Кр. 2 В. 14. Однобит

I

№1

1) $\sum_{n=1}^{\infty} \frac{n^2+3}{n^3\left(2+\sin\left(\frac{n\pi}{2}\right)\right)}$

$\lim_{n\to\infty} \sin\left(\frac{n\pi}{2}\right) \in [-1;1]$

$\lim_{n\to\infty} n^2+3 = n$

$\int_1^{\infty} \frac{x^2 dx}{x^3} = \int_1^{\infty} \frac{dx}{x} = \ln x \Big|_1^{\infty} = \ln\infty - 0 = \infty$ – расх.

2) $\sum_{n=1}^{\infty} \frac{1}{\sqrt[3]{n}+2} \cdot \operatorname{arctg} \frac{n+3}{n^2+5}$

$\lim_{n\to\infty} \operatorname{arctg}\left(\frac{n+3}{n^2+5}\right) = \frac{n+3}{n^2+5}$

$\lim_{n\to\infty} \sqrt[3]{n}+2 = \sqrt[3]{n}$

$\lim_{n\to\infty} n+3 = n$

$\lim_{n\to\infty} n^2+5 = n^2$

$\lim_{n\to\infty} \frac{n}{n^{\frac{7}{3}}} = \lim_{n\to\infty} \frac{1}{n^{\frac{4}{3}}}$ $\quad \frac{4}{3} > 1$ – сходится

3) $\sum_{n=1}^{\infty} \frac{n!}{(3n)!}$

$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{(n+1)!}{(3(n+1))!} \cdot \frac{(3n)!}{n!} = \lim_{n\to\infty} \frac{n+1}{(3n+1)(3n+2)(3n+3)} = 0 < 1$ – расх.

4) $\sum_{n=2}^{\infty} \left(\frac{n+1}{2n-3}\right)^{n^2}$

$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} \left(\frac{n+1}{2n-3}\right)^n = 0 < 1$ расх.

5) $\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$

$\sum_{n=1}^{\infty} \frac{|\cos n|}{n^2} \leq \sum_{n=1}^{\infty} \frac{1}{n^2}$; $2 > 1$ – сх.

II $\sum_{n=0}^{\infty} \left(-\frac{2}{3}\right)^n$, $\alpha = 0{,}1$

$1 - \frac{2}{3} + \frac{4}{9} - \frac{8}{27} + \frac{16}{81} - \ldots$

$1 > \frac{2}{3} > \frac{4}{9} > -\frac{8}{27} > \frac{16}{81} > \ldots$ $\quad \lim_{n\to\infty} \left(\frac{2}{3}\right)^n = 0$

$\left(\frac{2}{3}\right)^4 = \frac{16}{81} > 0{,}1$; $\left(\frac{2}{3}\right)^5 = \frac{32}{243} > 0{,}1$; $\left(\frac{2}{3}\right)^6 = \frac{64}{729} < 0{,}1$

$\sum_{n=0}^{5} \left(-\frac{2}{3}\right)^n = 1 - \frac{2}{3} + \frac{4}{9} - \frac{8}{27} + \frac{16}{81} - \frac{32}{243} = \frac{165}{243} \approx 0{,}679 \approx 0{,}7$

III $\sum_{n=1}^{\infty} \frac{e^{-nx}}{n}$