



Summer School  
**Fluid Dynamics of Sustainability  
and the Environment**  
Ecole Polytechnique (Paris), 1-12 July 2019

# Fluid-structure interactions Energy harvesting from flow-induced instabilities

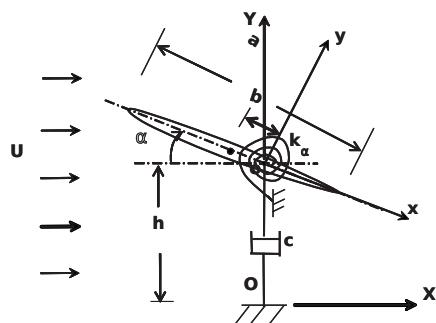
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olivier.doare@ensta.fr

# Energy harvesting from flow-induced vibrations

## Flapping wings

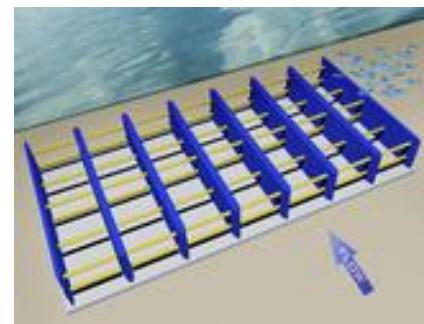


© 2003 - The Engineering Business Ltd



Peng & Zhu, Phys. Fluids, 2009

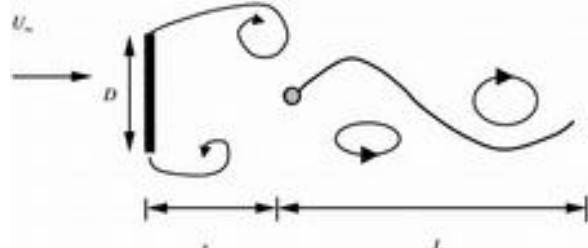
## Vortex Induced Vibrations (VIV)



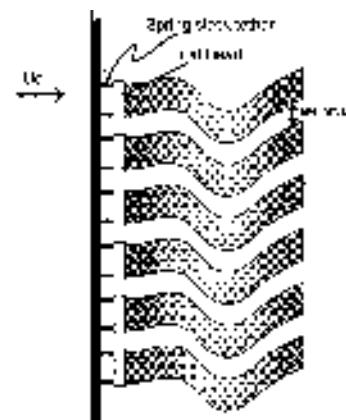
©2011 Vortex Hydro Energy



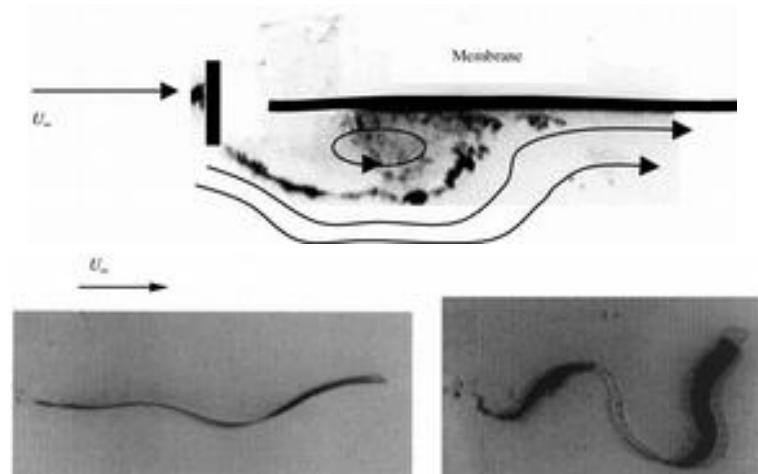
## Vibrations of slender structures in instationnary flows



Alen & Smith, JFS 2002



Techet et al., ISOPE 2002



Alen & Smith, JFS 2002

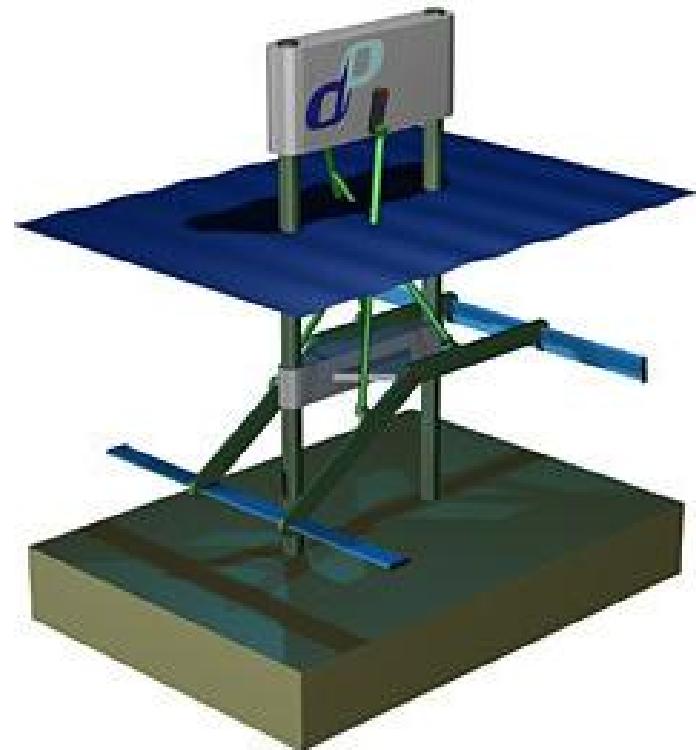
## Some commercial products

The STINGRAY project (Engineering Business Ltd.)



- Experimental project of the early 2000's
- 15m span 3m chord, displacement amplitude of 12m
- Displacement of the wing transmitted to an hydraulic motor through hydraulic cylinders
- Abandonned project because of economic viability

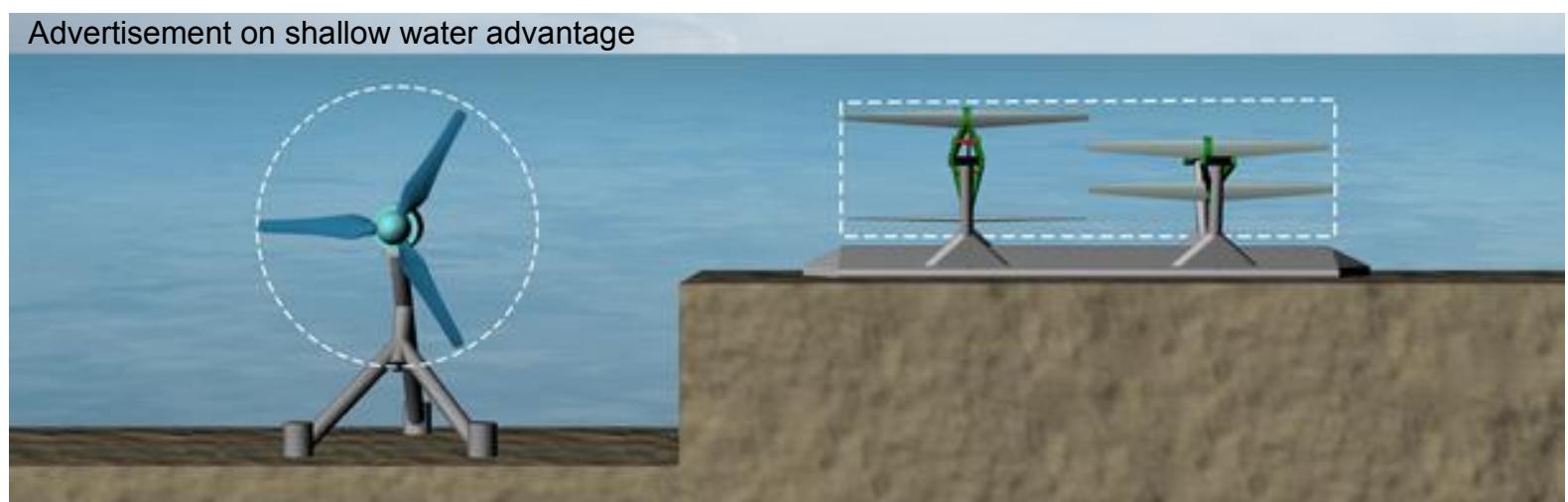
## Some commercial products



Pulse Generation

### PULSE-TIDAL

- Two wings are phase-locked, translation and rotation of each wing also phase-locked
- Movement transited to a generator through arms
- Generator can be put in or out the water
- Small vertical space





<http://www.biopowersystems.com/biostream.html>





[www.eel-energy.fr](http://www.eel-energy.fr)

# Flow induced instabilities

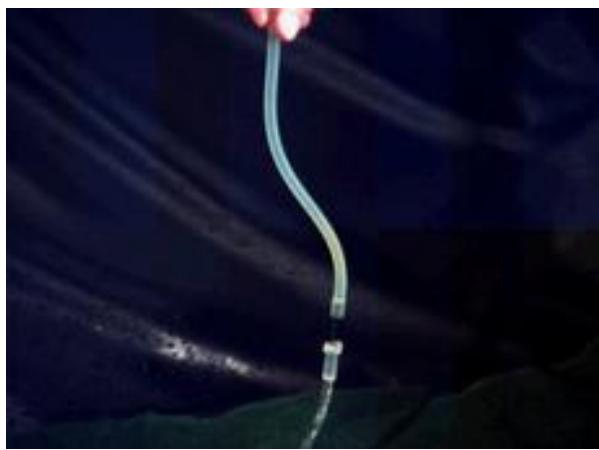
# Cross-flow instabilities



# Axial flow instabilities



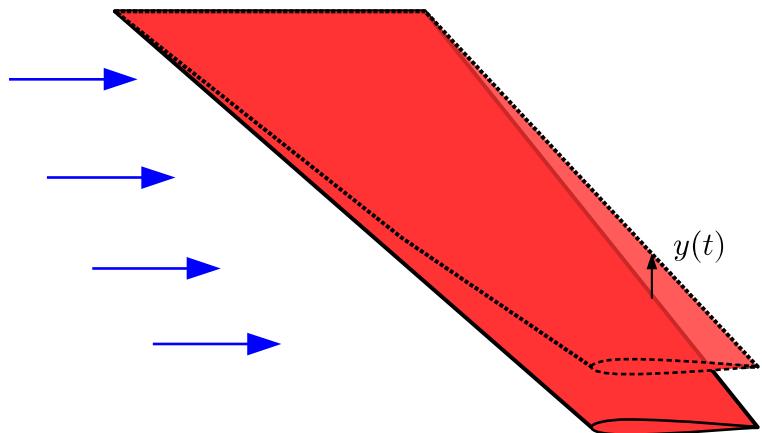
Fluttering flag



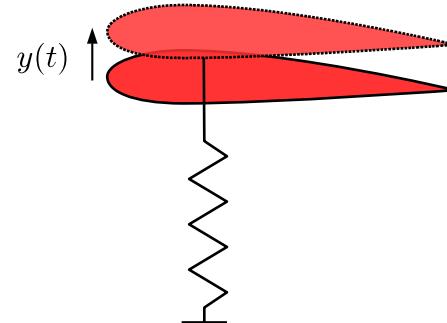
Fluttering pipe

- Objective : Overview of the different physical phenomena that may induce structural vibrations
- Part I : Cross-flow instabilities
- Part II : Axial flow instabilities

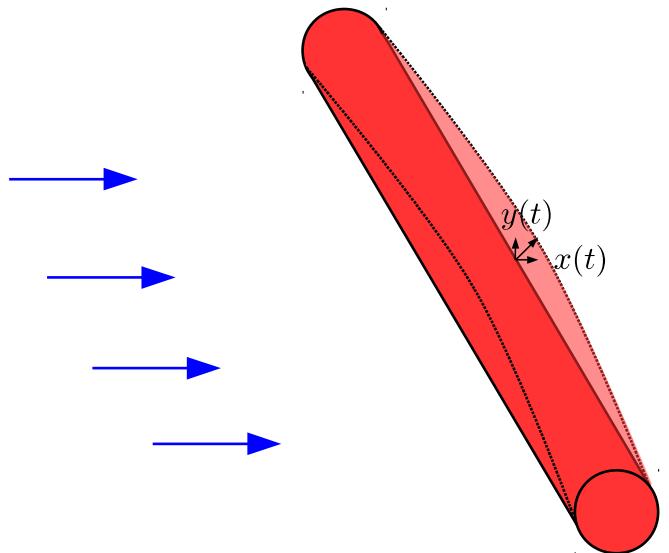
# Part I : Cross-flow instabilities



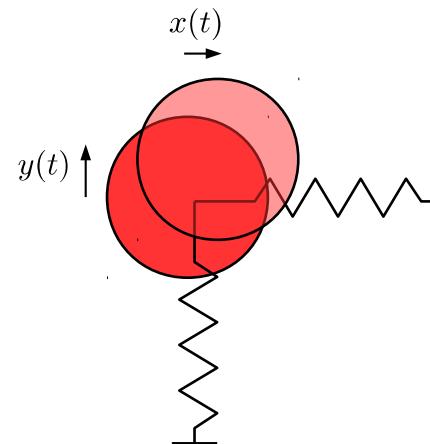
Flexural deformation of a wing



Translation of an airfoil profile + one stiffness

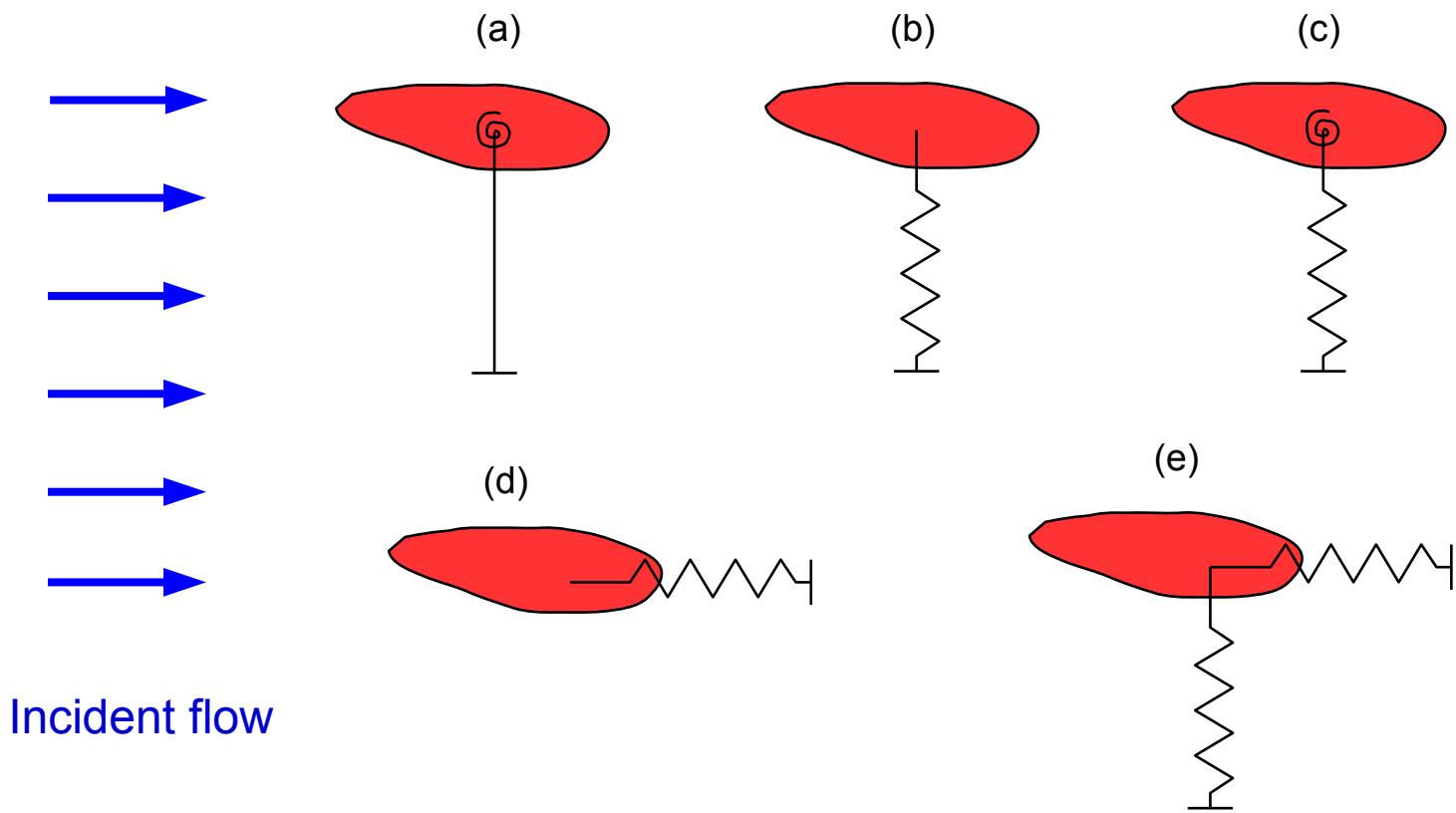


Two polarities flexural deformation of a cylinder

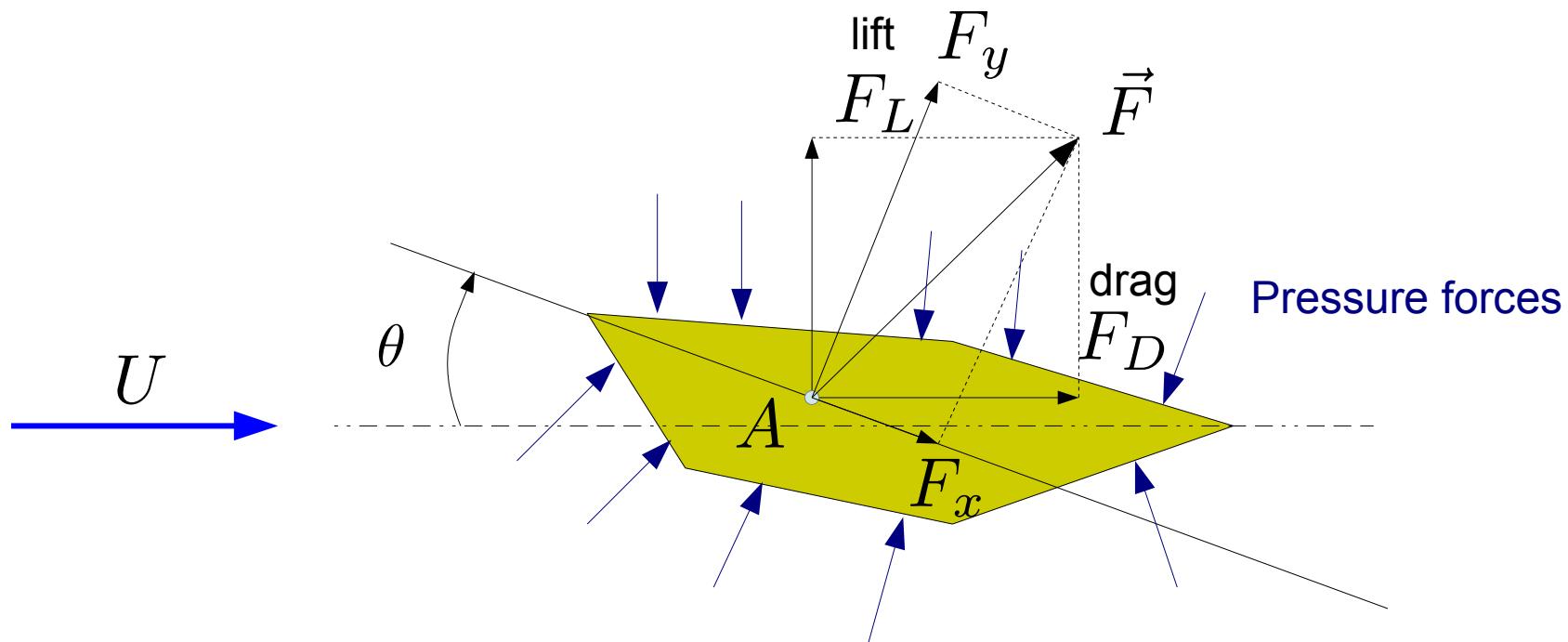


Translation of a circular section + two stiffnesses

# Cases considered here



# Aerodynamic efforts acting on a solid (2D)



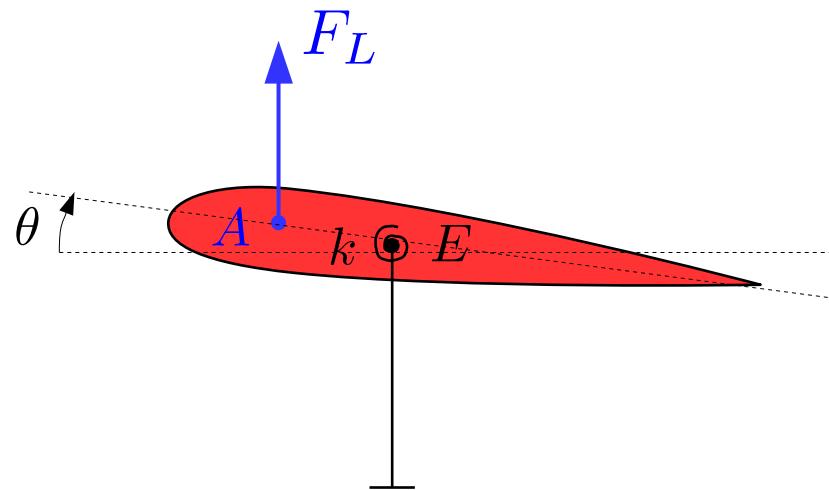
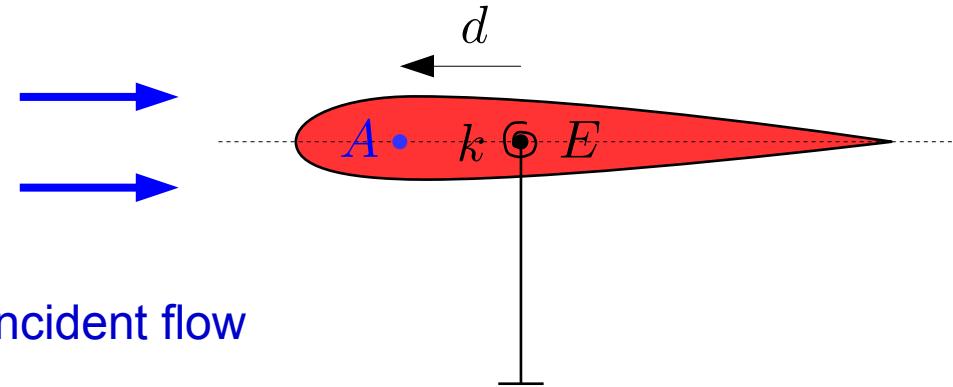
- Forces per unit length
- Definition of non-dimensional coefficients :

$$C_L = \frac{F_L}{\frac{1}{2} \rho U^2 L} \quad C_D = \frac{F_D}{\frac{1}{2} \rho U^2 L} \quad C_x = \frac{F_x}{\frac{1}{2} \rho U^2 L} \quad C_y = \frac{F_y}{\frac{1}{2} \rho U^2 L}$$

- Coefficients function of the Reynolds number and  $\theta$

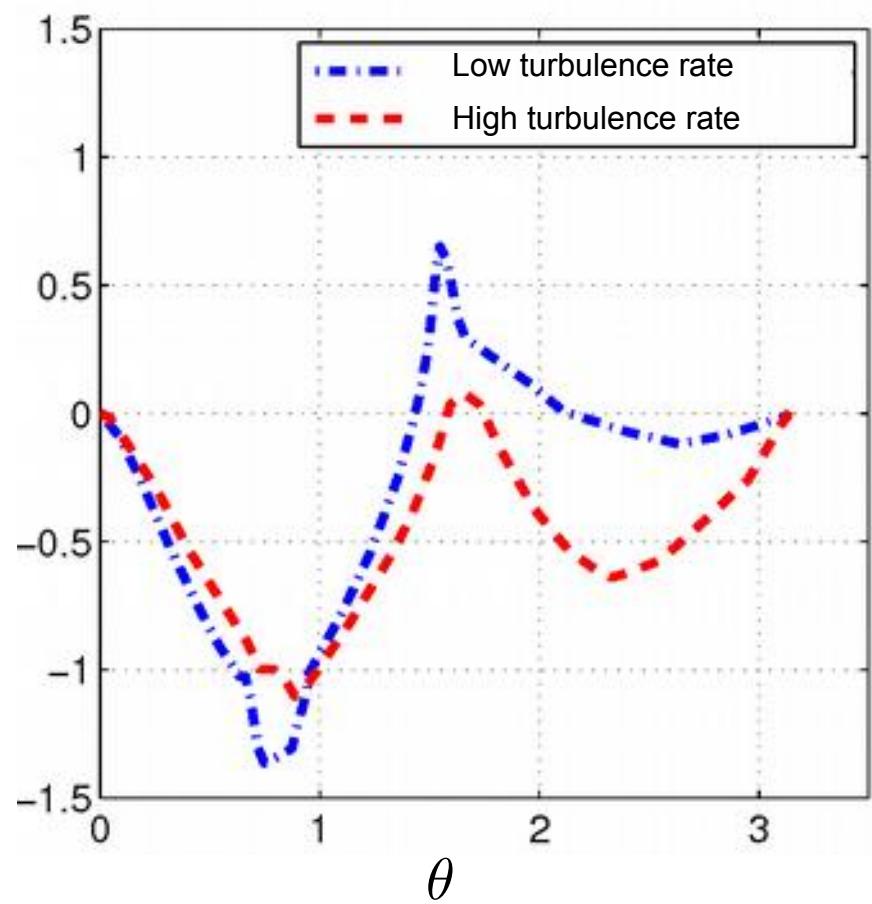
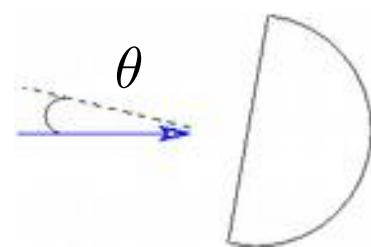
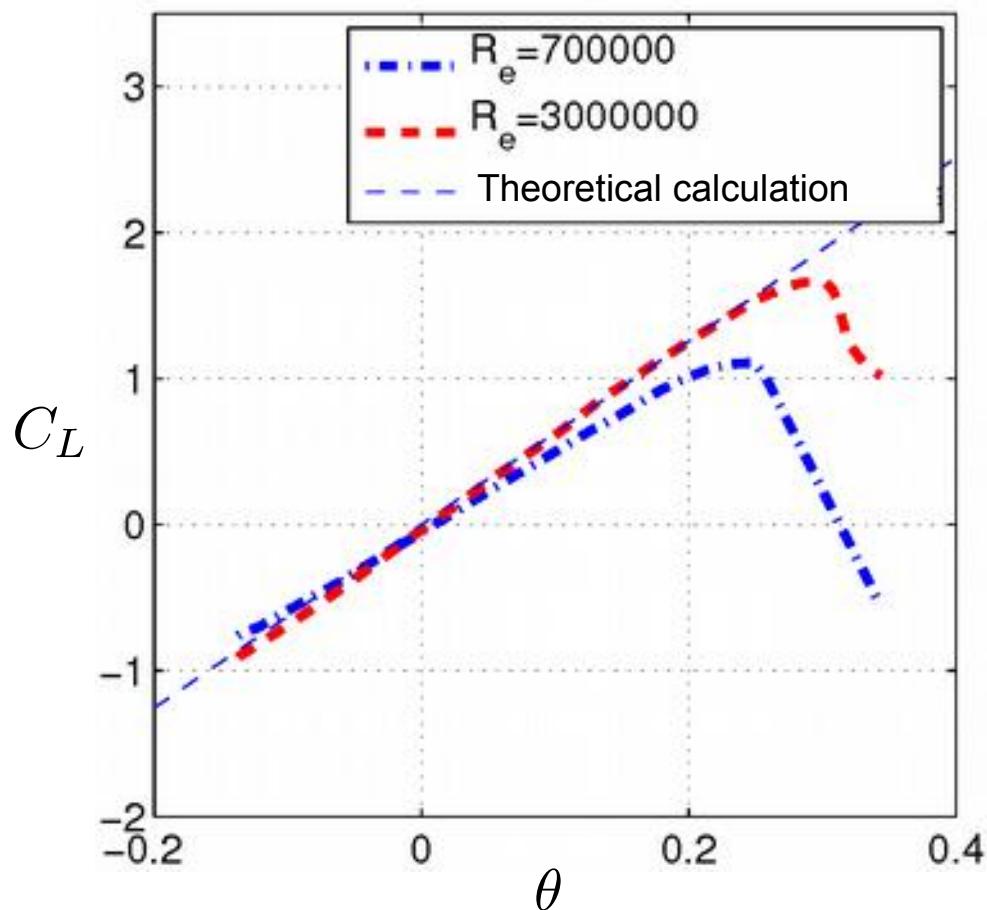
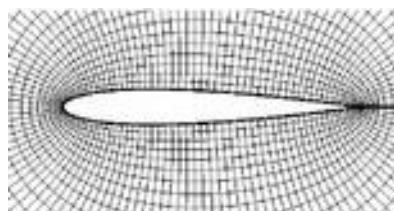
Buckling instability due to  
negative flow-induced stiffness

# Buckling instability due to negative flow-induced stiffness



- Lift force :  $F_L = \frac{1}{2} \rho U^2 S C_L$  with  $C_L = C_L(\theta, R_e)$
- Small angles of attack :  $C_L \sim \theta \frac{\partial C_L}{\partial \theta} = \theta C'_L$
- $M = dF_L$
- Angle of attack governed by :  $J\ddot{\theta} + \left[ k - \frac{1}{2} \rho U^2 S d C'_L \right] \theta = 0$

NACA0012



# Buckling instability due to negative flow-induced stiffness

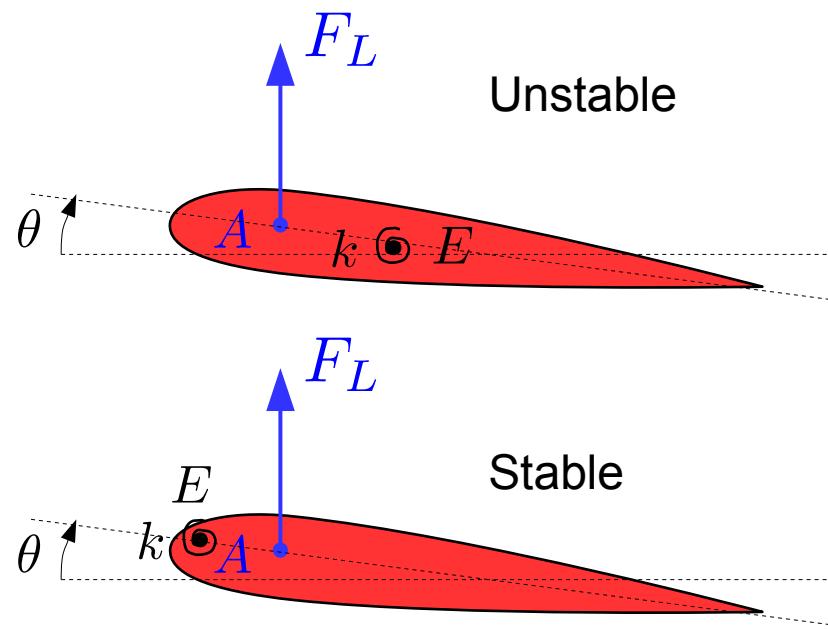
$$J\ddot{\theta} + \left[ k - \frac{1}{2}\rho U^2 S d C'_L \right] \theta = 0$$

- Negative stiffness if :  
→ (buckling instability)

$$U > \sqrt{\frac{2k}{\rho S d C'_L}}$$

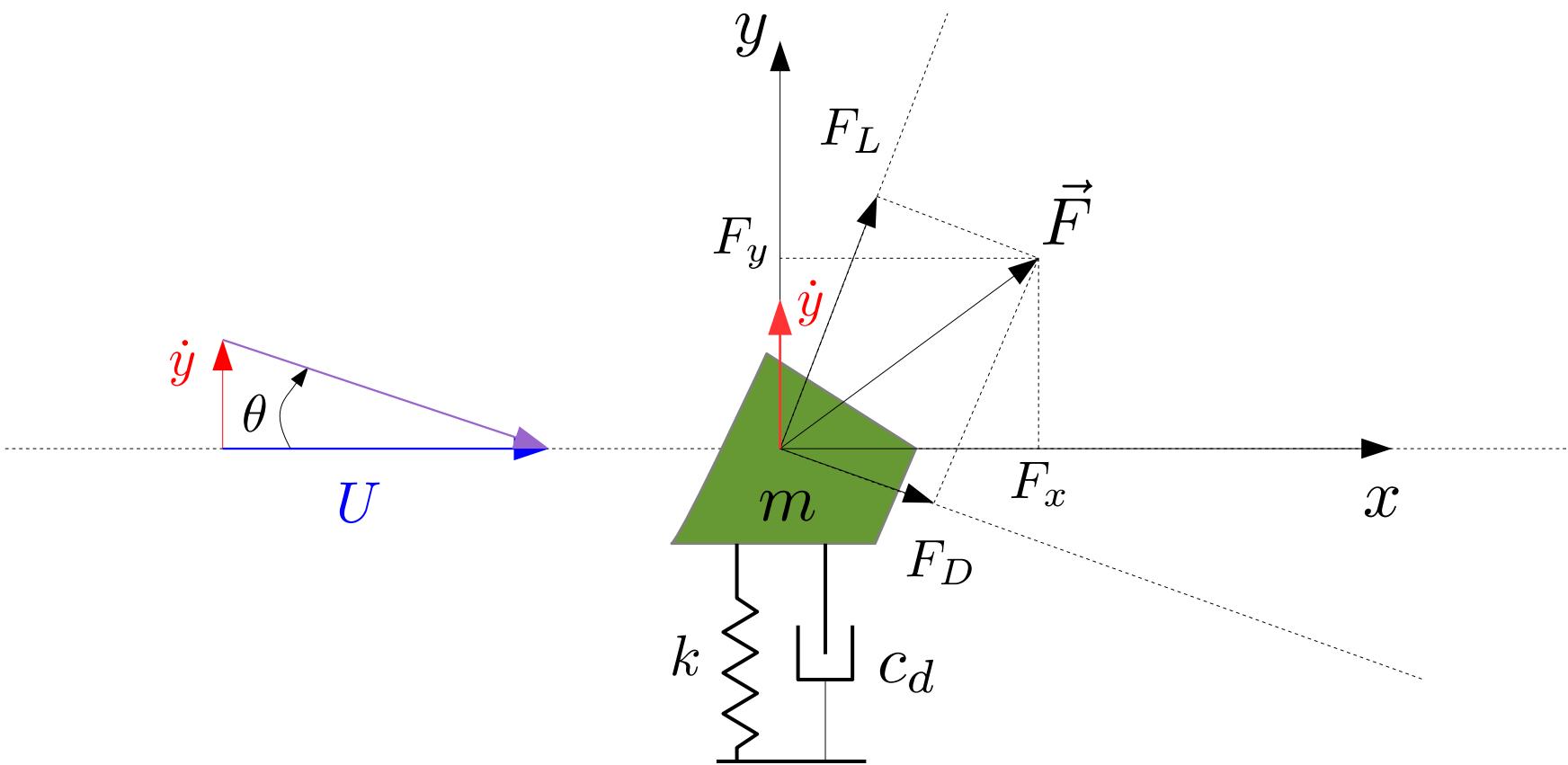


Weathercock



# Dynamic instability by negative flow-induced damping

# Added damping force

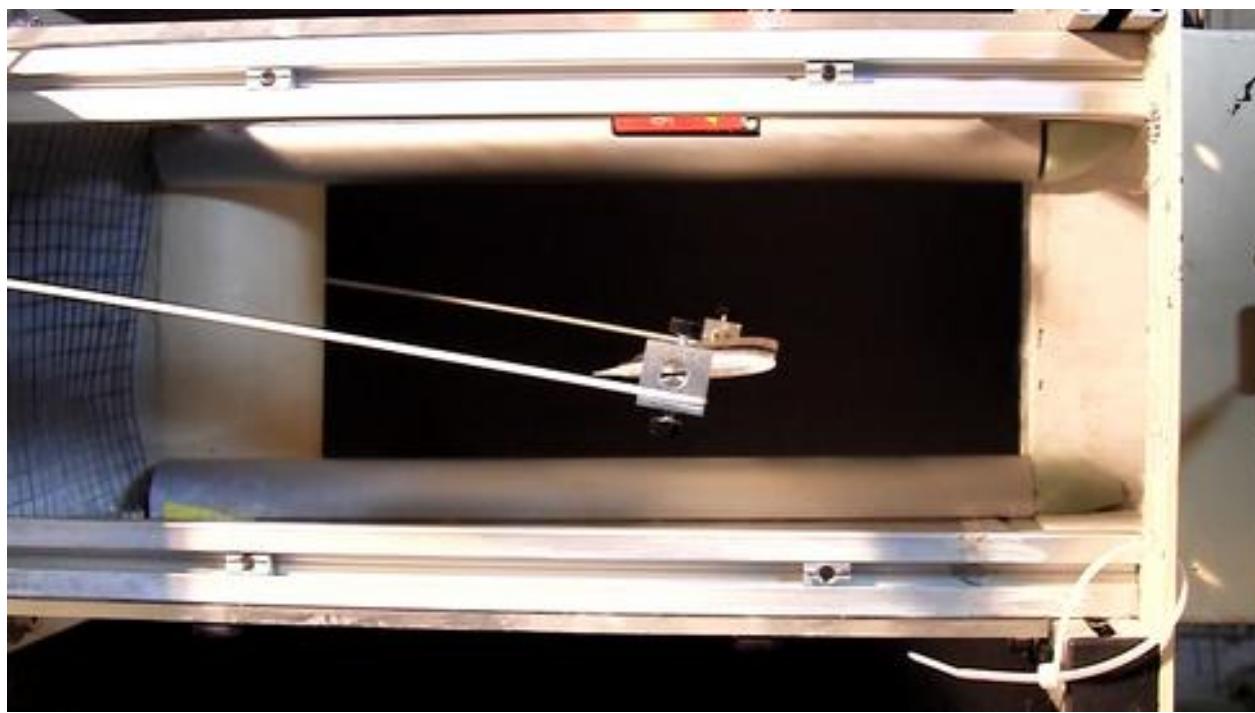
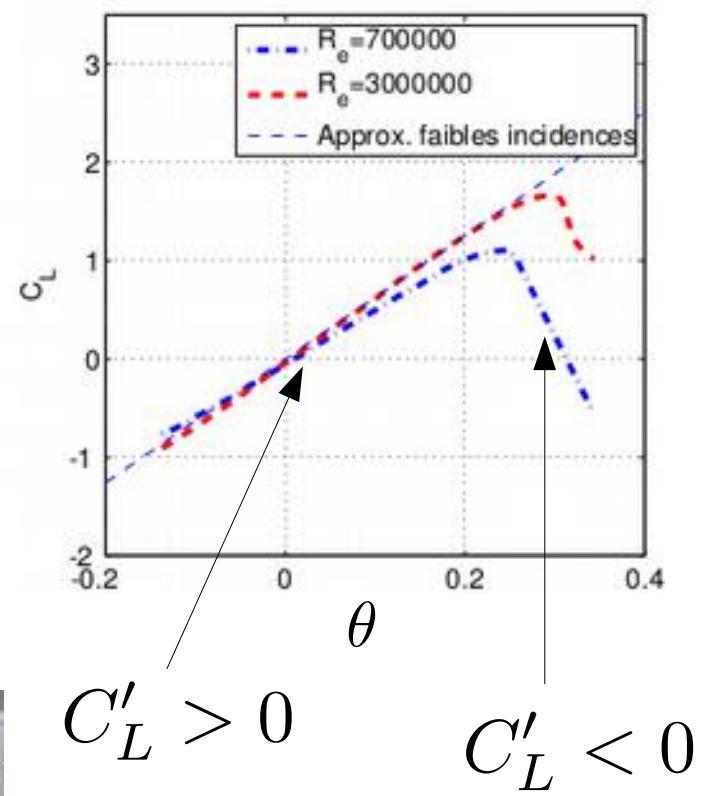
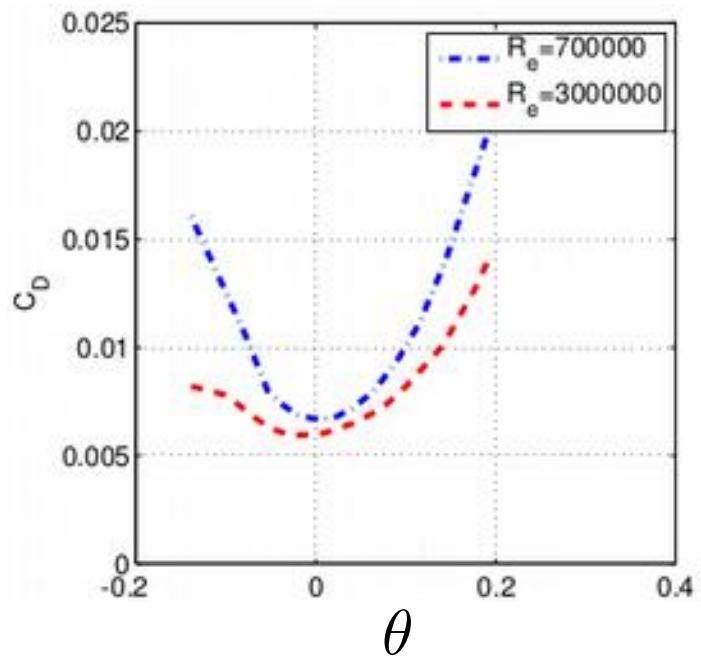


Instability criterion

$$F_y \sim -\frac{1}{2} \rho U^2 S \frac{\dot{y}}{U} (C'_L + C_D)$$

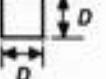
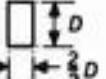
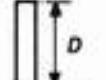
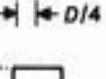
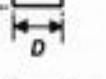
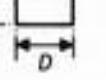
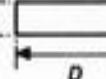
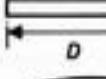
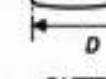
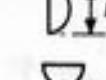
$$U > -\frac{2c_d}{\rho U S (C'_L + C_D)}$$

## NACA 0012 airfoil



# Other sections

$$-\frac{\partial C_L}{\partial \theta}$$

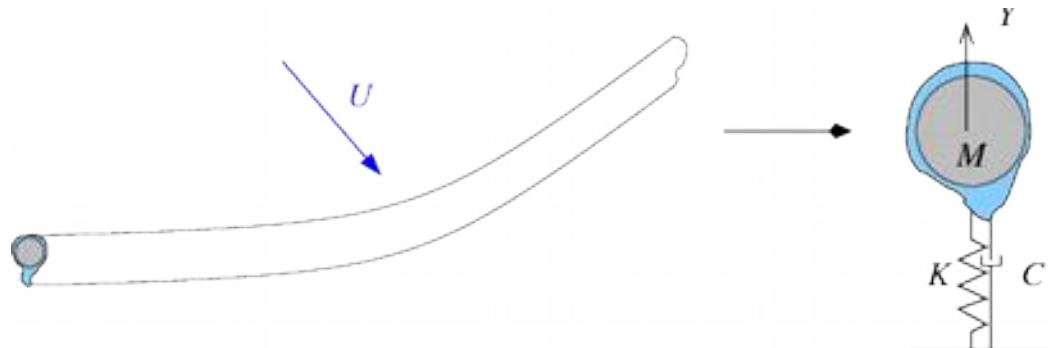
Section	Smooth flow	Turbulent flow <sup>b</sup>	Reynolds number
	3.0	3.5	$10^5$
	0.	-0.7	$10^5$
	-0.5	0.2	$10^5$
	-0.15	0.	$10^5$
	1.3	1.2	66 000
	2.8	-2.0	33 000
	-10.	—	2 000-20 000
	-6.3	-6.3	$>10^5$
	-6.3	-6.3	$>10^5$
	-0.1	0.	66 000
	-0.5	2.9	51 000
	0.66	—	75 000

(Blevins, 1990)

## Case of the square



# Ice on cables



[telegraph.co.uk](http://telegraph.co.uk)



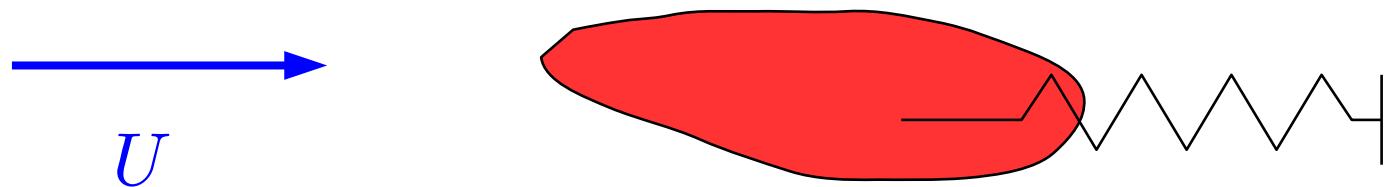
[icefree.ro](http://icefree.ro)



[edn.com](http://edn.com)

# Drag crisis instability

Oscillations in the direction of the flow

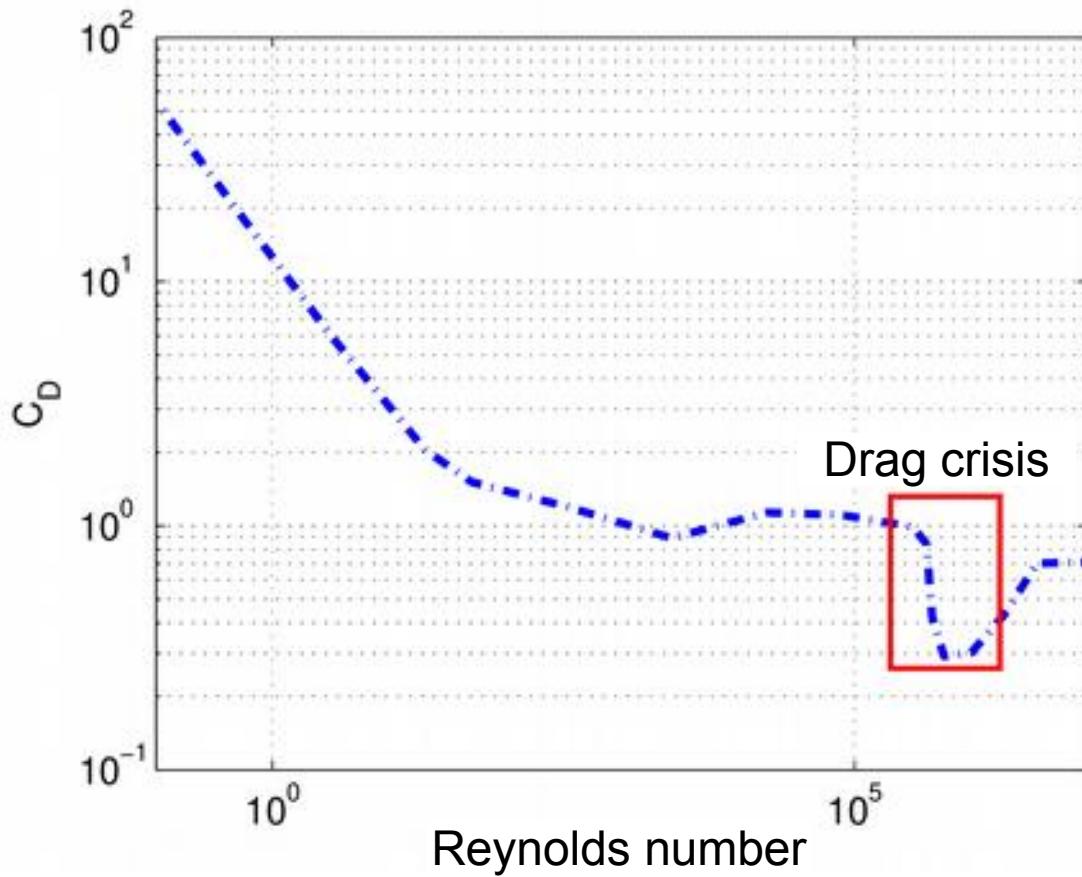


$$F_D = -\frac{1}{2}\rho U^2 S \left( C_D(R_E) + 2\frac{\dot{x}}{U}C_D(R_E) + \frac{\dot{x}}{U}R_E \frac{\partial C_D}{\partial R_E} \right)$$

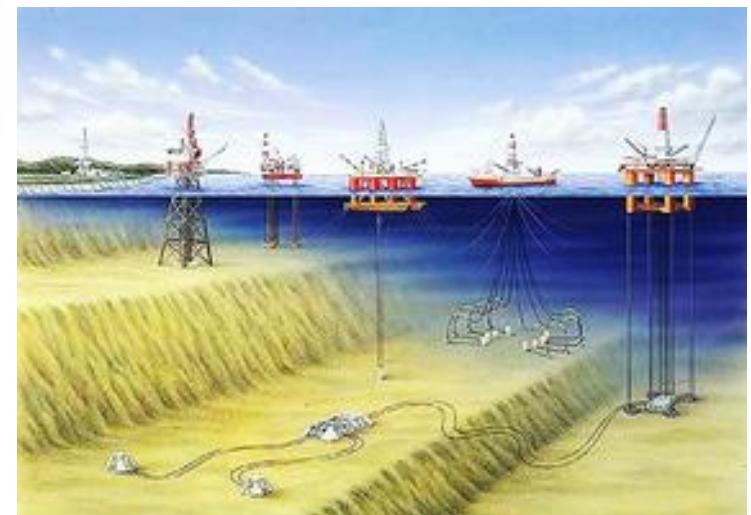
Instability if

$$2C_D(R_E) + R_E \frac{\partial C_D}{\partial R_E} < 0$$

## Case of the circular section



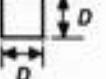
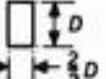
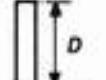
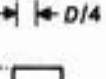
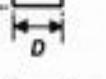
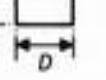
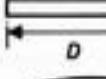
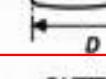
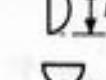
Common problem  
in the offshore industry



# Coupled mode flutter

# How to explain wing flutter ?

$$-\frac{\partial C_L}{\partial \theta}$$

Section	Smooth flow	Turbulent flow <sup>b</sup>	Reynolds number
	3.0	3.5	$10^5$
	0.	-0.7	$10^5$
	-0.5	0.2	$10^5$
	-0.15	0.	$10^5$
	1.3	1.2	66 000
	2.8	-2.0	33 000
	-10.	—	2 000–20 000
	-6.3	-6.3	$>10^5$
	-6.3	-6.3	$>10^5$
	-0.1	0.	66 000
	-0.5	2.9	51 000
	0.66	—	75 000

(Blevins, 1990)

But ...



Observation : the instability mechanism should involve flexural and torsional deformations.

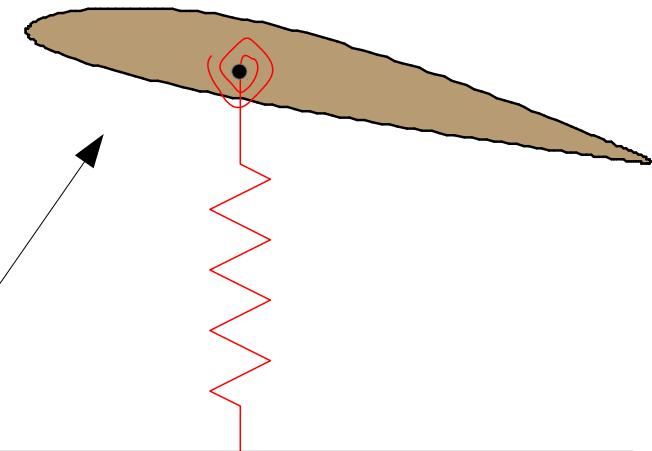
Thin profiles are stable  
with respect to galloping

## Example : flutter of a wing profile

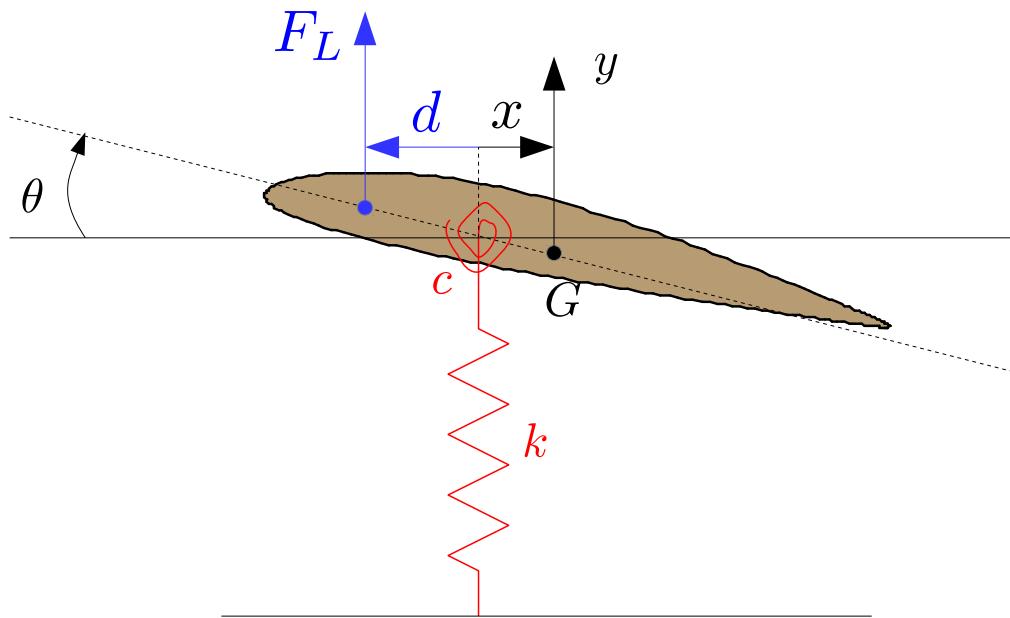


Coupled torsional and flexural modes of an airfoil

Equivalent 2D profile in translation and rotation



# The model



$G$	center of gravity
$m$	mass
$J$	moment of inertia

If \$G\$ is not at the elastic center, coupled flexural and torsional modes :

$$m\ddot{y} + ky + kx\theta = 0$$

$$J\ddot{\theta} + (c + kx^2)\theta + kxy = 0$$

With an incident flow :

$$\begin{bmatrix} m & 0 \\ 0 & J \end{bmatrix} \begin{bmatrix} \ddot{y} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} k & kx - \frac{1}{2}\rho U^2 S C'_L \\ kx & c + kx^2 - \frac{1}{2}\rho U^2 S(x + d) C'_L \end{bmatrix} \begin{bmatrix} y \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

# Eigenfrequencies and phase difference between eigenmodes components

$$\begin{bmatrix} m & 0 \\ 0 & J \end{bmatrix} \begin{bmatrix} \ddot{y} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} k & kx - \frac{1}{2}\rho U^2 S C'_L \\ kx & c + kx^2 - \frac{1}{2}\rho U^2 S(x+d) C'_L \end{bmatrix} \begin{bmatrix} y \\ \theta \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Dynamical equation of the form :

$$M\vec{q} + K\vec{q} = 0$$

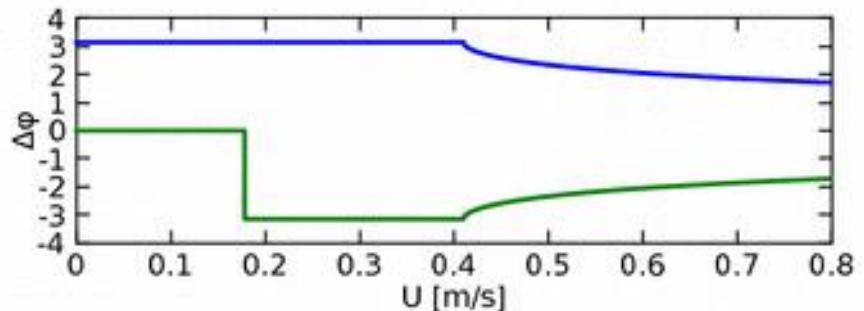
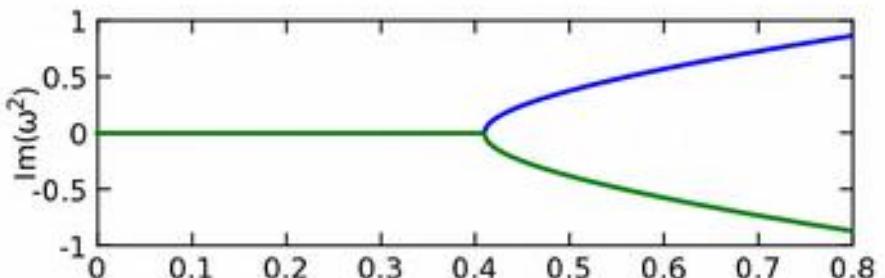
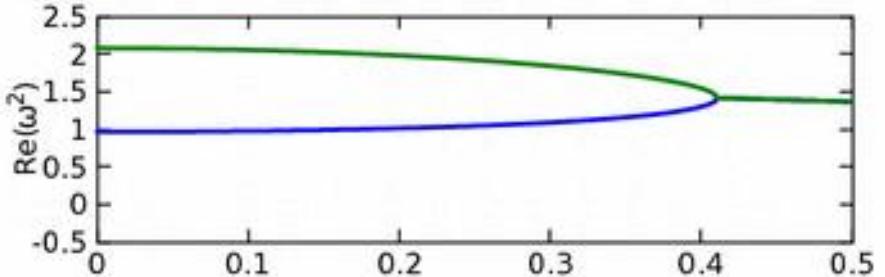
Solutions sought in the form :

$$\begin{aligned} \vec{q} &= \vec{q}_0 e^{i\omega t} \\ &= \vec{q}_0 e^{-\omega_i t} e^{i\omega_r t} \end{aligned}$$

Eigenvalue problem :

$$M^{-1} K \vec{q}_0 = \omega^2 \vec{q}_0$$

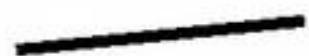
$$\rho = 1, m = 1, J = 1, k = 1, c = 2, d = 0.2, x = 0.2, s = 1$$



Negative phase difference

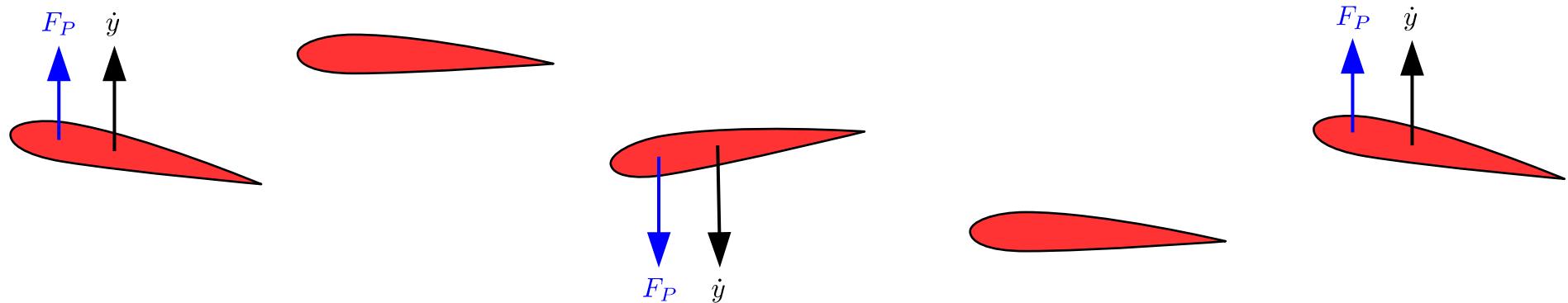


Positive phase difference

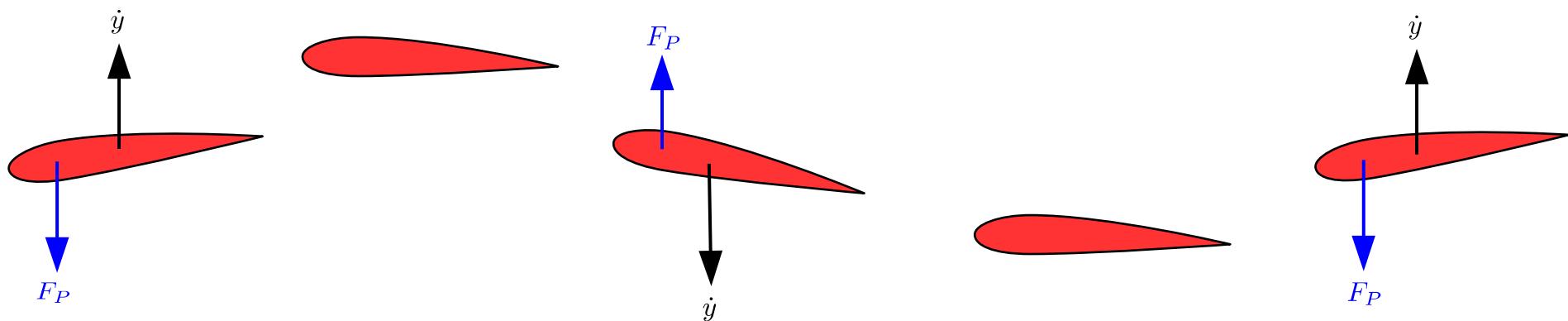


# Energy transfers

Unstable mode : positive work



Stable mode : negative work

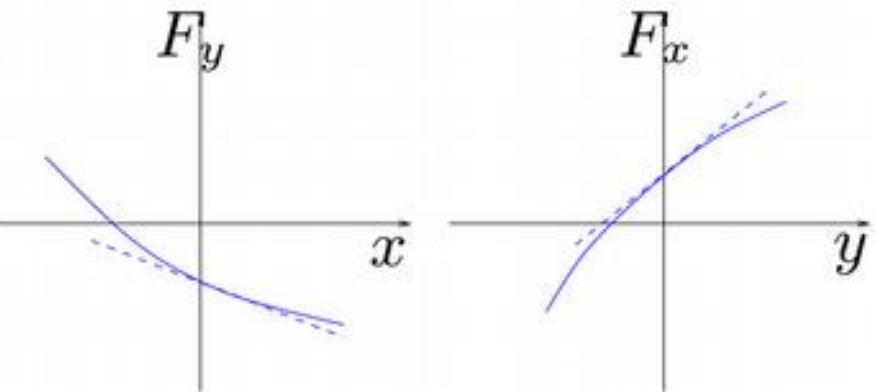
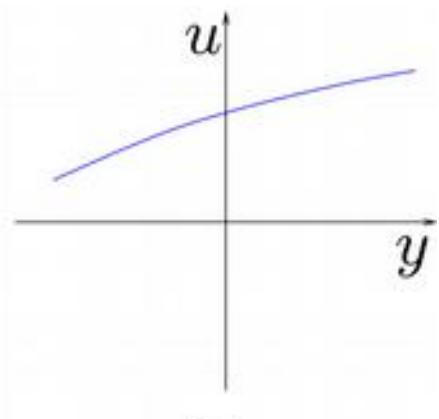
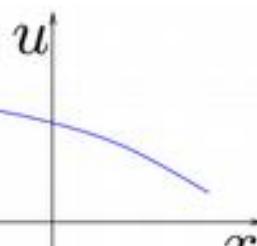
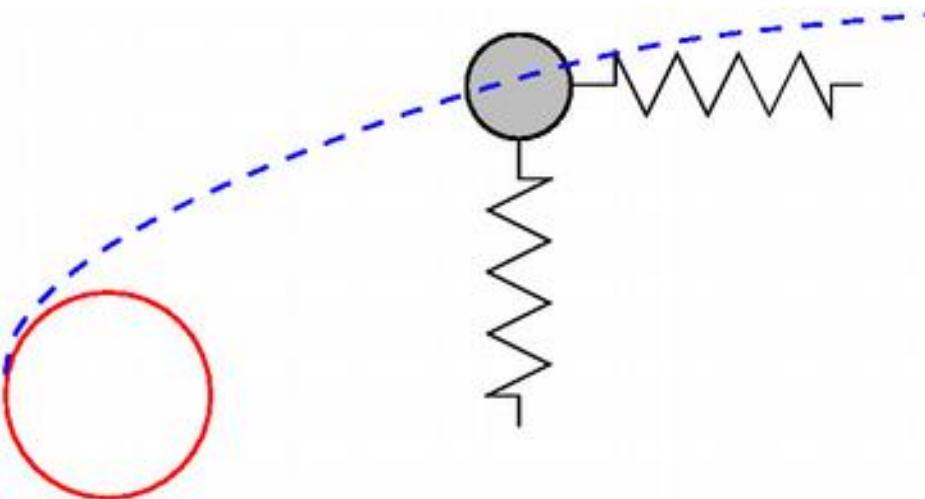


Work depends on the path : non-conservative force...

# Coupled mode flutter of cables

# Oscillations of a structure in the wake of another structure

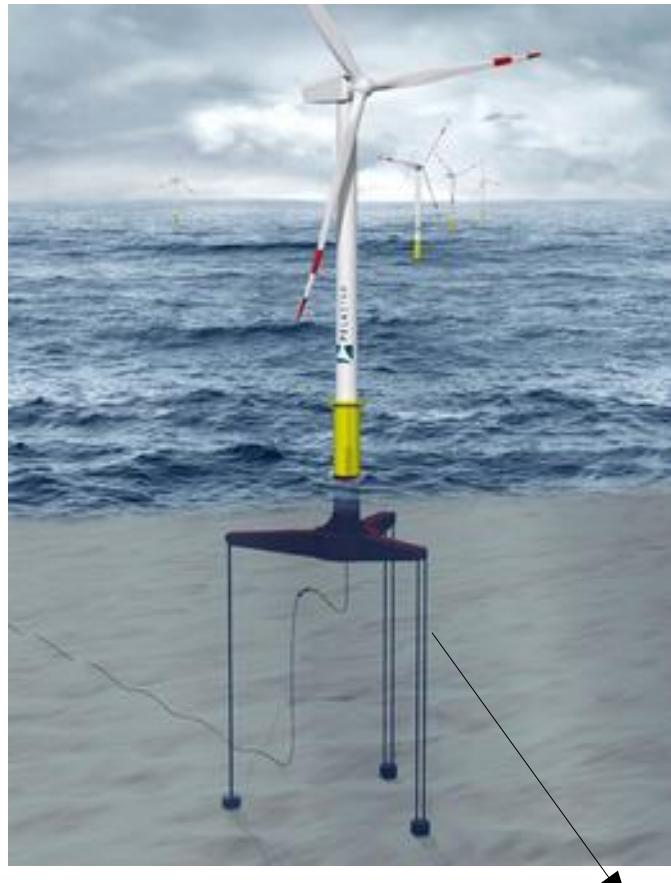
34



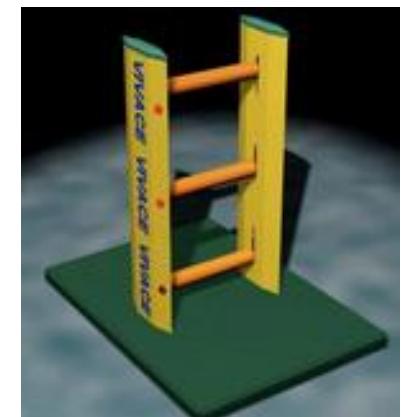
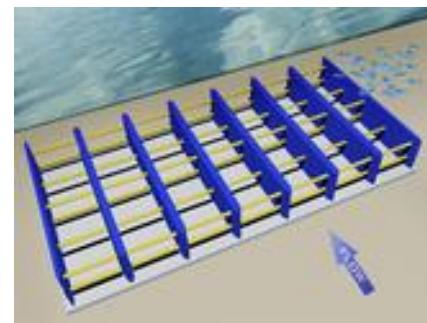
$$\begin{aligned} m\ddot{x} + k_x x &= x \frac{\partial F_x}{\partial x} + y \frac{\partial F_x}{\partial y} \\ m\ddot{y} + k_y y &= x \frac{\partial F_y}{\partial x} + y \frac{\partial F_y}{\partial y} \end{aligned}$$

# Vortex-induced vibration

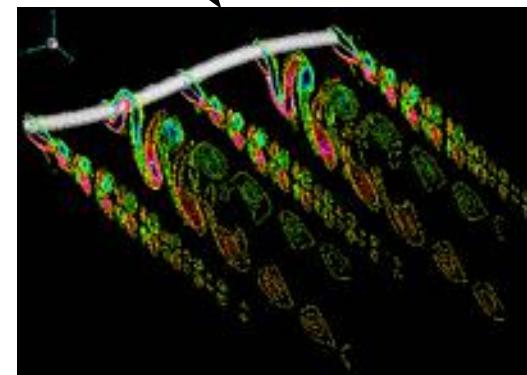
## Mooring lines in RME



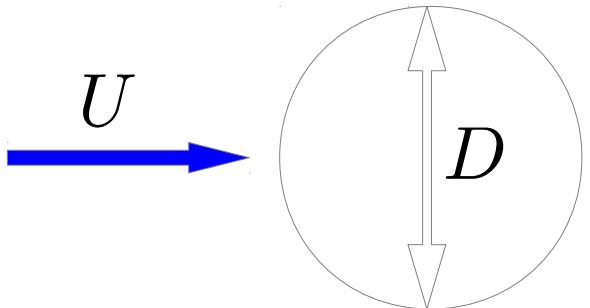
Energy harvesting



©2011 Vortex Hydro Energy



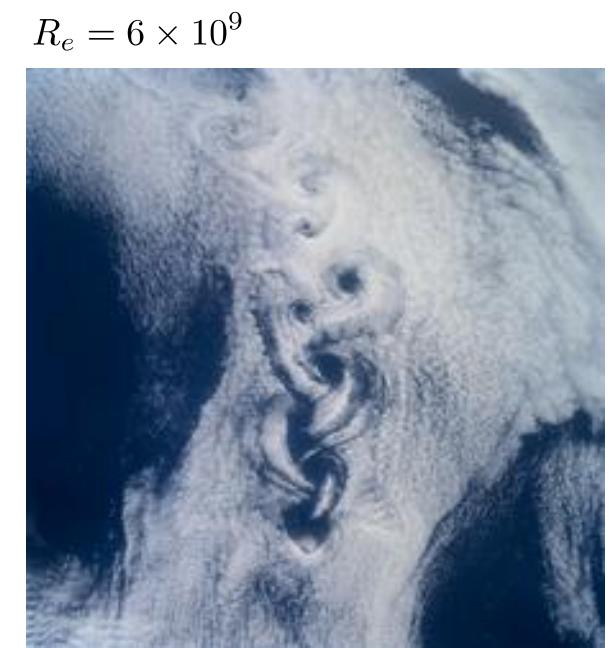
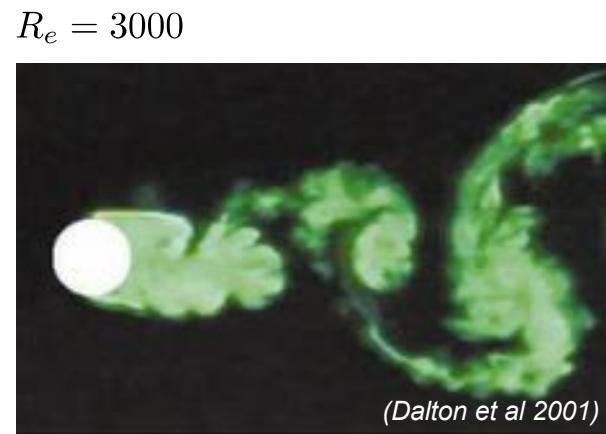
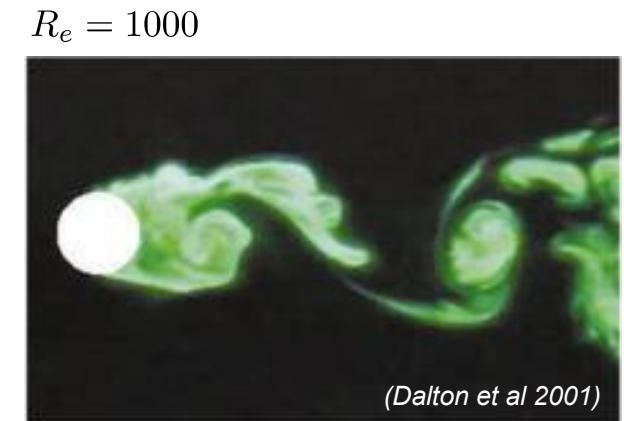
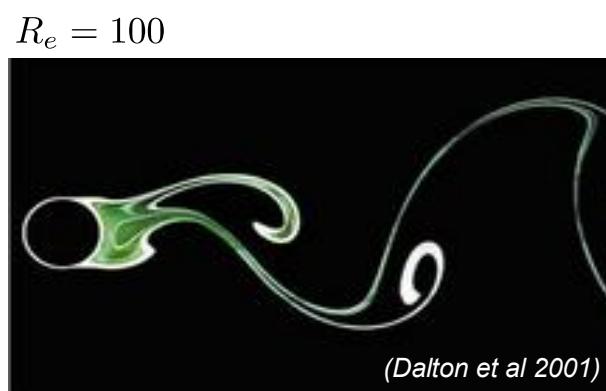
# Vortex shedding around a circular cylinder



$$R_e = \frac{UD}{\nu}$$



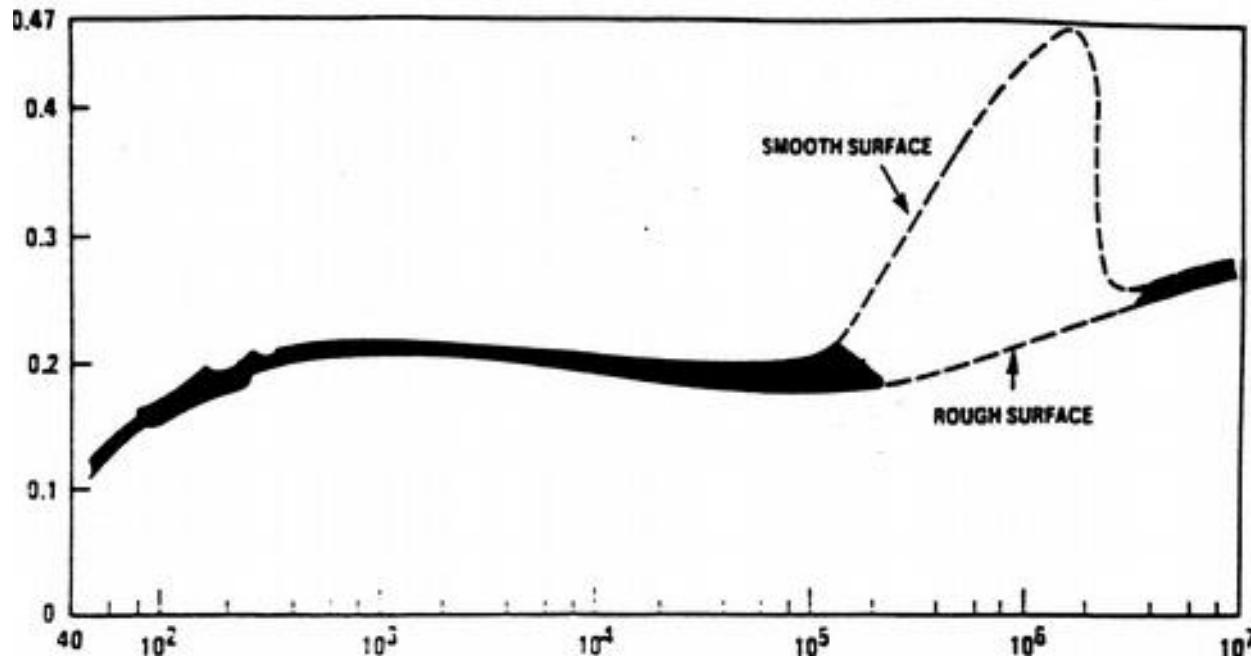
Thiria & Cadot



Rishiri island (source wikipedia)

# Frequency of the vortex street behind a cylinder

$$\text{Strouhal number } S_t = \frac{FD}{U}$$



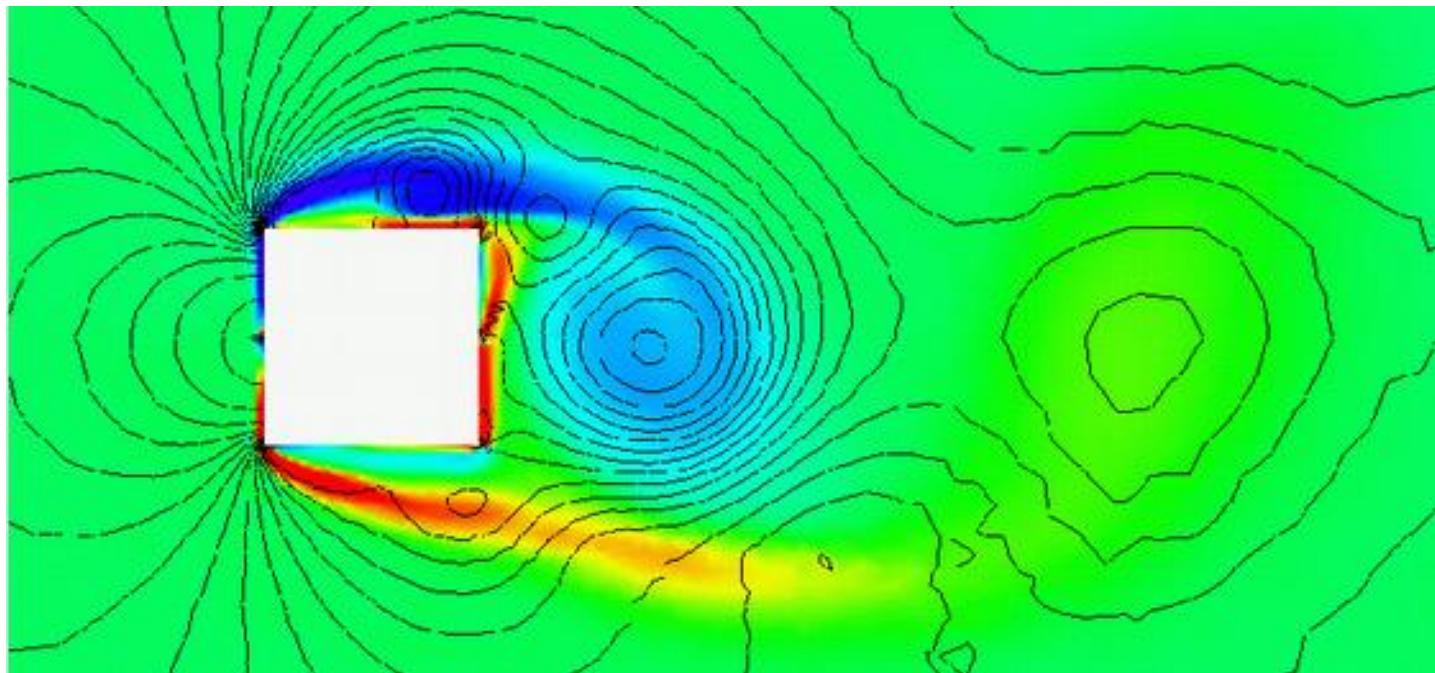
$$\text{Reynolds number } R_e = \frac{UD}{\nu}$$

Blevins, 1990

- Strouhal number almost constant (~0.2, 0.3)
- Frequency of the vortex shedding varies almost linearly with the flow velocity

## Other geometries

Square section   St ~ 0.16



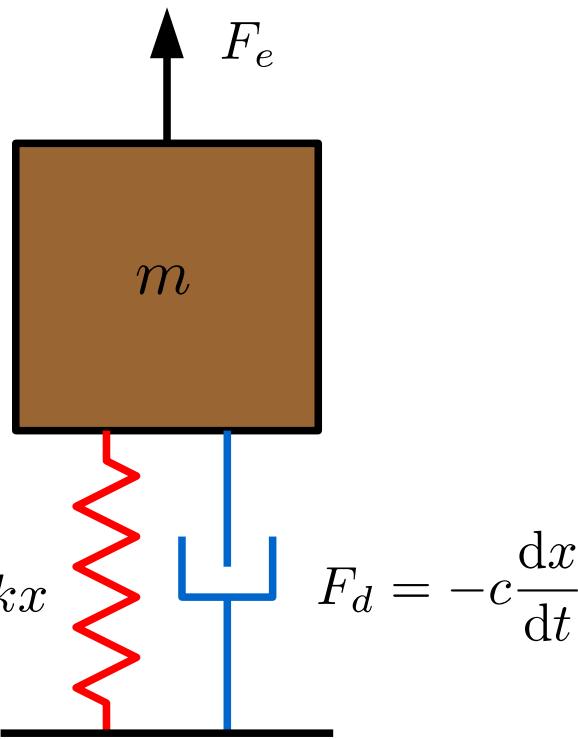
- Each cross-section has a unique Strouhal number
- Robust, generic, and predictable phenomenon

$$F = \frac{S_t U}{D}$$

This vortex shedding acts like a fluctuating force on the structure

Forcing → Vibrating structure → Response ?

# Harmonic oscillator : forced motion



$$m \frac{d^2x}{dt^2} + c \frac{dx}{dt} + kx = F_e$$

- Harmonic forcing :

$$F_e = \operatorname{Re}(F_0 e^{i\omega t}) \quad F_0 \in \mathbb{C}$$

- Hyp. : response at the same frequency :

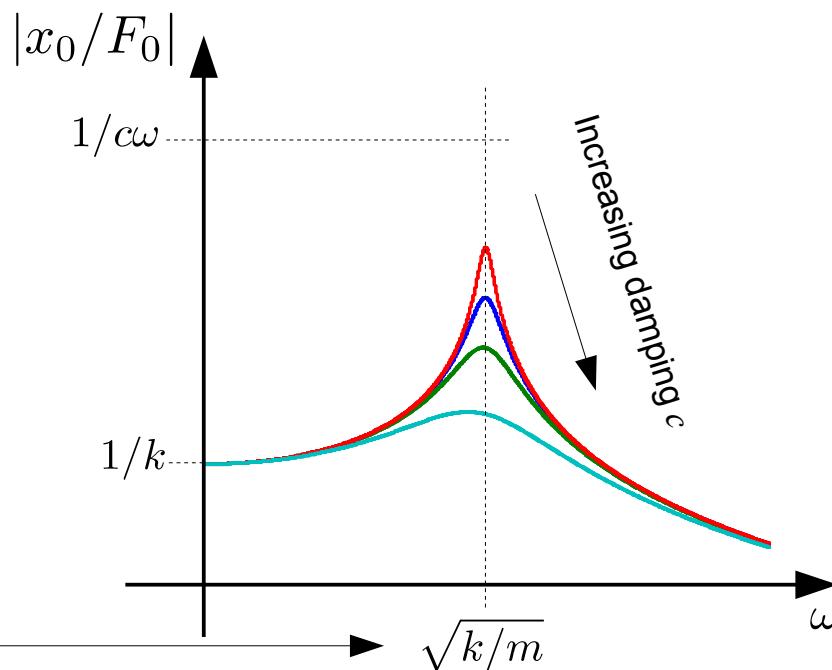
$$x = \operatorname{Re}(x_0 e^{i\omega t}) \quad x_0 \in \mathbb{C}$$

$$(-\omega^2 m + i\omega c + k)x_0 e^{i\omega t} = F_0 e^{i\omega t}$$

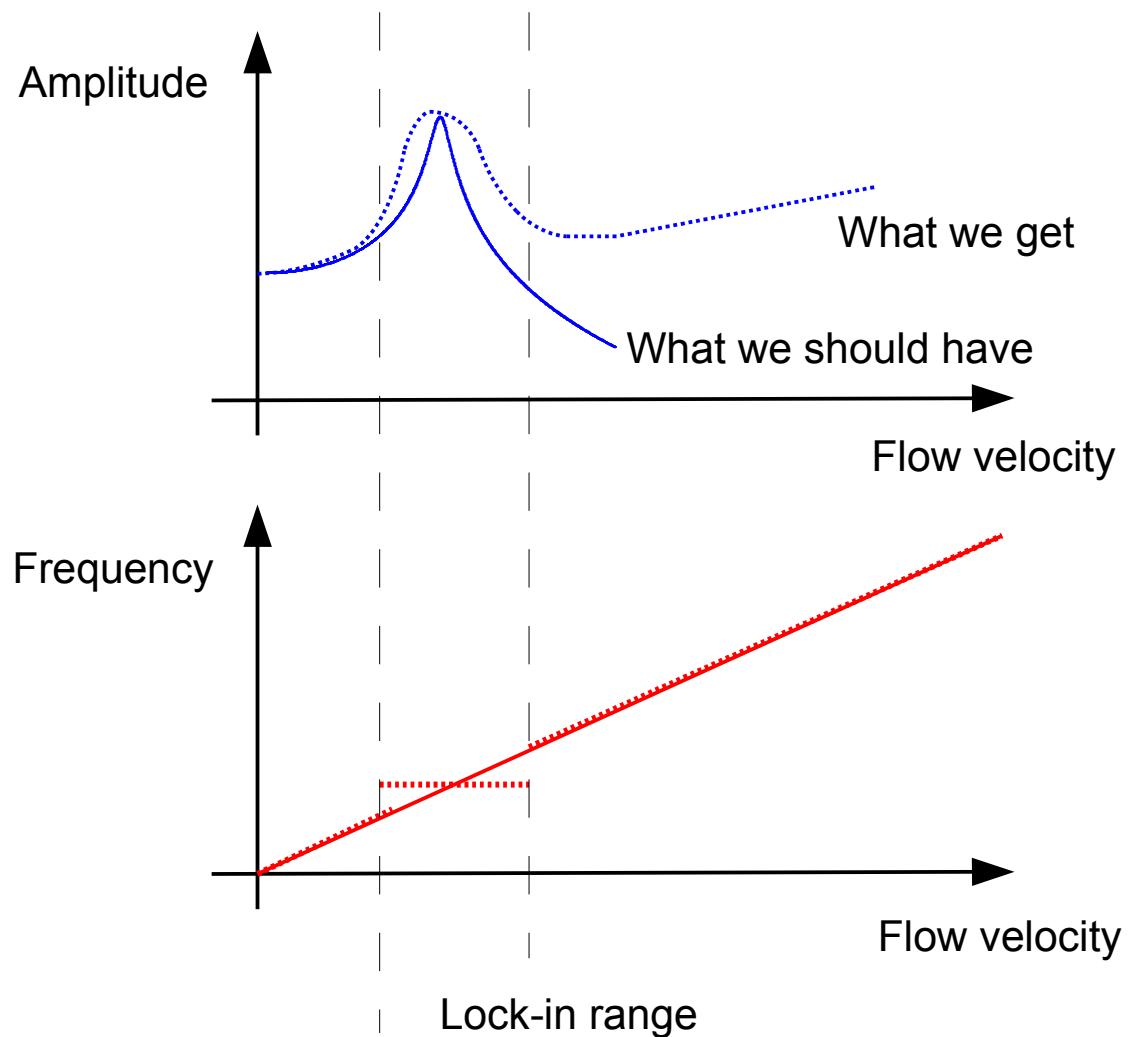
$$\frac{x_0}{F_0} = \frac{1}{m\left(\frac{k}{m} - \omega^2\right) + i\omega c}$$

(transfer function)

This is the frequency  
of free vibrating system



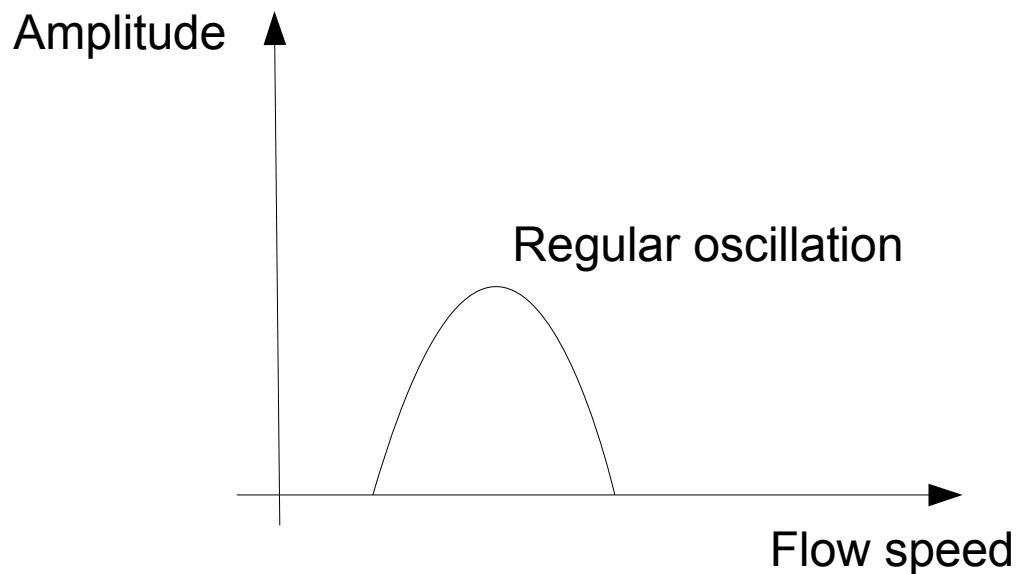
# Lock-in phenomenon



In the lock-in range, fluid-structure coupling is improved, frequency is locked to the eigenfrequency of the oscillator

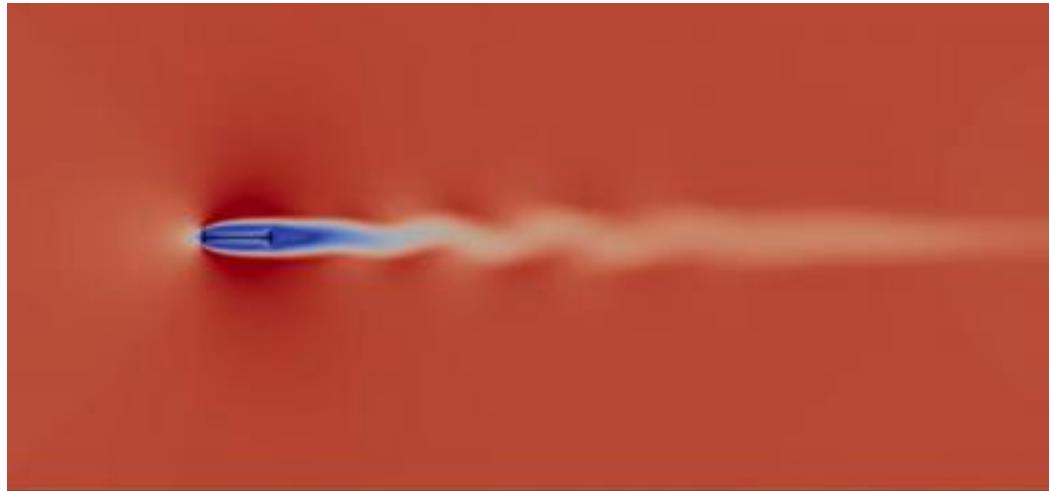
# Conclusion on vortex-induced vibration

VIV phenomenon predicts vibrations in a frequency range around the fundamental frequency of the oscillator



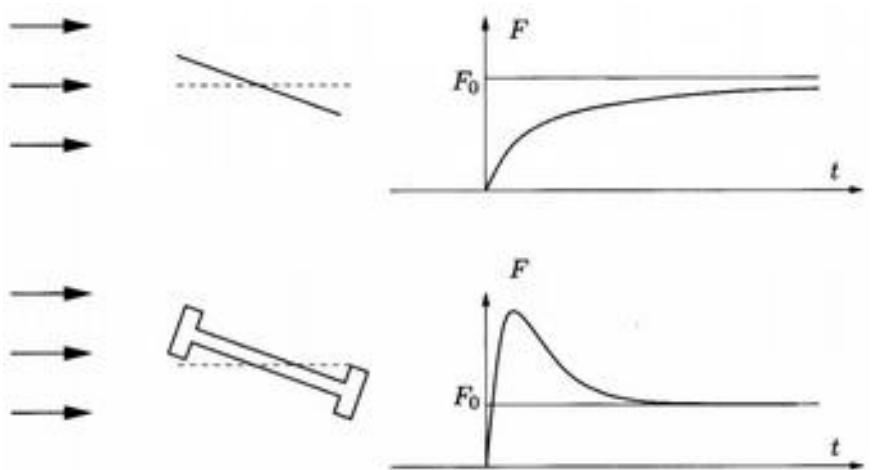
# Other instability phenomena





Not a VIV phenomenon !

Flow-structure instationnary dynamics

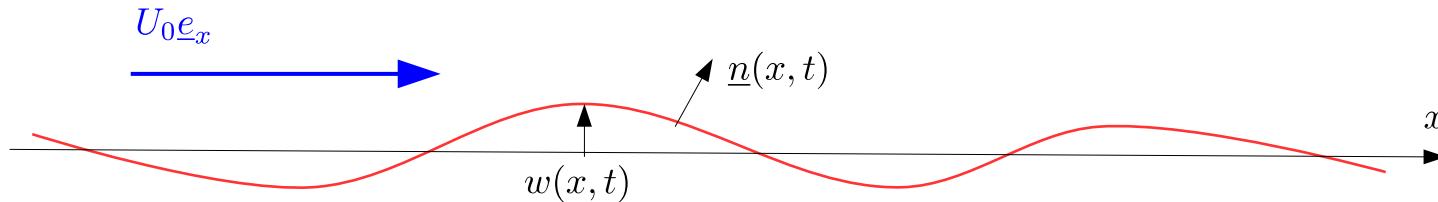




More general continuous problems

→ Axial flow instabilities

# Deformation of a boundary in presence of a potential flow



What kind of reactive forces are exerted on the solid ?

- For a displacement of the boundary of the form :

$$w(x, t) = q(t)\phi(x)$$

- Pressure in the fluid :

$$p = -\rho\varphi_2 \ddot{q} - \rho U_0 \left( \varphi_1 + \frac{\partial \varphi_2}{\partial x} \right) \dot{q} - \rho U_0^2 \varphi_1 q$$

# Deformation of a boundary in presence of a potential flow

- Pressure in the fluid :

$$p = -\rho\varphi_2 \ddot{q} - \rho U_0 \left( \varphi_1 + \frac{\partial \varphi_2}{\partial x} \right) \dot{q} - \rho U_0^2 \varphi_1 q$$

- Influence on the mode dynamics  $\rightarrow$  modal force calculation :

General form :  $f = \int_S (-p\underline{n}) \cdot \underline{\phi} \, dS$

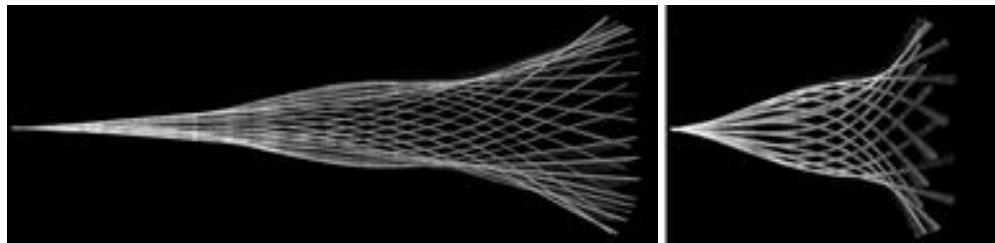
$\rightarrow$  Added mass, dissipation, stiffness

- If more than one mode is involved in the dynamics :

Modal force on mode  $i$  :  $f_i = \sum_j \int_S (-p_j \underline{n}) \cdot \underline{\phi}_i \, dS$

$\rightarrow$  Coupling through inertia, dissipation and stiffness terms

Plates in axial flow

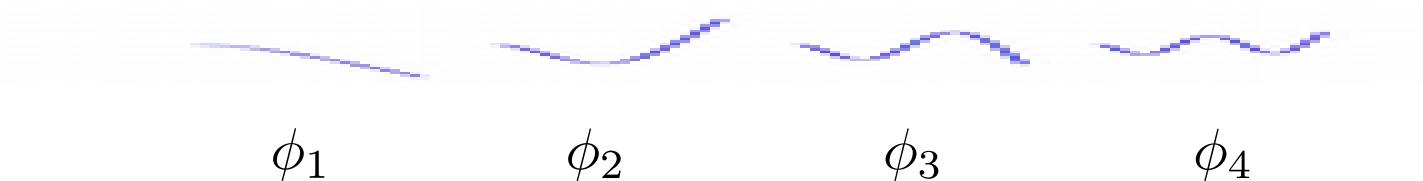


(Eloy et al, 2008)

Fluid-conveying pipe

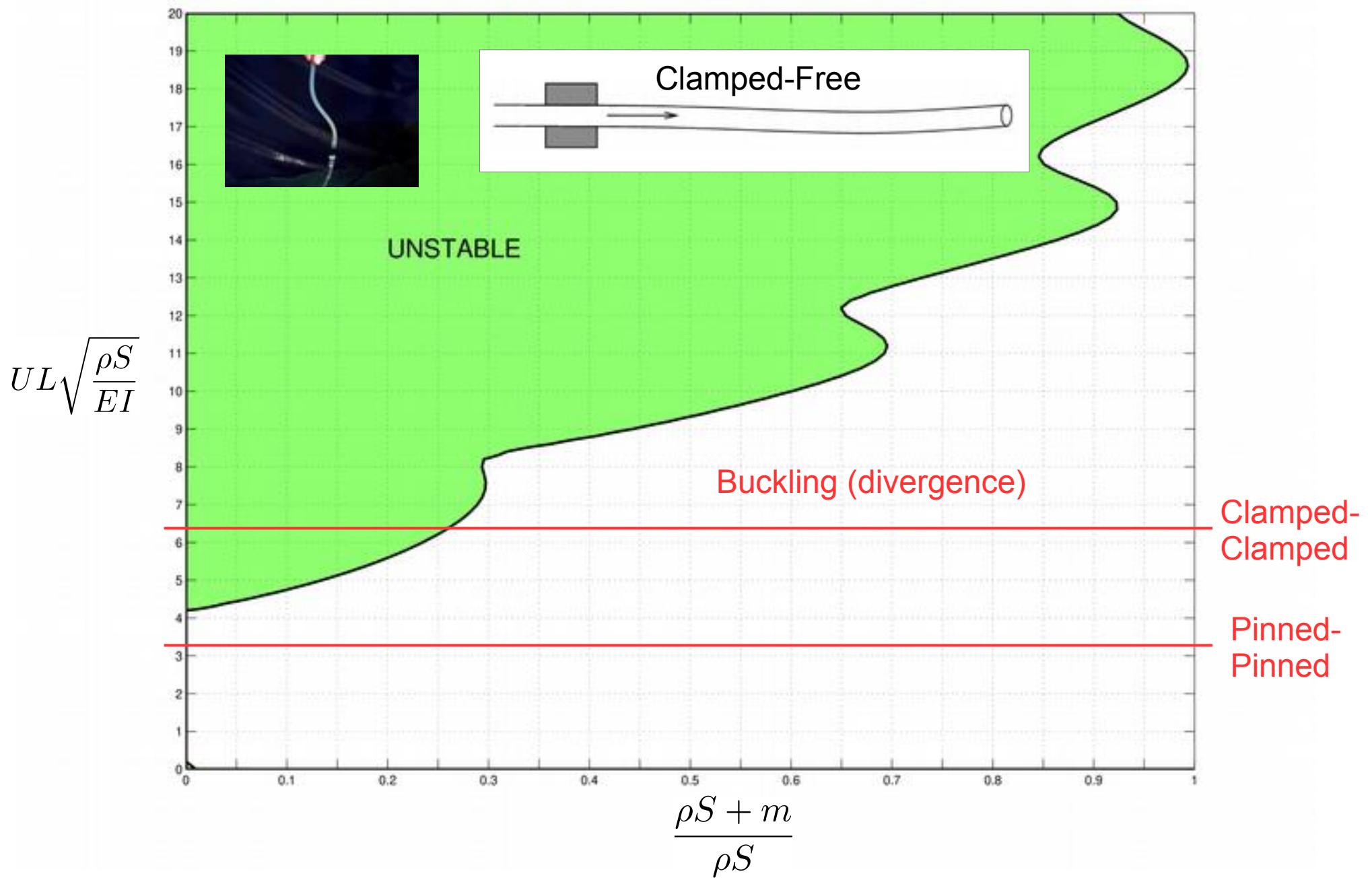


They can be modeled as beams in axial flow



The flow couples the modes dynamics through inertia, dissipation and stiffness terms  
→ instabilities

# Stability curve





# Conclusion

- **Flow-induced vibrations may have many different origins**

Negative induced stiffness

Negative induced damping

Coupled mode flutter

Vortex shedding

Other complex Flow-structure interactions

- **Surrounding flowing fluid may induce**

Added mass

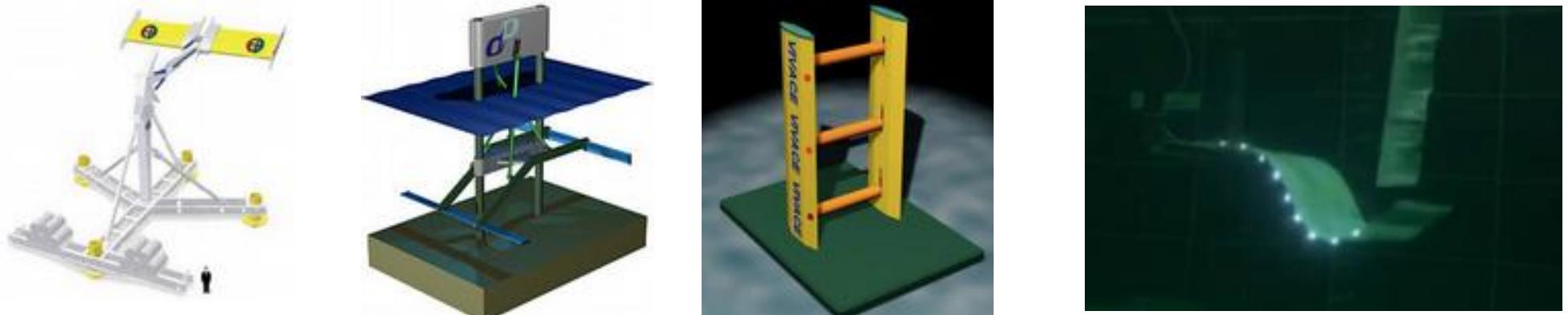
Added damping

Added stiffness

- **Axial flow systems**

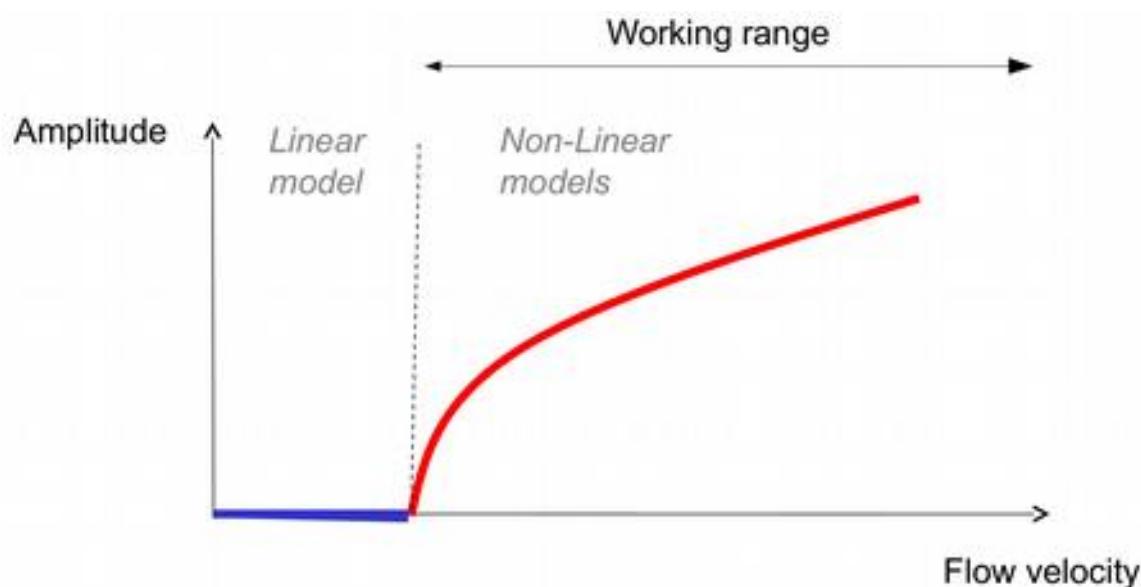
Mode coupling instabilities

# Perspectives



Most of the phenomena are explained by an instability induced by a flow-structure coupling  
 → linear phenomena

Energy harvesting → non linear permanent regime



1. The finite amplitude regime depends on the non linear phenomena and the dissipation induced by energy harvesting
2. An optimisation work is necessary to maximise the efficiency of the system