№456

$y'y''' = 2y''^2 \quad | : y'y''$

$\frac{y'''}{y''} = 2\frac{y''}{y'}$

$(\ln y'')' = 2(\ln y')'$

$\ln y'' = \ln y'^2 + \ln C_1$

$y'' = y'^2 C_1$

$y' = p(y);\ y'' = pp'$

$pp' = p^2 C_1$

$\frac{p'}{p} = C_1$

$\ln p = yC_1 + \ln C_2$

$p = e^{yC_1} \cdot C_2$

$y' = e^{yC_1} \cdot C_2$

$\frac{y'}{e^{yC_1}} = C_2$

$y' e^{-yC_1} = C_2$

$-\frac{e^{-yC_1}}{C_1} = xC_2 + C_3 \quad | \cdot(-C_1)$

$e^{-yC_1} = xC_4 + C_5$

$\ln e^{-yC_1} = \ln(xC_4 + C_5)$

$-yC_1 \ln e = \ln(xC_4 + C_5)$

$-yC_1 = \ln(xC_4 + C_5)$

$y = -\frac{\ln(xC_4 + C_5)}{C_1}$

$y=0$ – решение частное ($C_4=0$, $C_5=0$, $C_1 \neq 0$)

№458

$5y'''^2 - 3y''y^{IV} = 0 \quad | : y''y'''$

$5\frac{y'''}{y''} - 3\frac{y^{IV}}{y'''} = 0$

$5(\ln|y''|)' - 3(\ln|y'''|)' = 0$

$5\ln|y''| - 3\ln|y'''| = \ln C_1$

$-3\ln|y'''| = \ln C_1 - 5\ln|y''|$

$y'''^{-3} = \frac{C_1}{y''^5}$

$y'''^3 = \frac{y''^5}{C_1}$

$y'' = p(y);\ y''' = pp'$

$p^3p'^3 = \frac{p^5}{C_1}$

$p'^3 = \frac{p^2}{C_1}$

$p' = \frac{p^{2/3}}{C_2} \quad (C_2 = C_1^{1/3})$

$\frac{p'}{p^{2/3}} = \frac{1}{C_2}$

$\int p^{-2/3}dp = \int \frac{dx}{C_2}$

$3p^{1/3} = \frac{x}{C_2} + C_3$

$p = \frac{x^3}{C_2^3} + C_3^3$

$y'' = \frac{x^3}{C_4} + C_5 \quad (C_4 = C_2^3;\ C_5 = C_3^3)$

$y' = \frac{x^4}{4C_4} + xC_5$

$y = \frac{x^5}{20C_4} + \frac{x^2}{2}C_5$

$y=0$ – решение частное ($C_4=0$, $C_5=0$)

№460

$y'' = xy' + y + 1$

$y'' = (xy)' + (x)'$

$y' = xy + x + C_1$

$y' - xy = x + C_1$

1) $y' - xy = 0$

$\frac{y'}{y} = x$

$\ln|y| = \frac{x^2}{2} + C_2$

$y = C_2 e^{\frac{x^2}{2}}$

2) $y = C_2(x) e^{\frac{x^2}{2}}$

$y' = C_2' e^{\frac{x^2}{2}} + xC_2 e^{\frac{x^2}{2}}$

$C_2' e^{\frac{x^2}{2}} + xC_2 e^{\frac{x^2}{2}} - xC_2 e^{\frac{x^2}{2}} = x + C_1$

$C_2' = \frac{x + C_1}{e^{\frac{x^2}{2}}}$

$C_2 = C_1\int e^{-\frac{x^2}{2}}dx - \int e^{-\frac{x^2}{2}} d\left(-\frac{x^2}{2}\right) = C_1\int e^{-\frac{x^2}{2}}dx - e^{-\frac{x^2}{2}} + C_3$

3) $y = e^{\frac{x^2}{2}}\left(C_1\int e^{-\frac{x^2}{2}}dx - e^{-\frac{x^2}{2}} + C_3\right)$

$y=0$ – не решение

№462

$xy'' - y' = x^2yy' \quad | : x^2$

$\frac{y''}{x} - \frac{y'}{x^2} = yy'$

$\left(\frac{y'}{x}\right)' = \left(\frac{y^2}{2}\right)'$

$\frac{y'}{x} = \frac{y^2}{2} + C_1$

$\frac{y'}{x} = \frac{y^2 + 2C_1}{2}$

$\frac{y'}{y^2 + 2C_1} = \frac{x}{2}$

1) $\sqrt{\frac{2}{C_1}} \operatorname{arctg} \frac{y}{\sqrt{2C_1}} = \frac{x^2}{2} + C_2 \quad (C_1 > 0)$

2) $\frac{1}{\sqrt{-2C_1}} \ln\left|\frac{y - \sqrt{-2C_1}}{y + \sqrt{-2C_1}}\right| = \frac{x^2}{2} + C_2 \quad (C_1 < 0)$

3) $-\frac{2}{y} = \frac{x^2}{2} + C_2 \quad (C_1 = 0)$

№684

$xy'' + 2y' - xy = 0;\quad y_1 = \frac{e^x}{x}$

$y'' + \frac{2}{x}y' - y = 0$

$\left(\frac{y_2}{y_1}\right)' = \frac{C_1 e^{-\int p_1(x)dx}}{y_1^2}$

$\int p_1(x)dx = \int \frac{2}{x}dx = 2\ln x$

$\left(\frac{y_2}{y_1}\right)' = \frac{x^2 C_1 e^{-\ln x^2}}{e^{2x}} = \frac{x^2 C_1}{e^{2x}x^2} = \frac{C_1}{e^{2x}}$

$\frac{y_2}{y_1} = C_1\int e^{-2x}dx = -\frac{1}{2}C_1 e^{-2x} + C_2$

$y_2 = -\frac{1}{2}C_1 \frac{e^x}{x} e^{-2x} = -\frac{1}{2}C_1\frac{e^{-x}}{x} + \frac{e^x}{x}C_2$

$xy = C_1e^{-x} + C_2e^x$

№642

$6x+9,\ 8x+12$

$$\begin{vmatrix} 6x+9 & 8x+12 \\ 6 & 8 \\ 0 & 0 \end{vmatrix}$$

$\det = 0$

Ответ: Да

№644

$1,\ x,\ x^2$

$$\begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & 2x \\ 0 & 0 & 2 \end{vmatrix}$$

$\det = 1\begin{vmatrix} 1 & 2x \\ 0 & 2 \end{vmatrix} = 2 - 0 = 2$

Ответ: Нет

№646

$x^2+2,\ 3x^2-1,\ x+4$

$$\begin{vmatrix} x^2+2 & 3x^2-1 & x+4 \\ 2x & 6x & 1 \\ 2 & 6 & 0 \end{vmatrix} = (x+4)\begin{vmatrix} 2x & 6x \\ 2 & 6 \end{vmatrix} - 1\begin{vmatrix} x^2+2 & 3x^2-1 \\ 2 & 6 \end{vmatrix} =$$

$= (x+4)(12x - 12x) - (6x^2 + 12 - 6x^2 + 2) = -14$

Ответ: Нет.

№648

$e^x,\ e^{2x},\ e^{3x}$

$$\begin{vmatrix} e^x & e^{2x} & e^{3x} \\ e^x & 2e^{2x} & 3e^{3x} \\ e^x & 4e^{2x} & 9e^{3x} \end{vmatrix}$$

$\det = e^x\left(\begin{vmatrix} 2e^{2x} & 3e^{3x} \\ 4e^{2x} & 9e^{3x} \end{vmatrix} - \begin{vmatrix} e^{2x} & e^{3x} \\ 4e^{2x} & 9e^{3x} \end{vmatrix} + \begin{vmatrix} e^{2x} & e^{3x} \\ 2e^{2x} & 3e^{3x} \end{vmatrix}\right) = e^x\left(\underbrace{18e^{5x} - 12e^{5x} - 9e^{5x} + 4e^{5x} + 3e^{5x} - 2e^{5x}}_{2e^{5x}}\right) = 2e^{6x}$

Ответ: нет

№650

$1,\ \sin^2x,\ \cos 2x$

$$\begin{vmatrix} 1 & \sin^2x & \cos 2x \\ 0 & 2\sin x\cos x & -2\sin 2x \\ 0 & -2\sin^2x + 2\cos^2x & -4\cos 2x \end{vmatrix}$$

$\det = \begin{vmatrix} 2\sin x\cos x & -2\sin 2x \\ -2\sin^2x + 2\cos^2x & -4\cos 2x \end{vmatrix} = -4\sin 2x\cos 2x + 4\sin 2x\cos 2x = 0$

Ответ: Да

№646

$x(x-1)y'' - xy' + y = 0$

$y = x^n + \dots$

$x(x-1)n(n-1)x^{n-2} - xnx^{n-1} + x^n = 0$

$x^n(n(n-1) - n + 1) = 0$

$x^n(n^2 - 2n + 1) = 0$

$n = 1$

$y = x + b$

$x(x-1)y'' - xy' + y = 0$

$-x + x + b = 0$

$b = 0 \Rightarrow y_1 = x$

$\begin{vmatrix} x & y_2 \\ 1 & y_2' \end{vmatrix} = C_1 e^{-\int\frac{-x}{x(x-1)}dx}$

$-\int\frac{-x}{x(x-1)}dx = \ln|x-1|$

$\begin{vmatrix} x & y_2 \\ 1 & y_2' \end{vmatrix} = C_1(x-1)$

$xy_2' - y_2 = C_1(x-1)$

1) $xy_2' - y_2 = 0$

$\frac{y_2'}{y_2} = \frac{1}{x}$

$\ln|y_2| = \ln|x| + C_2$

$y_2 = xC_2$

2) $y_2' = C_2 + xC_2'$

$xC_2 + x^2C_2' - xC_2 = C_1(x-1)$

$C_2' = C_1\frac{x-1}{x^2}$

$C_2 = C_1\left(\ln|x| + \frac{1}{x}\right) + C_3$

3) $y = C_1(x\ln|x| + 1) + xC_3$

№682

$x^2(x+1)y'' - 2y = 0,\quad y_1 = 1 + \frac{1}{x}$

$y = z(x) + \frac{z(x)}{x}$

$y' = z'(x) + \frac{z'(x)}{x} - \frac{z(x)}{x^2}$

$y'' = z'' + \frac{z''}{x} - \frac{z'}{x^2} - \frac{z'}{x^2} + \frac{2z}{x^3}$

$(x^3 + x)\left(z'' + \frac{z''}{x} - \frac{2z'}{x^2} + \frac{2z}{x^3}\right) - 2z - \frac{2z}{x} = 0,$

$x^3z'' + xz'' + x^2z'' + z'' - 2xz' - \frac{2z'}{x} + 2z + \frac{2z}{x^2} - 2z - \frac{2z}{x} = 0$

$z''(x^3 + x^2 + x + 1) - z'\left(2x + \frac{2}{x}\right) + z\left(\frac{2}{x^2} - \frac{2}{x}\right) = 0$

$x(x+1)^2z'' - 2z'(x+1) = 0 \quad | : (x+1) \neq 0$

$x(x+1)z'' - 2z' = 0$

$z' = t,\ z'' = t'$

$x(x+1)t' = 2t$

$\frac{t'}{t} = \frac{2}{x(x+1)}$

$\frac{t'}{t} = \frac{2}{x} - \frac{2}{x+1}$

$\ln|t| = 2\ln|x| - 2\ln|x+1| + C_1$

$t = \frac{x^2C_1}{(x+1)^2};\quad z' = \frac{x^2C_1}{(x+1)^2}$

$z' = C_1\frac{(x-1+1)^2}{(x+1)^2}$

$z = C_1\int\left(1 - \frac{1}{x+1}\right)^2dx$

$z = C_1\int\left(1 - \frac{2}{x+1} + \frac{1}{(x+1)^2}\right)dx$

$z = C_1\left(x - 2\ln|x+1| - \frac{1}{x+1}\right) + C_2$

$y = \left(1 + \frac{1}{x}\right)\left(C_1\left(x - 2\ln|x+1| - \frac{1}{x+1}\right) + C_2\right)$