

## 125 Numbering Paths

### Background

Problems that process input and generate a simple “yes” or “no” answer are called decision problems. One class of decision problems, the NP-complete problems, are not amenable to general efficient solutions. Other problems may be simple as decision problems, but enumerating all possible “yes” answers may be very difficult (or at least time-consuming).

This problem involves determining the number of routes available to an emergency vehicle operating in a city of one-way streets.

### The Problem

Given the intersections connected by one-way streets in a city, you are to write a program that determines the number of different routes between each intersection. A route is a sequence of one-way streets connecting two intersections.

Intersections are identified by non-negative integers. A one-way street is specified by a pair of intersections. For example,  $j \ k$  indicates that there is a one-way street from intersection  $j$  to intersection  $k$ . Note that two-way streets can be modeled by specifying two one-way streets:  $j \ k$  and  $k \ j$ .

Consider a city of four intersections connected by the following one-way streets:

```
0  1
0  2
1  2
2  3
```

There is one route from intersection 0 to 1, two routes from 0 to 2 (the routes are  $0 \rightarrow 1 \rightarrow 2$  and  $0 \rightarrow 2$ ), one route from 2 to 3, and no other routes.

It is possible for an infinite number of different routes to exist. For example if the intersections above are augmented by the street  $3 \ 2$ , there is still only one route from 0 to 1, but there are infinitely many different routes from 0 to 2. This is because the street from 2 to 3 and back to 2 can be repeated yielding a different sequence of streets and hence a different route. Thus the route  $0 \rightarrow 2 \rightarrow 3 \rightarrow 2 \rightarrow 3 \rightarrow 2$  is a different route than  $0 \rightarrow 2 \rightarrow 3 \rightarrow 2$ .

### The Input

The input is a sequence of city specifications. Each specification begins with the number of one-way streets in the city followed by that many one-way streets given as pairs of intersections. Each pair  $j \ k$  represents a one-way street from intersection  $j$  to intersection  $k$ . In all cities, intersections are numbered sequentially from 0 to the “largest” intersection. All integers in the input are separated by whitespace. The input is terminated by end-of-file.

There will never be a one-way street from an intersection to itself. No city will have more than 30 intersections.

### The Output

For each city specification, a square matrix of the number of different routes from intersection  $j$  to intersection  $k$  is printed. If the matrix is denoted  $M$ , then  $M[j][k]$  is the number of different routes from intersection  $j$  to intersection  $k$ . The matrix  $M$  should be printed in row-major order, one row per line.

Each matrix should be preceded by the string “`matrix for city  $k$` ” (with  $k$  appropriately instantiated, beginning with 0).

If there are an infinite number of different paths between two intersections a -1 should be printed. **DO NOT** worry about justifying and aligning the output of each matrix. All entries in a row should be separated by whitespace.

### Sample Input

```
7 0 1 0 2 0 4 2 4 2 3 3 1 4 3
5
0 2
0 1 1 5 2 5 2 1
9
0 1 0 2 0 3
0 4 1 4 2 1
2 0
3 0
3 1
```

### Sample Output

```
matrix for city 0
0 4 1 3 2
0 0 0 0 0
0 2 0 2 1
0 1 0 0 0
0 1 0 1 0
matrix for city 1
0 2 1 0 0 3
0 0 0 0 0 1
0 1 0 0 0 2
0 0 0 0 0 0
0 0 0 0 0 0
0 0 0 0 0 0
matrix for city 2
-1 -1 -1 -1 -1
0 0 0 0 1
-1 -1 -1 -1 -1
-1 -1 -1 -1 -1
0 0 0 0 0
```