \dot{x} + \frac{\partial E}{\partial \dot{x}} \cdot \ddot{x}, \quad \frac{\partial E}{\partial x} = -mg \left( \frac{x_d}{x} \right)^k + mg$$

$$\frac{\partial E}{\partial \dot{x}} = m \dot{x}$$

$$\begin{aligned}
 \dot{E} &= \overbrace{\left(-mg\left(\frac{x_d}{x}\right)^k + mg\right)}^{\partial E / \partial x} \dot{x} + \overbrace{mx''}^{\partial E / \partial \dot{x}} \underbrace{\left(-g\left(1 - \left(\frac{x_d}{x}\right)^k\right)\right)}_{\dot{x}''} \\
 &= -mg\dot{x}\left(\frac{x_d}{x}\right)^k + mg\dot{x} - m\dot{x}g + m\dot{x}g\left(\frac{x_d}{x}\right)^k \\
 &= \underline{\underline{0}} \quad \text{f.e.d.}
 \end{aligned}$$

b) See attached code. We ~~probably~~ probably have energy loss because machine precision, so the derivative of  $E$  is not perfect 0.