9.75 Let $Y_1, Y_2, \ldots, Y_n$ be a random sample from the probability density function given by

$$f(y \mid \theta) = \begin{cases} \dfrac{\Gamma(2\theta)}{[\Gamma(\theta)]^2}(y^{\theta-1})(1-y)^{\theta-1}, & 0 \le y \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

Find the method-of-moments estimator for θ.

9.76 Let $X_1, X_2, X_3, \ldots$ be independent Bernoulli random variables such that $P(X_i = 1) = p$ and $P(X_i = 0) = 1 - p$ for each $i = 1, 2, 3, \ldots$. Let the random variable Y denote the number of trials necessary to obtain the first success—that is, the value of i for which $X_i = 1$ first occurs. Then Y has a geometric distribution with $P(Y = y) = (1 - p)^{y-1}p$, for $y = 1, 2, 3, \ldots$. Find the method-of-moments estimator of p based on this single observation Y.

level of the control variable.

7. It is estimated that over 90,000 students will apply to the top 30 M.B.A. programs in the United States this year.
   a. Using the concept of net present value and opportunity cost, explain when it is rational for an individual to pursue an M.B.A. degree.
   b. What would you expect to happen to the number of applicants if the starting salaries of managers with M.B.A. degrees remained constant but salaries of managers without such degrees increased by 15 percent? Why?
8. Jaynet spends \$20,000 per year on painting supplies and storage space. She recently received two job offers from a famous marketing firm—one offer was for \$100,000 per year, and the other was for \$90,000. However, she turned both jobs down to continue a painting career. If Jaynet sells 20 paintings per year at a price of \$10,000 each:
   a. What are her accounting profits?
   b. What are her economic profits?
9. Suppose the total benefit derived from a given decision, Q, is $B(Q) = 25Q - Q^2$ and the corresponding total cost is $C(Q) = 5 + Q^2$, so that $MB(Q) = 25 - 2Q$ and $MC(Q) = 2Q$.
   a. What is total benefit when $Q = 2$? $Q = 10$?
   b. What is marginal benefit when $Q = 2$? $Q = 10$?
   c. What level of Q maximizes total benefit?
   d. What is total cost when $Q = 2$? $Q = 10$?
   e. What is marginal cost when $Q = 2$? $Q = 10$?
   f. What level of Q minimizes total cost?
   g. What level of Q maximizes net benefits?

18. From California to New York, legislative bodies across the United States are considering eliminating or reducing the surcharges that banks impose on noncustomers who make $10 million in withdrawals from other banks' ATM machines. On average, noncustomers earn a wage of $20 per hour and pay ATM fees of $2.75 per transaction. It is estimated that banks would be willing to maintain services for 4 million transactions at $0.75 per transaction, while noncustomers would attempt to conduct 16 million transactions at that price. Estimates suggest that, for every 1 million gap between the desired and available transactions, a typical consumer will have to spend an extra minute traveling to another machine to withdraw cash. Based on this information, use a graph to carefully illustrate the impact of legislation that would place a $0.75 cap on the fees banks can charge for noncustomer transactions.

19. Rapel Valley in Chile is renowned for its ability to produce high-quality wine at a fraction of the cost of many other vineyards around the world. Rapel Valley produces over 20 million bottles of wine annually, of which 5 million are exported to the United States. Each bottle entering the United States is subjected to a $0.50 per bottle excise tax, which generates about $2.5 million in tax revenues. Strong La Niña weather patterns have caused unusually cold temperatures, devastating many of the wine producers in that region of Chile. How will La Niña affect the price of Chilean wine? Assuming La Niña does

***9.29** Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample of size n from a power family distribution (see Exercise 6.17). Then the methods of Section 6.7 imply that $Y_{(n)} = \max(Y_1, Y_2, \ldots, Y_n)$ has the distribution function given by

$$F_{(n)}(y) = \begin{cases} 0, & y < 0, \\ (y/\theta)^{\alpha n}, & 0 \le y \le \theta, \\ 1, & y > \theta. \end{cases}$$

Use the method described in Exercise 9.26 to show that $Y_{(n)}$ is a consistent estimator of θ.

9.30 Let $Y_1, Y_2, \ldots, Y_n$ be independent random variables, each with probability density function

$$f(y) = \begin{cases} 3y^2, & 0 \le y \le 1, \\ 0, & \text{elsewhere.} \end{cases}$$

Show that $\overline{Y}$ converges in probability to some constant and find the constant.

9.31 If $Y_1, Y_2, \ldots, Y_n$ denote a random sample from a gamma distribution with parameters α and β, show that $\overline{Y}$ converges in probability to some constant and find the constant.

9.32 Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from the probability density function

$$f(y) = \begin{cases} \dfrac{2}{y^2}, & y \ge 2, \\ 0, & \text{elsewhere.} \end{cases}$$

Does the law of large numbers apply to $\overline{Y}$ in this case? Why or why not?

14. You are the manager of an organization in America that distributes blood to hospitals in all 50 states and the District of Columbia. A recent report indicates that nearly 50 Americans contract HIV each year through blood transfusions. Although every pint of blood donated in the United States undergoes a battery of nine different tests, existing screening methods can detect only the antibodies produced by the body's immune system—not foreign agents in the blood. Since it takes weeks or even months for these antibodies to build up in the blood, newly infected HIV donors can pass along the virus through blood that has passed existing screening tests. Happily, researchers have developed a series of new tests aimed at detecting and removing infections from donated blood before it is used in transfusions. The obvious benefit of these tests is the reduced incidence of infection through blood transfusions. The report indicates that the current price of decontaminated blood is \$80 per pint. However, if the new screening methods are adopted, the demand and supply for decontaminated blood will change to $Q^d = 175 - P$ and $Q^s = 2P - 200$. What price do you expect to prevail if the new screening methods are adopted? How many units of blood will be used in the United States? What is the level of consumer and producer surplus? Illustrate your findings in a graph.

12. Fund A accumulates at a rate of 12% convertible monthly. Fund B accumulates with a force of interest $\delta_t = t/6$. At time $t = 0$ equal deposits are made in each fund. Find the next time that the two funds are equal.

2.5 Unknown rate of interest

13. Find the nominal rate of interest convertible semiannually at which the accumulated value of \$1000 at the end of 15 years is \$3000.

14. Find an expression for the exact effective rate of interest at which payments of \$300 at the present, \$200 at the end of one year, and \$100 at the end of two years will accumulate to \$700 at the end of two years.

9.21 Let $Y_1, Y_2, \ldots, Y_n$ be a random sample of size n from a normal population with mean μ and variance σ^2. Assuming that $n = 2k$ for some integer k, one possible estimator for σ^2 is given by

$$\hat{\sigma}^2 = \frac{1}{2k}\sum_{i=1}^{k}(Y_{2i} - Y_{2i-1})^2.$$

a Show that $\hat{\sigma}^2$ is an unbiased estimator for σ^2.

b Show that $\hat{\sigma}^2$ is a consistent estimator for σ^2.

9.22 Refer to Exercise 9.21. Suppose that $Y_1, Y_2, \ldots, Y_n$ is a random sample of size n from a Poisson-distributed population with mean λ. Again, assume that $n = 2k$ for some integer k. Consider

$$\hat{\lambda} = \frac{1}{2k}\sum_{i=1}^{k}(Y_{2i} - Y_{2i-1})^2.$$

a Show that $\hat{\lambda}$ is an unbiased estimator for λ.

b Show that $\hat{\lambda}$ is a consistent estimator for λ.

9.23 Refer to Exercise 9.21. Suppose that $Y_1, Y_2, \ldots, Y_n$ is a random sample of size n from a population for which the first four moments are finite. That is, $m_1' = E(Y_1) < \infty$, $m_2' = E(Y_1^2) < \infty$, $m_3' = E(Y_1^3) < \infty$, and $m_4' = E(Y_1^4) < \infty$. (*Note:* This assumption is valid for the normal and Poisson distributions in Exercises 9.21 and 9.22, respectively.) Again, assume

9.15 Refer to Exercise 9.3. Show that both $\hat{\theta}_1$ and $\hat{\theta}_2$ are consistent estimators for θ.

9.16 Refer to Exercise 9.5. Is $\hat{\sigma}_2^2$ a consistent estimator of σ^2?

9.17 Suppose that $X_1, X_2, \ldots, X_n$ and $Y_1, Y_2, \ldots, Y_n$ are independent random samples from populations with means μ_1 and μ_2 and variances σ_1^2 and σ_2^2, respectively. Show that $\overline{X} - \overline{Y}$ is a consistent estimator of $\mu_1 - \mu_2$.

9.18 In Exercise 9.17, suppose that the populations are normally distributed with $\sigma_1^2 = \sigma_2^2 = \sigma^2$. Show that

$$\frac{\sum_{i=1}^{n}(X_i - \overline{X})^2 + \sum_{i=1}^{n}(Y_i - \overline{Y})^2}{2n - 2}$$

is a consistent estimator of σ^2.

9.19 Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from the probability density function

$$f(y) = \begin{cases} \theta y^{\theta-1}, & 0 < y < 1, \\ 0, & \text{elsewhere,} \end{cases}$$

where $\theta > 0$. Show that $\overline{Y}$ is a consistent estimator of $\theta/(\theta + 1)$.

9.20 If Y has a binomial distribution with n trials and success probability p, show that Y/n is a consistent estimator of p.

***9.8** Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from a probability density function $f(y)$, which has unknown parameter θ. If $\hat{\theta}$ is an unbiased estimator of θ, then under very general conditions

$$V(\hat{\theta}) \geq I(\theta), \quad \text{where } I(\theta) = \left[nE\left(-\frac{\partial^2 \ln f(Y)}{\partial \theta^2} \right) \right]^{-1}.$$

(This is known as the Cramer–Rao inequality.) If $V(\hat{\theta}) = I(\theta)$, the estimator $\hat{\theta}$ is said to be *efficient*.[1]

a Suppose that $f(y)$ is the normal density with mean μ and variance σ^2. Show that $\overline{Y}$ is an efficient estimator of μ.

b This inequality also holds for discrete probability functions $p(y)$. Suppose that $p(y)$ is the Poisson probability function with mean λ. Show that $\overline{Y}$ is an efficient estimator of λ.

11. You are the manager of a midsized company that assembles personal computers. You purchase most components—such as random access memory (RAM)—in a competitive market. Based on your marketing research, consumers earning

over $75,000 purchase 1.3 times more RAM than consumers with lower incomes. One morning, you pick up a copy of *The Wall Street Journal* and read an article indicating that a new technological breakthrough will permit manufacturers to produce RAM at a lower unit cost. Based on this information, what can you expect to happen to the price you pay for random access memory? Would your answer change if, in addition to this technological breakthrough, the article indicated that consumer incomes are expected to grow over the next two years as the economy pulls out of recession? Explain.

***9.93** Let $Y_1, Y_2, \ldots, Y_n$ be a random sample from a population with density function

$$f(y \mid \theta) = \begin{cases} \dfrac{2\theta^2}{y^3}, & \theta < y < \infty, \\ 0, & \text{elsewhere.} \end{cases}$$

In Exercise 9.53, you showed that $Y_{(1)} = \min(Y_1, Y_2, \ldots, Y_n)$ is sufficient for θ.

a Find the MLE for θ. [*Hint:* See Example 9.16.]

b Find a function of the MLE in part (a) that is a pivotal quantity.

c Use the pivotal quantity from part (b) to find a $100(1-\alpha)\%$ confidence interval for θ.

***9.94** Suppose that $\hat{\theta}$ is the MLE for a parameter θ. Let $t(\theta)$ be a function of θ that possesses a unique inverse [that is, if $\beta = t(\theta)$, then $\theta = t^{-1}(\beta)$]. Show that $t(\hat{\theta})$ is the MLE of $t(\theta)$.

***9.95** A random sample of n items is selected from the large number of items produced by a certain production line in one day. Find the MLE of the ratio R, the proportion of defective items divided by the proportion of good items.

9.96 Consider a random sample of size n from a normal population with mean μ and variance σ^2, both unknown. Derive the MLE of σ.

19. Rapel Valley in Chile is renowned for its ability to produce high-quality wine at a fraction of the cost of many other vineyards around the world. Rapel Valley produces over 20 million bottles of wine annually, of which 5 million are exported to the United States. Each bottle entering the United States is subjected to a $0.50 per bottle excise tax, which generates about $2.5 million in tax revenues. Strong La Niña weather patterns have caused unusually cold temperatures, devastating many of the wine producers in that region of Chile. How will La Niña affect the price of Chilean wine? Assuming La Niña does

not impact the California wine-producing region, how will La Niña impact the market for Californian wines?

9.85 Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from the density function given by

$$f(y \mid \alpha, \theta) = \begin{cases} \left(\dfrac{1}{\Gamma(\alpha)\theta^{\alpha}}\right) y^{\alpha-1} e^{-y/\theta}, & y > 0, \\ 0, & \text{elsewhere,} \end{cases}$$

where $\alpha > 0$ is known.

a Find the MLE $\hat{\theta}$ of θ.

b Find the expected value and variance of $\hat{\theta}$.

c Show that $\hat{\theta}$ is consistent for θ.

d What is the best (minimal) sufficient statistic for θ in this problem?

e Suppose that $n = 5$ and $\alpha = 2$. Use the minimal sufficient statistic to construct a 90% confidence interval for θ. [*Hint:* Transform to a χ^2 distribution.]

9.86 Suppose that $X_1, X_2, \ldots, X_m$, representing yields per acre for corn variety A, constitute a random sample from a normal distribution with mean μ_1 and variance σ^2. Also, $Y_1\ Y_2, \ldots, Y_n$, representing yields for corn variety B, constitute a random sample from a normal distribution with mean μ_2 and variance σ^2. If the X's and Y's are independent, find the MLE for the common variance σ^2. Assume that μ_1 and μ_2 are unknown.

9.87 A random sample of 100 voters selected from a large population revealed 30 favoring candidate A, 38 favoring candidate B, and 32 favoring candidate C. Find MLEs for the proportions of voters in the population favoring candidates A, B, and C, respectively. Estimate the difference between the fractions favoring A and B and place a 2-standard-deviation bound on the error of estimation.

9.56 Refer to Exercise 9.38(b). Find an MVUE of σ^2.

9.57 Refer to Exercise 9.18. Is the estimator of σ^2 given there an MVUE of σ^2?

9.58 Refer to Exercise 9.40. Use $\sum_{i=1}^{n} Y_i^2$ to find an MVUE of θ.

9.59 The number of breakdowns Y per day for a certain machine is a Poisson random variable with mean λ. The daily cost of repairing these breakdowns is given by $C = 3Y^2$. If $Y_1, Y_2, \ldots, Y_n$ denote the observed number of breakdowns for n independently selected days, find an MVUE for $E(C)$.

9.60 Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from the probability density function

$$f(y \mid \theta) = \begin{cases} \theta y^{\theta-1}, & 0 < y < 1, \ \theta > 0, \\ 0, & \text{elsewhere.} \end{cases}$$

a Show that this density function is in the (one-parameter) exponential family and that $\sum_{i=1}^{n} -\ln(Y_i)$ is sufficient for θ. (See Exercise 9.45.)

b If $W_i = -\ln(Y_i)$, show that W_i has an exponential distribution with mean $1/\theta$.

c Use methods similar to those in Example 9.10 to show that $2\theta \sum_{i=1}^{n} W_i$ has a χ^2 distribution with $2n$ df.

d Show that

$$E\left(\frac{1}{2\theta \sum_{i=1}^{n} W_i}\right) = \frac{1}{2(n-1)}.$$

[*Hint:* Recall Exercise 4.112.]

e What is the MVUE for θ?

21. Seventy-two percent of the members of the United Food and Commercial Workers Local 655 voted to strike against Stop 'n Shop in the St. Louis area. In fear of similar union responses, two of Stop 'n Shop's larger rivals in the St. Louis market—Dierberg's and Schnuck's—decided to lock out its union employees. The actions of these supermarkets, not surprisingly, caused Local 655 union members to picket and boycott each of the supermarkets' locations. While the manager of Mid Towne IGA—one of many smaller competing grocers—viewed the strike as unfortunate for both sides, he was quick to point out that the strike provided an opportunity for his store to increase market share. To take advantage of the strike, the manager of Mid Towne IGA increased newspaper advertising by pointing out that Mid Towne employed Local 655 union members and that it operated under a different contract than "other" grocers in the area. Use a graph to describe the expected impact of advertising on Mid Towne IGA (how the equilibrium price and quantity change). Identify the type of advertising in which Mid Towne IGA engaged. Do you believe the impact of advertising will be permanent? Explain.

16. As a marketing manager for one of the world's largest automakers, you are responsible for the advertising campaign for a new energy-efficient sports utility vehicle. Your support team has prepared the following table, which summarizes the (year-end) profitability, estimated number of vehicles sold, and average estimated selling price for alternative levels of advertising. The accounting department projects that the best alternative use for the funds used in the advertising campaign is an investment returning 10 percent. In light of the staggering cost of advertising (which accounts for the lower projected profits in years 1 and 2 for the high and moderate advertising intensities), the team leader recommends a low advertising intensity in order to maximize the value of the firm. Do you agree? Explain.

Profitability by Advertising Intensity

	Profits (in millions)			***Units Sold (in thousands)***			***Average Selling Price***		
	Year 1	**Year 2**	**Year 3**	**Year 1**	**Year 2**	**Year 3**	**Year 1**	**Year 2**	**Year 3**
Advertising Intensity									
High	$15	$ 90	$270	10	60	120	$24,000	$25,500	$26,000
Moderate	30	75	150	5	12.5	25	24,500	24,750	25,000
Low	70	105	126	4	6	7.2	24,800	24,850	24,900

12. You are the manager of a firm that produces and markets a generic type of soft drink in a competitive market. In addition to the large number of generic products in your market, you also compete against major brands such as Coca-Cola and Pepsi. Suppose that, due to the successful lobbying efforts of sugar producers in the United States, Congress is going to levy a \$0.50 per pound tariff on all imported raw sugar—the primary input for your product. In addition, Coke and Pepsi plan to launch an aggressive advertising campaign designed to persuade consumers that their branded products are superior to generic soft drinks. How will these events impact the equilibrium price and quantity of generic soft drinks?
13. Some have argued that higher cigarette prices do not deter smoking. While there are many arguments both for and against this view, some find the following argument to be the most persuasive of all: "The laws of supply and demand indicate that higher prices are ineffective in reducing smoking. In particular, higher cigarette prices will reduce the demand for cigarettes. This reduction in demand will push the equilibrium price back down to its original level. Since the equilibrium price will remain unchanged, smokers will consume the same number of cigarettes." Do you agree or disagree with this view? Explain.

17. The head of the accounting department at a major software manufacturer has asked you to put together a pro forma statement of the company's value under several possible growth scenarios and the assumption that the company's many divisions will remain a single entity forever. The manager is concerned that, despite the fact that the firm's competitors are comparatively small, collectively their annual revenue growth has exceeded 50 percent over each of the last five years. She has requested that the value projections be based on the firm's current profits of $2.5 billion (which have yet to be paid out to stockholders) and the average interest rate over the past 20 years (8 percent) in each of the following profit growth scenarios:
    a. Profits grow at an annual rate of 10 percent. (This one is tricky.)
    b. Profits grow at an annual rate of 3 percent.
    c. Profits grow at an annual rate of 0 percent.
    d. Profits decline at an annual rate of 3 percent.
18. Suppose one of your clients is four years away from retirement and has only $1,500 in pretax income to devote to either a Roth or a traditional IRA. The traditional IRA permits investors to contribute the full $1,500 since contributions to these accounts are tax-deductible, but they must pay taxes on all future distributions. In contrast, contributions to a Roth IRA are not tax-deductible, meaning that at a tax rate of 25 percent, an investor is able to contribute only $1,125 after taxes; however, the earnings of a Roth IRA grow tax-free. Your company has decided to waive the one-time set-up fee of $25 to

10. An owner can lease her building for \$100,000 per year for three years. The explicit cost of maintaining the building is \$35,000, and the implicit cost is \$50,000. All revenues are received, and costs are borne, at the end of each year. If the interest rate is 4 percent, determine the present value of the stream of:
    a. Accounting profits.
    b. Economic profits.

ID APPLICATIONS

11. You've recently learned that the company where you work is being sold for \$275,000. The company's income statement indicates current profits of \$10,000, which have yet to be paid out as dividends. Assuming the company will remain a "going concern" indefinitely and that the interest rate will remain constant at 10 percent, at what constant rate does the owner believe that profits will grow? Does this seem reasonable?
12. You are in the market for a new refrigerator for your company's lounge, and you have narrowed the search down to two models. The energy efficient

22. A bill for $100 is purchased for $96 three months before it is due. Find:

   a) The nominal rate of discount convertible quarterly earned by the purchaser.

   b) The annual effective rate of interest earned by the purchaser.

23. A two-year certificate of deposit pays an annual effective rate of 9%. The purchaser is offered two options for prepayment penalties in the event of early withdrawal:

   A – a reduction in the rate of interest to 7%.

   B – loss of three months interest.

   In order to assist the purchaser in deciding which option to select, compute the ratio of the proceeds under Option A to those under Option B if the certificate of deposit is surrendered:

   a) At the end of 6 months.

   b) At the end of 18 months.

24. The ABC Bank has an early withdrawal policy for certificates of deposit (CDs) which states that interest still be credited for the entire length the money actually stays with the bank, but that the CD nominal interest rate will be reduced by 1.8% for the same number of months as the CD is redeemed early. An incoming college freshman invests $5000 in a two-year CD with a nominal rate of interest equal to 5.4% compounded monthly on September 1 at the beginning of the freshman year. The student intended to leave the money on deposit for the full two-year term to help finance the junior and senior years, but finds the need to withdraw it on May 1 of the sophomore year. Find the amount that the student will receive for the CD on that date.

Does the law of large numbers apply to Y in this case? Why or why not?

9.33 An experimenter wishes to compare the numbers of bacteria of types A and B in samples of water. A total of n independent water samples are taken, and counts are made for each sample. Let X_i denote the number of type A bacteria and Y_i denote the number of type B bacteria for sample i. Assume that the two bacteria types are sparsely distributed within a water sample so that $X_1, X_2, \dots, X_n$ and $Y_1, Y_2, \dots, Y_n$ can be considered independent random samples from Poisson distributions with means λ_1 and λ_2, respectively. Suggest an estimator of $\lambda_1/(\lambda_1+\lambda_2)$. What properties does your estimator have?

9.34 The Rayleigh density function is given by

$$f(y) = \begin{cases} \left(\dfrac{2y}{\theta}\right)e^{-y^2/\theta}, & y > 0, \\ 0, & \text{elsewhere.} \end{cases}$$

In Exercise 6.34(a), you established that Y^2 has an exponential distribution with mean θ. If $Y_1, Y_2, \dots, Y_n$ denote a random sample from a Rayleigh distribution, show that $W_n = \frac{1}{n}\sum_{i=1}^{n} Y_i^2$ is a consistent estimator for θ.

9.35 Let $Y_1, Y_2, \dots$ be a sequence of random variables with $E(Y_i) = \mu$ and $V(Y_i) = \sigma_i^2$. Notice that the σ_i^2's are not all equal.

- **a** What is $E(\overline{Y}_n)$?
- **b** What is $V(\overline{Y}_n)$?
- **c** Under what condition (on the σ_i^2's) can Theorem 9.1 be applied to show that $\overline{Y}_n$ is a consistent estimator for μ?

15. As a result of increased tensions in the Middle East, oil production is down by 1.21 million barrels per day—a 5 percent reduction in the world's supply of

crude oil. Explain the likely impact of this event on the market for gasoline and the market for small cars.

13. You are the human resources manager for a famous retailer, and you are trying to convince the president of the company to change the structure of employee compensation. Currently, the company's retail sales staff is paid a flat hourly wage of $18 per hour for each eight-hour shift worked. You propose a new pay structure whereby each salesperson in a store would be compensated $8 per hour, plus five-tenths of 1 percent of that store's daily profits. Assume that, when run efficiently, each store's maximum daily profits are $40,000. Outline the arguments that support your proposed plan.
14. Tara is considering leaving her current job, which pays $56,000 per year, to start a new company that manufactures a line of special pens for personal digital assistants. Based on market research, she can sell about 160,000 units during the first year at a price of $20 per unit. With annual overhead costs and operating expenses amounting to $3,160,000, Tara expects a profit margin of 25 percent. This margin is 6 percent larger than that of her largest competitor, Pens, Inc.
    a. If Tara decides to embark on her new venture, what will her accounting costs be during the first year of operation? Her implicit costs? Her opportunity costs?
    b. Suppose that Tara's estimated selling price is lower than originally projected during the first year. How much revenue would she need in order to earn positive accounting profits? Positive economic profits?

9.63 Let $Y_1, Y_2, \ldots, Y_n$ be a random sample from a population with density function

$$f(y \mid \theta) = \begin{cases} \dfrac{3y^2}{\theta^3}, & 0 \le y \le \theta, \\ 0, & \text{elsewhere.} \end{cases}$$

Exerc

In Exercise 9.52 you showed that $Y_{(n)} = \max(Y_1, Y_2, \ldots, Y_n)$ is sufficient for θ.

a Show that $Y_{(n)}$ has probability density function

$$f_{(n)}(y \mid \theta) = \begin{cases} \dfrac{3ny^{3n-1}}{\theta^{3n}}, & 0 \le y \le \theta, \\ 0, & \text{elsewhere.} \end{cases}$$

b Find the MVUE of θ.

15. Approximately 14 million Americans are addicted to drugs and alcohol. The federal government estimates that these addicts cost the U.S. economy $300 billion in medical expenses and lost productivity. Despite the enormous potential market, many biotech companies have shied away from funding research and development (R&D) initiatives to find a cure for drug and alcohol addiction. Your firm—DrugAbuse Sciences (DAS)—is a notable exception. It has spent $170 million to date working on a cure, but is now at a crossroads. It can either abandon its program or invest another $30 million today. Unfortunately, the firm's opportunity cost of funds is 7 percent and it will take another five years before final approval from the Federal Drug Administration is achieved and the product is actually sold. Expected (year-end) profits from selling the drug are presented in the accompanying table. Should DAS continue with its plan to bring the drug to market, or should it abandon the project? Explain.

Year-End Profit Projections

Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9
$0	$0	$0	$0	$15,000,000	$16,500,000	$18,150,000	$19,965,000	$21,961,500

***9.28** Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample of size n from a Pareto distribution (see Exercise 6.18). Then the methods of Section 6.7 imply that $Y_{(1)} = \min(Y_1, Y_2, \ldots, Y_n)$ has the distribution function given by

$$F_{(1)}(y) = \begin{cases} 0, & y \leq \beta, \\ 1 - (\beta/y)^{\alpha n}, & y > \beta. \end{cases}$$

Use the method described in Exercise 9.26 to show that $Y_{(1)}$ is a consistent estimator of β.

1. In return for payments of \$2000 at the end of four years and \$5000 at the end of ten years, an investor agrees to pay \$3000 immediately and to make an additional payment at the end of three years. Find the amount of the additional payment if $i^{(4)} = .06$.

2. You have an inactive credit cart with a \$1000 outstanding unpaid balance. This particular credit card charges interest at the rate of 18% compounded monthly. You are able to make a payment of \$200 one month from today and \$300 two months from today. Find the amount that you will have to pay three months from today to completely pay off this credit card debt. (Note: Work this problem with an equation of value. You will learn an alternative approach for this type of problem in Chapter 5.)

3. At a certain interest rate the present value of the following two payment patterns are equal:

   (*i*) \$200 at the end of 5 years plus \$500 at the end of 10 years.

   (*ii*) \$400.94 at the end of 5 years.

   At the same interest rate \$100 invested now plus \$120 invested at the end of 5 years will accumulate to P at the end of 10 years. Calculate P.

4. An investor makes three deposits into a fund, at the end of 1, 3, and 5 years. The amount of the deposit at time t is $100(1.025)^t$. Find the size of the fund at the end of 7 years, if the nominal rate of discount convertible quarterly is 4/41.

9.69 Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from the probability density function

$$f(y \mid \theta) = \begin{cases} (\theta+1)y^{\theta}, & 0 < y < 1;\ \theta > -1, \\ 0, & \text{elsewhere.} \end{cases}$$

Find an estimator for θ by the method of moments. Show that the estimator is consistent. Is the estimator a function of the sufficient statistic $-\sum_{i=1}^{n} \ln(Y_i)$ that we can obtain from the factorization criterion? What implications does this have?

9.70 Suppose that $Y_1, Y_2, \ldots, Y_n$ constitute a random sample from a Poisson distribution with mean λ. Find the method-of-moments estimator of λ.

9.71 If $Y_1, Y_2, \ldots, Y_n$ denote a random sample from the normal distribution with known mean $\mu = 0$ and unknown variance σ^2, find the method-of-moments estimator of σ^2.

9.72 If $Y_1, Y_2, \ldots, Y_n$ denote a random sample from the normal distribution with mean μ and variance σ^2, find the method-of-moments estimators of μ and σ^2.

9.73 An urn contains θ black balls and $N - \theta$ white balls. A sample of n balls is to be selected without replacement. Let Y denote the number of black balls in the sample. Show that $(N/n)Y$ is the method-of-moments estimator of θ.

9.74 Let $Y_1, Y_2, \ldots, Y_n$ constitute a random sample from the probability density function given by

$$f(y \mid \theta) = \begin{cases} \left(\dfrac{2}{\theta^2}\right)(\theta - y), & 0 \le y \le \theta, \\ 0, & \text{elsewhere.} \end{cases}$$

a Find an estimator for θ by using the method of moments.

b Is this estimator a sufficient statistic for θ?

20. A sum of $10,000 is invested for the months of July and August at 6% simple interest. Find the amount of interest earned:

a) Assuming exact simple interest.

b) Assuming ordinary simple interest.

c) Assuming the Banker's Rule.

21. *a*) Show that the Banker's Rule is always more favorable to the lender than is exact simple interest.

b) Show that the Banker's Rule is usually more favorable to the lender than is ordinary simple interest.

c) Find a counterexample in (*b*) for which the opposite relationship holds.

22. Florida, like several other states, has passed a law that prohibits "price gouging" immediately before, during, or after the declaration of a state of emergency. Price gouging is defined as " . . . selling necessary commodities such as food, gas, ice, oil, and lumber at a price that grossly exceeds the average selling price for the 30 days prior to the emergency." Many consumers attempt to stock up on emergency supplies, such as bottled water, immediately before and after a hurricane or other natural disaster hits an area. Also, many supply shipments to retailers are interrupted during a natural disaster. Assuming that the law is strictly enforced, what are the economic effects of the price gouging statute? Explain carefully.

23. In a recent speech, the governor of your state announced: "One of the biggest causes of juvenile delinquency in this state is the high rate of unemployment among 16 to 19 year olds. The low wages offered by employers in the state have given fewer teenagers the incentive to find summer employment. Instead of working all summer, the way we used to, today's teenagers slack off and cause trouble. To address this problem, I propose to raise the state's minimum wage by $1.50 per hour. This will give teens the proper incentive to go out and find meaningful employment when they are not in school." Evaluate the governor's plan to reduce juvenile delinquency.

9.1 In Exercise 8.8, we considered a random sample of size 3 from an exponential distribution with density function given by

$$f(y) = \begin{cases} (1/\theta)e^{-y/\theta}, & 0 < y, \\ 0, & \text{elsewhere,} \end{cases}$$

and determined that $\hat{\theta}_1 = Y_1$, $\hat{\theta}_2 = (Y_1 + Y_2)/2$, $\hat{\theta}_3 = (Y_1 + 2Y_2)/3$, and $\hat{\theta}_5 = \overline{Y}$ are all unbiased estimators for θ. Find the efficiency of $\hat{\theta}_1$ relative to $\hat{\theta}_5$, of $\hat{\theta}_2$ relative to $\hat{\theta}_5$, and of $\hat{\theta}_3$ relative to $\hat{\theta}_5$.

9.2 Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from a population with mean μ and variance σ^2. Consider the following three estimators for μ:

$$\hat{\mu}_1 = \frac{1}{2}(Y_1 + Y_2), \qquad \hat{\mu}_2 = \frac{1}{4}Y_1 + \frac{Y_2 + \cdots + Y_{n-1}}{2(n-2)} + \frac{1}{4}Y_n, \qquad \hat{\mu}_3 = \overline{Y}.$$

a Show that each of the three estimators is unbiased.

b Find the efficiency of $\hat{\mu}_3$ relative to $\hat{\mu}_2$ and $\hat{\mu}_1$, respectively.

9.3 Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from the uniform distribution on the interval $(\theta, \theta + 1)$. Let

$$\hat{\theta}_1 = \overline{Y} - \frac{1}{2} \quad \text{and} \quad \hat{\theta}_2 = Y_{(n)} - \frac{n}{n+1}.$$

a Show that both $\hat{\theta}_1$ and $\hat{\theta}_2$ are unbiased estimators of θ.

b Find the efficiency of $\hat{\theta}_1$ relative to $\hat{\theta}_2$.

b. The instant after it pays out current profits as dividends.

5. What is the value of a preferred stock that pays a perpetual dividend of $75 at the end of each year when the interest rate is 4 percent?

9.23 Refer to Exercise 9.21. Suppose that $Y_1, Y_2, \ldots, Y_n$ is a random sample of size n from a population for which the first four moments are finite. That is, $m'_1 = E(Y_1) < \infty$, $m'_2 = E(Y_1^2) < \infty$, $m'_3 = E(Y_1^3) < \infty$, and $m'_4 = E(Y_1^4) < \infty$. (*Note:* This assumption is valid for the normal and Poisson distributions in Exercises 9.21 and 9.22, respectively.) Again, assume

that $n = 2k$ for some integer k. Consider

$$\hat{\sigma}^2 = \frac{1}{2k}\sum_{i=1}^{k}(Y_{2i} - Y_{2i-1})^2.$$

a Show that $\hat{\sigma}^2$ is an unbiased estimator for σ^2.

b Show that $\hat{\sigma}^2$ is a consistent estimator for σ^2.

c Why did you need the assumption that $m'_4 = E(Y_1^4) < \infty$?

1. Levi Strauss & Co. paid $46,532 for a 110-year-old pair of Levi's jeans—the oldest known pair of blue jeans—by outbidding several other bidders in an eBay Internet auction. Does this situation best represent producer–producer rivalry, consumer–consumer rivalry, or producer–consumer rivalry? Explain.
2. What is the maximum amount you would pay for an asset that generates an income of $150,000 at the end of each of five years if the opportunity cost of using funds is 9 percent?
3. Suppose that the total benefit and total cost from an activity are, respectively, given by the following equations: $B(Q) = 150 + 28Q - 5Q^2$ and $C(Q) = 100 + 8Q$. (Note: $MB(Q) = 28 - 10Q$ and $MC(Q) = 8$.)
   a. Write out the equation for the net benefits.
   b. What are the net benefits when $Q = 1$? $Q = 5$?
   c. Write out the equation for the marginal net benefits.
   d. What are the marginal net benefits when $Q = 1$? $Q = 5$?
   e. What level of Q maximizes net benefits?
   f. At the value of Q that maximizes net benefits, what is the value of marginal net benefits?
4. A firm's current profits are $550,000. These profits are expected to grow indefinitely at a constant annual rate of 5 percent. If the firm's opportunity cost of funds is 8 percent, determine the value of the firm:
   a. The instant before it pays out current profits as dividends.
   b. The instant after it pays out current profits as dividends.

32. An investor deposits \$10,000 in a bank. During the first year, the bank credits an annual effective rate of interest i. During the second year, the bank credits an annual effective rate of interest $i-.05$. At the end of two years the account balance is \$12,093.75. What would the account balance have been at the end of three years, if the annual effective rate of interest were $i+.09$ for each of the three years? Answer to the nearest dollar.

33. A signs a one-year note for \$1000 and receives \$920 from the bank. At the end of six months A makes a payment of \$288. Assuming simple discount, to what amount does this reduce the face amount of the note?

9.4 Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample of size n from a uniform distribution on the interval $(0, \theta)$. If $Y_{(1)} = \min(Y_1, Y_2, \ldots, Y_n)$, the result of Exercise 8.18 is that $\hat{\theta}_1 = (n+1)Y_{(1)}$ is an unbiased estimator for θ. If $Y_{(n)} = \max(Y_1, Y_2, \ldots, Y_n)$, the results of Example 9.1 imply that $\hat{\theta}_2 = [(n+1)/n]Y_{(n)}$ is another unbiased estimator for θ. Show that the efficiency of $\hat{\theta}_1$ to $\hat{\theta}_2$ is $1/n^2$. Notice that this implies that $\hat{\theta}_2$ is a markedly superior estimator.

9.5 Suppose that $Y_1, Y_2, \ldots, Y_n$ is a random sample from a normal distribution with mean μ and variance σ^2. Two unbiased estimators of σ^2 are

$$\hat{\sigma}_1^2 = S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \overline{Y})^2 \quad \text{and} \quad \hat{\sigma}_2^2 = \frac{1}{2}(Y_1 - Y_2)^2.$$

Find the efficiency of $\hat{\sigma}_1^2$ relative to $\hat{\sigma}_2^2$.

9.6 Suppose that $Y_1, Y_2, \ldots, Y_n$ denote a random sample of size n from a Poisson distribution with mean λ. Consider $\hat{\lambda}_1 = (Y_1 + Y_2)/2$ and $\hat{\lambda}_2 = \overline{Y}$. Derive the efficiency of $\hat{\lambda}_1$ relative to $\hat{\lambda}_2$.

9.7 Suppose that $Y_1, Y_2, \ldots, Y_n$ denote a random sample of size n from an exponential distribution with density function given by

$$f(y) = \begin{cases} (1/\theta)e^{-y/\theta}, & 0 < y, \\ 0, & \text{elsewhere.} \end{cases}$$

9.45 Suppose that $Y_1, Y_2, \ldots, Y_n$ is a random sample from a probability density function in the (one-parameter) exponential family so that

$$f(y \mid \theta) = \begin{cases} a(\theta)b(y)e^{-[c(\theta)d(y)]}, & a \le y \le b, \\ 0, & \text{elsewhere,} \end{cases}$$

where a and b do not depend on θ. Show that $\sum_{i=1}^{n} d(Y_i)$ is sufficient for θ.

9.46 If $Y_1, Y_2, \ldots, Y_n$ denote a random sample from an exponential distribution with mean β, show that $f(y \mid \beta)$ is in the exponential family and that $\overline{Y}$ is sufficient for β.

9.47 Refer to Exercise 9.43. If θ is known, show that the power family of distributions is in the exponential family. What is a sufficient statistic for α? Does this contradict your answer to Exercise 9.43?

9.48 Refer to Exercise 9.44. If β is known, show that the Pareto distribution is in the exponential family. What is a sufficient statistic for α? Argue that there is no contradiction between your answer to this exercise and the answer you found in Exercise 9.44.

***9.49** Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from the uniform distribution over the interval $(0, \theta)$. Show that $Y_{(n)} = \max(Y_1, Y_2, \ldots, Y_n)$ is sufficient for θ.

***9.50** Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from the uniform distribution over the interval (θ_1, θ_2). Show that $Y_{(1)} = \min(Y_1, Y_2, \ldots, Y_n)$ and $Y_{(n)} = \max(Y_1, Y_2, \ldots, Y_n)$ are jointly sufficient for θ_1 and θ_2.

sufficient for θ_1 and θ_2.

***9.51** Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from the probability density function

$$f(y \mid \theta) = \begin{cases} e^{-(y-\theta)}, & y \geq \theta, \\ 0, & \text{elsewhere.} \end{cases}$$

Show that $Y_{(1)} = \min(Y_1, Y_2, \ldots, Y_n)$ is sufficient for θ.

***9.52** Let $Y_1, Y_2, \ldots, Y_n$ be a random sample from a population with density function

$$f(y \mid \theta) = \begin{cases} \dfrac{3y^2}{\theta^3}, & 0 \leq y \leq \theta, \\ 0, & \text{elsewhere.} \end{cases}$$

Show that $Y_{(n)} = \max(Y_1, Y_2, \ldots, Y_n)$ is sufficient for θ.

***9.53** Let $Y_1, Y_2, \ldots, Y_n$ be a random sample from a population with density function

$$f(y \mid \theta) = \begin{cases} \dfrac{2\theta^2}{y^3}, & \theta < y < \infty, \\ 0, & \text{elsewhere.} \end{cases}$$

Show that $Y_{(1)} = \min(Y_1, Y_2, \ldots, Y_n)$ is sufficient for θ.

***9.54** Let $Y_1, Y_2, \ldots, Y_n$ denote independent and identically distributed random variables from a power family distribution with parameters α and θ. Then, as in Exercise 9.43, if $\alpha, \theta > 0$,

$$f(y \mid \alpha, \theta) = \begin{cases} \alpha y^{\alpha-1}/\theta^{\alpha}, & 0 \le y \le \theta, \\ 0, & \text{elsewhere.} \end{cases}$$

Show that $\max(Y_1, Y_2, \ldots, Y_n)$ and $\prod_{i=1}^{n} Y_i$ are jointly sufficient for α and θ.

***9.55** Let $Y_1, Y_2, \ldots, Y_n$ denote independent and identically distributed random variables from a Pareto distribution with parameters α and β. Then, as in Exercise 9.44, if $\alpha, \beta > 0$,

$$f(y \mid \alpha, \beta) = \begin{cases} \alpha\beta^{\alpha} y^{-(\alpha+1)}, & y \ge \beta, \\ 0, & \text{elsewhere.} \end{cases}$$

Show that $\prod_{i=1}^{n} Y_i$ and $\min(Y_1, Y_2, \ldots, Y_n)$ are jointly sufficient for α and β.

9.61 Refer to Exercise 9.49. Use $Y_{(n)}$ to find an MVUE of θ. (See Example 9.1.)

9.62 Refer to Exercise 9.51. Find a function of $Y_{(1)}$ that is an MVUE for θ.

9.24 Let $Y_1,\ Y_2,\ Y_3, \ldots Y_n$ be independent standard normal random variables.

a What is the distribution of $\sum_{i=1}^{n} Y_i^2$?

b Let $W_n = \frac{1}{n}\sum_{i=1}^{n} Y_i^2$. Does W_n *converge in probability* to some constant? If so, what is the value of the constant?

9.25 Suppose that $Y_1,\ Y_2, \ldots,\ Y_n$ denote a random sample of size n from a normal distribution with mean μ and variance 1. Consider the first observation Y_1 as an estimator for μ.

a Show that Y_1 is an unbiased estimator for μ.

b Find $P(|Y_1 - \mu| \leq 1)$.

c Look at the basic definition of consistency given in Definition 9.2. Based on the result of part (b), is Y_1 a consistent estimator for μ?

***9.92** Let $Y_1, Y_2, \ldots, Y_n$ be a random sample from a population with density function

$$f(y \mid \theta) = \begin{cases} \dfrac{3y^2}{\theta^3}, & 0 \le y \le \theta, \\ 0, & \text{elsewhere.} \end{cases}$$

In Exercise 9.52, you showed that $Y_{(n)} = \max(Y_1, Y_2, \ldots, Y_n)$ is sufficient for θ.

a Find the MLE for θ. [*Hint:* See Example 9.16.]

b Find a function of the MLE in part (a) that is a pivotal quantity. [*Hint:* see Exercise 9.63.]

c Use the pivotal quantity from part (b) to find a $100(1 - \alpha)\%$ confidence interval for θ.

***9.26** It is sometimes relatively easy to establish consistency or lack of consistency by appealing directly to Definition 9.2, evaluating $P(|\hat{\theta}_n - \theta| \le \varepsilon)$ directly, and then showing that $\lim_{n\to\infty} P(|\hat{\theta}_n - \theta| \le \varepsilon) = 1$. Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample of size n from a uniform distribution on the interval $(0, \theta)$. If $Y_{(n)} = \max(Y_1, Y_2, \ldots, Y_n)$, we showed in Exercise 6.74 that the probability distribution function of $Y_{(n)}$ is given by

$$F_{(n)}(y) = \begin{cases} 0, & y < 0, \\ (y/\theta)^n, & 0 \le y \le \theta, \\ 1, & y > \theta. \end{cases}$$

a For each $n \ge 1$ and every $\varepsilon > 0$, it follows that $P(|Y_{(n)} - \theta| \le \varepsilon) = P(\theta - \varepsilon \le Y_{(n)} \le \theta + \varepsilon)$. If $\varepsilon > \theta$, verify that $P(\theta - \varepsilon \le Y_{(n)} \le \theta + \varepsilon) = 1$ and that, for every positive $\varepsilon < \theta$, we obtain $P(\theta - \varepsilon \le Y_{(n)} \le \theta + \varepsilon) = 1 - [(\theta - \varepsilon)/\theta]^n$.

b Using the result from part (a), show that $Y_{(n)}$ is a consistent estimator for θ by showing that, for every $\varepsilon > 0$, $\lim_{n\to\infty} P(|Y_{(n)} - \theta| \le \varepsilon) = 1$.

***9.27** Use the method described in Exercise 9.26 to show that, if $Y_{(1)} = \min(Y_1, Y_2, \ldots, Y_n)$ when $Y_1, Y_2, \ldots, Y_n$ are independent uniform random variables on the interval $(0, \theta)$, then $Y_{(1)}$ is *not* a consistent estimator for θ. [*Hint:* Based on the methods of Section 6.7, $Y_{(1)}$ has the distribution function

$$F_{(1)}(y) = \begin{cases} 0, & y < 0, \\ 1 - (1 - y/\theta)^n, & 0 \le y \le \theta, \\ 1, & y > \theta.] \end{cases}$$

5. Whereas the choice of a comparison date has no effect on the answer obtained with compound interest, the same cannot be said of simple interest. Find the amount to be paid at the end of 10 years which is equivalent to two payments of $100 each, the first to be paid immediately and the second to be paid at the end of 5 years. Assume 5% simple interest is earned from the date each payment is made and use a comparison date of:

a) The end of 10 years.

b) The end of 15 years.

9.103 A random sample of size n is taken from a population with a Rayleigh distribution. As in Exercise 9.34, the Rayleigh density function is

$$f(y) = \begin{cases} \left(\dfrac{2y}{\theta}\right)e^{-y^2/\theta}, & y > 0, \\ 0, & \text{elsewhere.} \end{cases}$$

a Find the MLE of θ.

***b** Find the approximate variance of the MLE obtained in part (a).

9.104 Suppose that $Y_1, Y_2, \ldots, Y_n$ constitute a random sample from the density function

$$f(y \mid \theta) = \begin{cases} e^{-(y-\theta)}, & y > \theta, \\ 0, & \text{elsewhere} \end{cases}$$

where θ is an unknown, positive constant.

a Find an estimator $\hat{\theta}_1$ for θ by the method of moments.

b Find an estimator $\hat{\theta}_2$ for θ by the method of maximum likelihood.

c Adjust $\hat{\theta}_1$ and $\hat{\theta}_2$ so that they are unbiased. Find the efficiency of the adjusted $\hat{\theta}_1$ relative to the adjusted $\hat{\theta}_2$.

9.105 Refer to Exercise 9.38(b). Under the conditions outlined there, find the MLE of σ^2.

***9.106** Suppose that $Y_1, Y_2, \ldots, Y_n$ denote a random sample from a Poisson distribution with mean λ. Find the MVUE of $P(Y_i = 0) = e^{-\lambda}$. [*Hint:* Make use of the Rao–Blackwell theorem.]

9.37 Let $X_1, X_2, \ldots, X_n$ denote n independent and identically distributed *Bernoulli* random variables such that

$$P(X_i = 1) = p \quad \text{and} \quad P(X_i = 0) = 1 - p,$$

for each $i = 1, 2, \ldots, n$. Show that $\sum_{i=1}^{n} X_i$ is sufficient for p by using the factorization criterion given in Theorem 9.4.

9.38 Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from a normal distribution with mean μ and variance σ^2.

a If μ is unknown and σ^2 is known, show that $\overline{Y}$ is sufficient for μ.

b If μ is known and σ^2 is unknown, show that $\sum_{i=1}^{n}(Y_i - \mu)^2$ is sufficient for σ^2.

c If μ and σ^2 are both unknown, show that $\sum_{i=1}^{n} Y_i$ and $\sum_{i=1}^{n} Y_i^2$ are jointly sufficient for μ and σ^2. [Thus, it follows that $\overline{Y}$ and $\sum_{i=1}^{n}(Y_i - \overline{Y})^2$ or $\overline{Y}$ and S^2 are also jointly sufficient for μ and σ^2.]

27. A bank offers the following certificates of deposit (CDs):

Term in years	Nominal annual interest rate (convertible semiannually)
1	5%
2	6%
3	7%
4	8%

The bank does not permit early withdrawal, and all CDs mature at the end of the term. During the next six years the bank will continue to offer these CDs. An investor deposits $1000 in the bank. Calculate the maximum amount that can be withdrawn at the end of six years.

Miscellaneous problems

28. A store is running a promotion during which customers have two options for payment. Option One is to pay 90% of the purchase price two months after the date of sale. Option Two is to deduct X % off the purchase price and pay cash on the date of sale. Determine X such that a customer would be indifferent between the two options when valuing them using an effective annual interest rate of 8%.

29. A manufacturer sells a product to a retailer who has the option of paying 30% below the retail price immediately, or 25% below the retail price in six months. Find the annual effective rate of interest at which the retailer would be indifferent between the two options.

9. The supply curve for product X is given by $Q_x^s = -340 + 10P_x$.
   a. Find the inverse supply curve.
   b. How much surplus do producers receive when $Q_x = 350$? When $Q_x = 1{,}000$?
10. Consider a market where supply and demand are given by $Q_x^s = -10 + P_x$ and $Q_x^d = 56 - 2P_x$. Suppose the government imposes a price floor of \$25, and agrees to purchase any and all units consumers do not buy at the floor price of \$25 per unit.
   a. Determine the cost to the government of buying firms' unsold units.
   b. Compute the lost social welfare (deadweight loss) that stems from the \$25 price floor.

9.83 Suppose that $Y_1, Y_2, \ldots, Y_n$ constitute a random sample from a uniform distribution with probability density function

$$f(y \mid \theta) = \begin{cases} \dfrac{1}{2\theta + 1}, & 0 \le y \le 2\theta + 1, \\ 0, & \text{otherwise.} \end{cases}$$

a Obtain the MLE of θ.

b Obtain the MLE for the *variance* of the underlying distribution.

9.84 A certain type of electronic component has a lifetime Y (in hours) with probability density function given by

$$f(y \mid \theta) = \begin{cases} \left(\dfrac{1}{\theta^2}\right) y e^{-y/\theta}, & y > 0, \\ 0, & \text{otherwise.} \end{cases}$$

That is, Y has a gamma distribution with parameters $\alpha = 2$ and θ. Let $\hat{\theta}$ denote the MLE of θ. Suppose that three such components, tested independently, had lifetimes of 120, 130, and 128 hours.

a Find the MLE of θ.

b Find $E(\hat{\theta})$ and $V(\hat{\theta})$.

c Suppose that θ actually equals 130. Give an approximate bound that you might expect for the error of estimation.

d What is the MLE for the variance of Y?

9.88 Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from the probability density function

$$f(y \mid \theta) = \begin{cases} (\theta + 1)y^{\theta}, & 0 < y < 1,\ \theta > -1, \\ 0, & \text{elsewhere.} \end{cases}$$

Find the MLE for θ. Compare your answer to the method-of-moments estimator found in Exercise 9.69.

9.89 It is known that the probability p of tossing heads on an unbalanced coin is either 1/4 or 3/4. The coin is tossed twice and a value for Y, the number of heads, is observed. For each possible value of Y, which of the two values for p (1/4 or 3/4) maximizes the probability that $Y = y$? Depending on the value of y actually observed, what is the MLE of p?

9.90 A random sample of 100 men produced a total of 25 who favored a controversial local issue. An independent random sample of 100 women produced a total of 30 who favored the issue. Assume that p_M is the true underlying proportion of men who favor the issue and that p_W is the true underlying proportion of women who favor of the issue. If it actually is true that $p_W = p_M = p$, find the MLE of the common proportion p.

***9.91** Find the MLE of θ based on a random sample of size n from a uniform distribution on the interval $(0, 2\theta)$.

1. The X-Corporation produces a good (called *X*) that is a normal good. Its competitor, Y-Corp., makes a substitute good that it markets under the name "*Y*." Good *Y* is an inferior good.
   a. How will the demand for good *X* change if consumer incomes increase?
   b. How will the demand for good *Y* change if consumer incomes decrease?
   c. How will the demand for good *X* change if the price of good *Y* decreases?
   d. Is good *Y* a lower-quality product than good *X*? Explain.
2. Good *X* is produced in a competitive market using input A. Explain what would happen to the supply of good *X* in each of the following situations:
   a. The price of input A increases.
   b. An excise tax of $1 is imposed on good *X*.

   c. An ad valorem tax of 5 percent is imposed on good *X*.
   d. A technological change reduces the cost of producing additional units of good *X*.

21. Brazil points to its shrimp-farming industry as an example of how it can compete in world markets. One decade ago, Brazil exported a meager 400 tons of shrimp. Today, Brazil exports more than 58,000 tons of shrimp, with approximately one-third of that going to the United States. Brazilian shrimp farmers, however, potentially face a new challenge in the upcoming years. The Southern Shrimp Alliance—a U.S. organization representing shrimpers—filed a dumping complaint alleging that Brazil and five other shrimp-producing countries are selling shrimp below "fair market value." The organization is calling for the United States to impose a 300 percent tariff on all shrimp entering the United States' borders. Brazilian producers and the other five countries named in the complaint counter that they have a natural competitive advantage such as lower labor costs, availability of cheap land, and a more favorable climate, resulting in a higher yield per acre and permitting three harvests per year. In what many see as a bold move, the American Seafood Distributors Association—an organization representing supermarkets, shrimp processors, and restaurants—has supported Brazilian and other foreign producers, arguing that it is the Southern Shrimp Alliance that is engaging in unfair trade practices. Describe the various rivalries depicted in this scenario, and then use the five forces framework to analyze the industry.

***9.98** Refer to Exercise 9.97. What is the approximate variance of the MLE?

***9.99** Consider the distribution discussed in Example 9.18. Use the method presented in Section 9.8 to derive a $100(1-\alpha)\%$ confidence interval for $t(p) = p$. Is the resulting interval familiar to you?

***9.100** Suppose that $Y_1, Y_2, \ldots, Y_n$ constitute a random sample of size n from an exponential distribution with mean θ. Find a $100(1-\alpha)\%$ confidence interval for $t(\theta) = \theta^2$.

***9.101** Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample of size n from a Poisson distribution with mean λ. Find a $100(1-\alpha)\%$ confidence interval for $t(\lambda) = e^{-\lambda} = P(Y = 0)$.

***9.102** Refer to Exercises 9.97 and 9.98. If a sample of size 30 yields $\overline{y} = 4.4$, find a 95% confidence interval for p.

***9.109** Suppose that n integers are drawn at random and *with replacement* from the integers $1, 2, \ldots, N$. That is, each sampled integer has probability $1/N$ of taking on any of the values $1, 2, \ldots, N$, and the sampled values are independent.

a Find the method-of-moments estimator $\hat{N}_1$ of N.

b Find $E(\hat{N}_1)$ and $V(\hat{N}_1)$.

***9.110** Refer to Exercise 9.109.

a Find the MLE $\hat{N}_2$ of N.

b Show that $E(\hat{N}_2)$ is approximately $[n/(n+1)]N$. Adjust $\hat{N}_2$ to form an estimator $\hat{N}_3$ that is approximately unbiased for N.

c Find an approximate variance for $\hat{N}_3$ by using the fact that for large N the variance of the largest sampled integer is approximately

$$\frac{nN^2}{(n+1)^2(n+2)}.$$

d Show that for large N and $n > 1$, $V(\hat{N}_3) < V(\hat{N}_1)$.

***9.111** Refer to Exercise 9.110. Suppose that enemy tanks have serial numbers $1, 2, \ldots, N$. A spy randomly observed five tanks (with replacement) with serial numbers 97, 64, 118, 210, and 57. Estimate N and place a bound on the error of estimation.

16. You are an assistant to a senator who chairs an ad hoc committee on reforming taxes on telecommunication services. Based on your research, AT&T has spent over \$15 million on related paperwork and compliance costs. Moreover, depending on the locale, telecom taxes can amount to as much as 25 percent of a consumer's phone bill. These high tax rates on telecom services have become quite controversial, due to the fact that the deregulation of the telecom industry has led to a highly competitive market. Your best estimates indicate that, based on current tax rates, the monthly market demand for telecommunication services is given by $Q^d = 250 - 5P$ and the market supply (including taxes) is $Q^s = 4P - 110$ (both in millions), where P is the monthly price of telecommunication services. The senator is considering tax reform that would dramatically cut tax rates, leading to a supply function under the new tax policy of $Q^s = 4.171P - 110$. How much money would a typical consumer save each month as a result of the proposed legislation?

17. G.R. Dry Foods Distributors specializes in the wholesale distribution of dry goods, such as rice and dry beans. The firm's manager is concerned about an article he read in this morning's *The Wall Street Journal* indicating that the incomes of individuals in the lowest income bracket are expected to increase by 10 percent over the next year. While the manager is pleased to see this group of individuals doing well, he is concerned about the impact this will have on G.R. Dry Foods. What do you think is likely to happen to the price of the products G.R. Dry Foods sells? Why?

6. Find how long \$1000 should be left to accumulate at 6% effective in order that it will amount to twice the accumulated value of another \$1000 deposited at the same time at 4% effective.

7. You invest \$3000 today and plan to invest another \$2000 two years from today. You plan to withdraw \$5000 in n years and another \$5000 in $n+5$ years, exactly liquidating your investment account at that time. If the effective rate of discount is equal to 6%, find n.

8. The present value of two payments of \$100 each to be made at the end of n years and $2n$ years is \$100. If $i = .08$, find n.

9. A payment of n is made at the end of n years, $2n$ at the end of $2n$ years, ..., n^2 at the end of n^2 years. Find the value of t by the method of equated time.

10. You are asked to develop a *rule of n* to approximate how long it takes money to triple. Find n, where n is a positive integer.

11. A deposits 10 today and another 30 in five years into a fund paying simple interest of 11% per year. B will make the same two deposits, but the 10 will be deposited n years from today and the 30 will be deposited $2n$ years from today. B's deposits earn an annual effective rate of 9.15%. At the end of 10 years, the accumulated value of B's deposits equals the accumulated value of A's deposits. Calculate n.

***9.65** In this exercise, we illustrate the direct use of the Rao–Blackwell theorem. Let $Y_1, Y_2, \dots, Y_n$ be independent Bernoulli random variables with

$$p(y_i \mid p) = p^{y_i}(1-p)^{1-y_i}, \qquad y_i = 0, 1.$$

That is, $P(Y_i = 1) = p$ and $P(Y_i = 0) = 1 - p$. Find the MVUE of $p(1-p)$, which is a term in the variance of Y_i or $W = \sum_{i=1}^{n} Y_i$, by the following steps.

a Let

$$T = \begin{cases} 1, & \text{if } Y_1 = 1 \text{ and } Y_2 = 0, \\ 0, & \text{otherwise.} \end{cases}$$

Show that $E(T) = p(1-p)$.

b Show that

$$P(T = 1 \mid W = w) = \frac{w(n-w)}{n(n-1)}.$$

c Show that

$$E(T \mid W) = \frac{n}{n-1}\left[\frac{W}{n}\left(1 - \frac{W}{n}\right)\right] = \frac{n}{n-1}\overline{Y}(1-\overline{Y})$$

and hence that $n\overline{Y}(1-\overline{Y})/(n-1)$ is the MVUE of $p(1-p)$.

9.112 Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from a Poisson distribution with mean λ and define

$$W_n = \frac{\overline{Y} - \lambda}{\sqrt{\overline{Y}/n}}.$$

a Show that the distribution of W_n converges to a standard normal distribution.

b Use W_n and the result in part (a) to derive the formula for an approximate 95% confidence interval for λ.

9.64 Let $Y_1, Y_2, \ldots, Y_n$ be a random sample from a normal distribution with mean μ and variance 1.

a Show that the MVUE of μ^2 is $\widehat{\mu^2} = \overline{Y}^2 - 1/n$.

b Derive the variance of $\widehat{\mu^2}$.

9.96 Consider a random sample of size n from a normal population with mean μ and variance σ^2, both unknown. Derive the MLE of σ.

9.97 The geometric probability mass function is given by

$$p(y \mid p) = p(1-p)^{y-1}, \qquad y = 1, 2, 3, \ldots.$$

A random sample of size n is taken from a population with a geometric distribution.

a Find the method-of-moments estimator for p.

b Find the MLE for p.

30. You deposit \$1000 into a bank account. The bank credits interest at a nominal annual rate of i convertible semiannually for the first 7 years and a nominal annual of $2i$ convertible quarterly for all years thereafter. The accumulated amount in the account at the end of 5 years is X. The accumulated amount in the account at the end of 10.5 years is 1980. Calculate X to the nearest dollar.

31. Fund A accumulates at 6% effective and Fund B accumulates at 8% effective. At the end of 20 years the total of the two funds is \$2000. At the end of 10 years the amount in Fund A is half that in Fund B. What is the total of the two funds at the end of 5 years? Answer to the nearest dollar.

***9.67** Refer to Exercise 9.66. Suppose that a sample of size n is taken from a normal population with mean μ and variance σ^2. Show that $\sum_{i=1}^{n} Y_i$, and $\sum_{i=1}^{n} Y_i^2$ jointly form minimal sufficient statistics for μ and σ^2.

***9.68** Suppose that a statistic U has a probability density function that is positive over the interval $a \le u \le b$ and suppose that the density depends on a parameter θ that can range over the interval $\alpha_1 \le \theta \le \alpha_2$. Suppose also that $g(u)$ is continuous for u in the interval $[a, b]$. If $E[g(U) \mid \theta] = 0$ for all θ in the interval $[\alpha_1, \alpha_2]$ implies that $g(u)$ is identically zero, then the family of density functions $\{f_U(u \mid \theta),\ \alpha_1 \le \theta \le \alpha_2\}$ is said to be *complete*. (All statistics that we employed in Section 9.5 have complete families of density functions.) Suppose that U is a sufficient statistic for θ, and $g_1(U)$ and $g_2(U)$ are both unbiased estimators of θ. Show that, if the family of density functions for U is complete, $g_1(U)$ must equal $g_2(U)$, and thus there is a *unique* function of U that is an unbiased estimator of θ.

Coupled with the Rao–Blackwell theorem, the property of completeness of $f_U(u \mid \theta)$, along with the sufficiency of U, assures us that there is a unique minimum-variance unbiased estimator (UMVUE) of θ.

3. Suppose the supply function for product X is given by $Q_x^s = -50 + 0.5P_x - 5P_z$.
   a. How much of product X is produced when $P_x = \$500$ and $P_z = \$30$?
   b. How much of product X is produced when $P_x = \$50$ and $P_z = \$30$?
   c. Suppose $P_z = \$30$. Determine the supply function and inverse supply function for good X. Graph the inverse supply function.
4. The demand for good X is given by

$$Q_x^d = 1{,}200 - \frac{1}{2}P_x + \frac{1}{4}P_y - 8P_z + \frac{1}{10}M$$

   Research shows that the prices of related goods are given by $P_y = \$5{,}900$ and $P_z = \$90$, while the average income of individuals consuming this product is $M = \$55{,}000$.
   a. Indicate whether goods Y and Z are substitutes or complements for good X.
   b. Is X an inferior or a normal good?
   c. How many units of good X will be purchased when $P_x = \$4{,}910$?
   d. Determine the demand function and inverse demand function for good X. Graph the demand curve for good X.
5. The demand curve for product X is given by $Q_x^d = 460 - 4P_x$.
   a. Find the inverse demand curve.
   b. How much consumer surplus do consumers receive when $P_x = \$35$?
   c. How much consumer surplus do consumers receive when $P_x = \$25$?
   d. In general, what happens to the level of consumer surplus as the price of a good falls?

9.39 Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from a Poisson distribution with parameter λ. Show by conditioning that $\sum_{i=1}^{n} Y_i$ is sufficient for λ.

9.40 Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from a Rayleigh distribution with parameter θ. (Refer to Exercise 9.34.) Show that $\sum_{i=1}^{n} Y_i^2$ is sufficient for θ.

9.41 Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from a Weibull distribution with known m and unknown α. (Refer to Exercise 6.26.) Show that $\sum_{i=1}^{n} Y_i^m$ is sufficient for α.

9.42 If $Y_1, Y_2, \ldots, Y_n$ denote a random sample from a geometric distribution with parameter p, show that $\overline{Y}$ is sufficient for p.

9.43 Let $Y_1, Y_2, \ldots, Y_n$ denote independent and identically distributed random variables from a power family distribution with parameters α and θ. Then, by the result in Exercise 6.17, if $\alpha, \theta > 0$,

$$f(y \mid \alpha, \theta) = \begin{cases} \alpha y^{\alpha-1}/\theta^{\alpha}, & 0 \le y \le \theta, \\ 0, & \text{elsewhere.} \end{cases}$$

If θ is known, show that $\prod_{i=1}^{n} Y_i$ is sufficient for α.

9.44 Let $Y_1, Y_2, \ldots, Y_n$ denote independent and identically distributed random variables from a Pareto distribution with parameters α and β. Then, by the result in Exercise 6.18, if $\alpha, \beta > 0$,

$$f(y \mid \alpha, \beta) = \begin{cases} \alpha\beta^{\alpha} y^{-(\alpha+1)}, & y \ge \beta, \\ 0, & \text{elsewhere.} \end{cases}$$

If β is known, show that $\prod_{i=1}^{n} Y_i$ is sufficient for α.

18. Suppose one of your clients is four years away from retirement and has only \$1,500 in pretax income to devote to either a Roth or a traditional IRA. The traditional IRA permits investors to contribute the full \$1,500 since contributions to these accounts are tax-deductible, but they must pay taxes on all future distributions. In contrast, contributions to a Roth IRA are not tax-deductible, meaning that at a tax rate of 25 percent, an investor is able to contribute only \$1,125 after taxes; however, the earnings of a Roth IRA grow tax-free. Your company has decided to waive the one-time set-up fee of \$25 to

open a Roth IRA; however, investors opening a traditional IRA must pay the \$25 set-up fee. Assuming that your client anticipates that her tax rate will remain at 17 percent in retirement and will earn a stable 8 percent return on her investments, will she prefer a traditional or a Roth IRA?

9.80 Suppose that $Y_1, Y_2, \ldots, Y_n$ denote a random sample from the Poisson distribution with mean λ.

a Find the MLE $\hat{\lambda}$ for λ.

b Find the expected value and variance of $\hat{\lambda}$.

c Show that the estimator of part (a) is consistent for λ.

d What is the MLE for $P(Y = 0) = e^{-\lambda}$?

9.81 Suppose that $Y_1, Y_2, \ldots, Y_n$ denote a random sample from an exponentially distributed population with mean θ. Find the MLE of the population variance θ^2. [*Hint:* Recall Example 9.9.]

9.82 Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from the density function given by

$$f(y \mid \theta) = \begin{cases} \left(\dfrac{1}{\theta}\right) r y^{r-1} e^{-y^r/\theta}, & \theta > 0, y > 0, \\ 0, & \text{elsewhere,} \end{cases}$$

where r is a known positive constant.

a Find a sufficient statistic for θ.

b Find the MLE of θ.

c Is the estimator in part (b) an MVUE for θ?

***9.106** Suppose that $Y_1, Y_2, \ldots, Y_n$ denote a random sample from a Poisson distribution with mean λ. Find the MVUE of $P(Y_i = 0) = e^{-\lambda}$. [*Hint:* Make use of the Rao–Blackwell theorem.]

9.107 Suppose that a random sample of length-of-life measurements, $Y_1, Y_2, \ldots, Y_n$, is to be taken of components whose length of life has an exponential distribution with mean θ. It is frequently of interest to estimate

$$\overline{F}(t) = 1 - F(t) = e^{-t/\theta},$$

the *reliability* at time t of such a component. For any fixed value of t, find the MLE of $\overline{F}(t)$.

9.77 Let $Y_1, Y_2, \ldots, Y_n$ denote independent and identically distributed uniform random variables on the interval $(0, 3\theta)$. Derive the method-of-moments estimator for θ.

9.78 Let $Y_1, Y_2, \ldots, Y_n$ denote independent and identically distributed random variables from a power family distribution with parameters α and $\theta = 3$. Then, as in Exercise 9.43, if $\alpha > 0$,

$$f(y|\alpha) = \begin{cases} \alpha y^{\alpha-1}/3^{\alpha}, & 0 \le y \le 3, \\ 0, & \text{elsewhere.} \end{cases}$$

Show that $E(Y_1) = 3\alpha/(\alpha + 1)$ and derive the method-of-moments estimator for α.

***9.79** Let $Y_1, Y_2, \ldots, Y_n$ denote independent and identically distributed random variables from a Pareto distribution with parameters α and β, where β is known. Then, if $\alpha > 0$,

$$f(y|\alpha, \beta) = \begin{cases} \alpha\beta^{\alpha} y^{-(\alpha+1)}, & y \ge \beta, \\ 0, & \text{elsewhere.} \end{cases}$$

Show that $E(Y_i) = \alpha\beta/(\alpha - 1)$ if $\alpha > 1$ and $E(Y_i)$ is undefined if $0 < \alpha < 1$. Thus, the method-of-moments estimator for α is undefined.

12. You are in the market for a new refrigerator for your company's lounge, and you have narrowed the search down to two models. The energy efficient

model sells for $500 and will save you $25 at the end of each of the next five years in electricity costs. The standard model has features similar to the energy efficient model but provides no future saving in electricity costs. It is priced at only $400. Assuming your opportunity cost of funds is 5 percent, which refrigerator should you purchase?

15. You can receive one of the following two payment streams:

    (*i*) 100 at time 0, 200 at time n, and 300 at time $2n$.

    (*ii*) 600 at time 10.

    At an annual effective interest rate of i, the present values of the two streams are equal. Given $v^n = 0.75941$, determine i.

16. It is known that an investment of \$1000 will accumulate to \$1825 at the end of 10 years. If it is assumed that the investment earns simple interest at rate i during the 1st year, $2i$ during the 2nd year, …, $10i$ during the 10th year, find i.

17. It is known that an amount of money will double itself in 10 years at a varying force of interest $\delta_t = kt$. Find an expression for k.

18. The sum of the accumulated value of 1 at the end of three years at a certain effective rate of interest i, and the present value of 1 to be paid at the end of three years at an effective rate of discount numerically equal to i is 2.0096. Find the rate i.

2.6 Determining time periods

19. If an investment was made on the day the United States entered World War II, i.e. December 7, 1941, and was terminated at the end of the war on August 8, 1945, for how many days was the money invested:

    a) On the actual/actual basis?

    b) On the 30/360 basis?

22. You are the manager of Local Electronics Shop (LES), a small brick-and-mortar retail camera and electronics store. One of your employees proposed a new online strategy whereby LES lists its products at Pricesearch.com—a price comparison Web site that allows consumers to view the prices of dozens of retailers selling the same items. Would you expect this strategy to enable LES to achieve sustainable economic profits? Explain.
23. Two months ago, the owner of a car dealership (and a current football star) significantly changed his sales manager's compensation plan. Under the old plan, the manager was paid a salary of $6,000 per month; under the new plan, she receives 2 percent of the sales price of each car sold. During the past two months, the number of cars sold increased by 40 percent, but the dealership's margins (and profits) significantly declined. According to the sales manager, "Consumers are driving harder bargains and I have had to authorize significantly lower prices to remain competitive." What advice would you give the owner of the dealership?

c Under what condition (on the σ_i^2's) can Theorem 9.1 be applied to show that $\overline{Y}_n$ is a consistent estimator for μ?

9.36 Suppose that Y has a binomial distribution based on n trials and success probability p. Then $\hat{p}_n = Y/n$ is an unbiased estimator of p. Use Theorem 9.3 to prove that the distribution of

$(\hat{p}_n - p)/\sqrt{\hat{p}_n\hat{q}_n/n}$ converges to a standard normal distribution. [*Hint:* Write Y as we did in Section 7.5.]

25. Many banks quote two rates of interest on certificates of deposit (CDs). If a bank quotes 5.1% compounded daily, find the ratio of the APY (annual percentage yield) to the quoted rate for this CD.

26. A savings and loan association pays 7% effective on deposits at the end of each year. At the end of every three years a 2% bonus is paid on the balance at that time. Find the effective rate of interest earned by an investor if the money is left on deposit:

a) Two years.

b) Three years.

c) Four years.

6. Suppose demand and supply are given by $Q^d = 50 - P$ and $Q^s = \frac{1}{2}P - 10$.
   a. What are the equilibrium quantity and price in this market?
   b. Determine the quantity demanded, the quantity supplied, and the magnitude of the surplus if a price floor of \$42 is imposed in this market.
   c. Determine the quantity demanded, the quantity supplied, and the magnitude of the shortage if a price ceiling of \$30 is imposed in this market. Also, determine the full economic price paid by consumers.
7. Suppose demand and supply are given by

$$Q_x^d = 7 - \frac{1}{2}P_x \quad \text{and} \quad Q_x^s = \frac{1}{4}P_x - \frac{1}{2}$$

   a. Determine the equilibrium price and quantity. Show the equilibrium graphically.
   b. Suppose a \$6 excise tax is imposed on the good. Determine the new equilibrium price and quantity.
   c. How much tax revenue does the government earn with the \$6 tax?
8. Use the accompanying graph to answer these questions.
   a. Suppose demand is D and supply is S^0. If a price ceiling of \$6 is imposed, what are the resulting shortage and full economic price?

9.1 In Exercise 8.8, we considered a random sample of size 3 from an exponential distribution with density function given by

$$f(y) = \begin{cases} (1/\theta)e^{-y/\theta}, & 0 < y, \\ 0, & \text{elsewhere}, \end{cases}$$

and determined that $\hat{\theta}_1 = Y_1$, $\hat{\theta}_2 = (Y_1 + Y_2)/2$, $\hat{\theta}_3 = (Y_1 + 2Y_2)/3$, and $\hat{\theta}_5 = \overline{Y}$ are all unbiased estimators for θ. Find the efficiency of $\hat{\theta}_1$ relative to $\hat{\theta}_5$, of $\hat{\theta}_2$ relative to $\hat{\theta}_5$, and of $\hat{\theta}_3$ relative to $\hat{\theta}_5$.

9.2 Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from a population with mean μ and variance σ^2. Consider the following three estimators for μ:

$$\hat{\mu}_1 = \frac{1}{2}(Y_1 + Y_2), \qquad \hat{\mu}_2 = \frac{1}{4}Y_1 + \frac{Y_2 + \cdots + Y_{n-1}}{2(n-2)} + \frac{1}{4}Y_n, \qquad \hat{\mu}_3 = \overline{Y}.$$

a Show that each of the three estimators is unbiased.

b Find the efficiency of $\hat{\mu}_3$ relative to $\hat{\mu}_2$ and $\hat{\mu}_1$, respectively.

9.3 Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from the uniform distribution on the interval $(\theta, \theta + 1)$. Let

$$\hat{\theta}_1 = \overline{Y} - \frac{1}{2} \quad \text{and} \quad \hat{\theta}_2 = Y_{(n)} - \frac{n}{n+1}.$$

a Show that both $\hat{\theta}_1$ and $\hat{\theta}_2$ are unbiased estimators of θ.

b Find the efficiency of $\hat{\theta}_1$ relative to $\hat{\theta}_2$.

***9.8** Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from a probability density function $f(y)$, which has unknown parameter θ. If $\hat{\theta}$ is an unbiased estimator of θ, then under very general conditions

$$V(\hat{\theta}) \geq I(\theta), \quad \text{where } I(\theta) = \left[nE\left(-\frac{\partial^2 \ln f(Y)}{\partial \theta^2} \right) \right]^{-1}.$$

(This is known as the Cramer–Rao inequality.) If $V(\hat{\theta}) = I(\theta)$, the estimator $\hat{\theta}$ is said to be *efficient.*[1]

a Suppose that $f(y)$ is the normal density with mean μ and variance σ^2. Show that $\overline{Y}$ is an efficient estimator of μ.

b This inequality also holds for discrete probability functions $p(y)$. Suppose that $p(y)$ is the Poisson probability function with mean λ. Show that $\overline{Y}$ is an efficient estimator of λ.

9.15 Refer to Exercise 9.3. Show that both $\hat{\theta}_1$ and $\hat{\theta}_2$ are consistent estimators for θ.

9.16 Refer to Exercise 9.5. Is $\hat{\sigma}_2^2$ a consistent estimator of σ^2?

9.17 Suppose that $X_1, X_2, \ldots, X_n$ and $Y_1, Y_2, \ldots, Y_n$ are independent random samples from populations with means μ_1 and μ_2 and variances σ_1^2 and σ_2^2, respectively. Show that $\overline{X} - \overline{Y}$ is a consistent estimator of $\mu_1 - \mu_2$.

9.18 In Exercise 9.17, suppose that the populations are normally distributed with $\sigma_1^2 = \sigma_2^2 = \sigma^2$. Show that

$$\frac{\sum_{i=1}^n (X_i - \overline{X})^2 + \sum_{i=1}^n (Y_i - \overline{Y})^2}{2n - 2}$$

is a consistent estimator of σ^2.

9.19 Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from the probability density function

$$f(y) = \begin{cases} \theta y^{\theta - 1}, & 0 < y < 1, \\ 0, & \text{elsewhere,} \end{cases}$$

where $\theta > 0$. Show that $\overline{Y}$ is a consistent estimator of $\theta/(\theta + 1)$.

9.20 If Y has a binomial distribution with n trials and success probability p, show that Y/n is a consistent estimator of p.

9.56 Refer to Exercise 9.38(b). Find an MVUE of σ^2.

9.57 Refer to Exercise 9.18. Is the estimator of σ^2 given there an MVUE of σ^2?

9.58 Refer to Exercise 9.40. Use $\sum_{i=1}^{n} Y_i^2$ to find an MVUE of θ.

9.59 The number of breakdowns Y per day for a certain machine is a Poisson random variable with mean λ. The daily cost of repairing these breakdowns is given by $C = 3Y^2$. If $Y_1, Y_2, \ldots, Y_n$ denote the observed number of breakdowns for n independently selected days, find an MVUE for $E(C)$.

9.60 Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample from the probability density function

$$f(y \mid \theta) = \begin{cases} \theta y^{\theta-1}, & 0 < y < 1, \ \theta > 0, \\ 0, & \text{elsewhere.} \end{cases}$$

a Show that this density function is in the (one-parameter) exponential family and that $\sum_{i=1}^{n} -\ln(Y_i)$ is sufficient for θ. (See Exercise 9.45.)

b If $W_i = -\ln(Y_i)$, show that W_i has an exponential distribution with mean $1/\theta$.

c Use methods similar to those in Example 9.10 to show that $2\theta \sum_{i=1}^{n} W_i$ has a χ^2 distribution with $2n$ df.

d Show that

$$E\left(\frac{1}{2\theta \sum_{i=1}^{n} W_i}\right) = \frac{1}{2(n-1)}.$$

[*Hint:* Recall Exercise 4.112.]

e What is the MVUE for θ?

5. Whereas the choice of a comparison date has no effect on the answer obtained with compound interest, the same cannot be said of simple interest. Find the amount to be paid at the end of 10 years which is equivalent to two payments of \$100 each, the first to be paid immediately and the second to be paid at the end of 5 years. Assume 5% simple interest is earned from the date each payment is made and use a comparison date of:

a) The end of 10 years.

b) The end of 15 years.

9.4 Let $Y_1, Y_2, \ldots, Y_n$ denote a random sample of size n from a uniform distribution on the interval $(0, \theta)$. If $Y_{(1)} = \min(Y_1, Y_2, \ldots, Y_n)$, the result of Exercise 8.18 is that $\hat{\theta}_1 = (n+1)Y_{(1)}$ is an unbiased estimator for θ. If $Y_{(n)} = \max(Y_1, Y_2, \ldots, Y_n)$, the results of Example 9.1 imply that $\hat{\theta}_2 = [(n+1)/n]Y_{(n)}$ is another unbiased estimator for θ. Show that the efficiency of $\hat{\theta}_1$ to $\hat{\theta}_2$ is $1/n^2$. Notice that this implies that $\hat{\theta}_2$ is a markedly superior estimator.

9.5 Suppose that $Y_1, Y_2, \ldots, Y_n$ is a random sample from a normal distribution with mean μ and variance σ^2. Two unbiased estimators of σ^2 are

$$\hat{\sigma}_1^2 = S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(Y_i - \overline{Y})^2 \quad \text{and} \quad \hat{\sigma}_2^2 = \frac{1}{2}(Y_1 - Y_2)^2.$$

Find the efficiency of $\hat{\sigma}_1^2$ relative to $\hat{\sigma}_2^2$.

9.6 Suppose that $Y_1, Y_2, \ldots, Y_n$ denote a random sample of size n from a Poisson distribution with mean λ. Consider $\hat{\lambda}_1 = (Y_1 + Y_2)/2$ and $\hat{\lambda}_2 = \overline{Y}$. Derive the efficiency of $\hat{\lambda}_1$ relative to $\hat{\lambda}_2$.

9.7 Suppose that $Y_1, Y_2, \ldots, Y_n$ denote a random sample of size n from an exponential distribution with density function given by

$$f(y) = \begin{cases} (1/\theta)e^{-y/\theta}, & 0 < y, \\ 0, & \text{elsewhere.} \end{cases}$$

9.6 Suppose that $Y_1, Y_2, \ldots, Y_n$ denote a random sample of size n from a Poisson distribution with mean λ. Consider $\hat{\lambda}_1 = (Y_1 + Y_2)/2$ and $\hat{\lambda}_2 = \overline{Y}$. Derive the efficiency of $\hat{\lambda}_1$ relative to $\hat{\lambda}_2$.

9.7 Suppose that $Y_1, Y_2, \ldots, Y_n$ denote a random sample of size n from an exponential distribution with density function given by

$$f(y) = \begin{cases} (1/\theta)e^{-y/\theta}, & 0 < y, \\ 0, & \text{elsewhere.} \end{cases}$$

In Exercise 8.19, we determined that $\hat{\theta}_1 = nY_{(1)}$ is an unbiased estimator of θ with $\text{MSE}(\hat{\theta}_1) = \theta^2$. Consider the estimator $\hat{\theta}_2 = \overline{Y}$ and find the efficiency of $\hat{\theta}_1$ relative to $\hat{\theta}_2$.