

# Computational Methods: Problem Set 2

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## Problem One

A competitive equilibrium is a set of prices  $\{p_t, r_t, w_t\}_{t=0}^{\infty}$ , an allocation  $\{k_t^d, n_t^d, y_t\}_{t=0}^{\infty}$  for the firm, and an allocation  $\{c_t, i_t, x_{t+1}, k_t^s, n_t^s\}_{t=0}^{\infty}$  for the household such that:

1. The firm's allocation solves:

$$\begin{aligned} \max_{k_t, n_t} \pi &= \sum_{t=0}^{\infty} p_t [y_t - r_t k_t - w_t n_t] \\ \text{s.t. } y_t &\leq F(k_t, n_t) \end{aligned}$$

2. The household's allocation solves:

$$\begin{aligned} \max_{c_t, n_t} \sum_{t=0}^{\infty} \beta^t u(c_t) \\ \text{s.t. } \sum_{t=0}^{\infty} p_t [c_t + i_t] &\leq \sum_{t=0}^{\infty} p_t [r_t k_t + w_t n_t] + \pi_t \\ x_{t+1} &= i_t, \quad 0 \leq n_t \leq 1, \quad 0 \leq k_t \leq x_t \\ k_0 &\text{ given, } c_t \geq 0, \quad x_{t+1} \geq 0 \\ \text{TVC: } \lim_{t \rightarrow \infty} p_t k_{t+1} &= 0 \end{aligned}$$

3. All markets clear for all  $t$ :  $k_t^d = k_t^s$ ,  $n_t^d = n_t^s$ ,  $c_t + i_t = y_t$

## Problem 2

The first order conditions are:

$$\begin{aligned} \beta^t c_t^{-\sigma} &= \lambda p_t \\ \beta^t \chi l_t^\eta &= \lambda p_t w_t \\ \lambda p_t &= \lambda p_{t+1} (1 - \delta + r_{t+1}) \\ z \alpha k^{\alpha-1} l_t^{1-\alpha} - r_t &= 0 \\ z(1 - \alpha) k^\alpha l_t^{-\alpha} - w_t &= 0 \\ \text{Market Clearing: } c_t + [k_{t+1} - (1 - \delta)k_t] &= z k_t^\alpha l_t^{1-\alpha} \end{aligned}$$

The first and third FOCs, as well as the assumption of steady state gives:

$$\bar{r} = \frac{1}{\beta} - 1 + \delta \tag{1}$$

The fourth FOC gives:

$$l_t^{1-\alpha} = \frac{r_t}{z \alpha k^{\alpha-1}} \tag{2}$$

Subbing equation 2 into the market clearing condition, as well as the assumption of steady state, gives:

$$\bar{c} = \left(\frac{\bar{r}}{\alpha} - \delta\right)\bar{k} \quad (3)$$

The first, second and fifth FOCs give:

$$c_t^{-\alpha} = \frac{\chi l_t^{\eta+1}}{z(1-\alpha)k_t^\alpha l_t^{1-\alpha}} \quad (4)$$

Subbing the second equation into the fourth gives:

$$\bar{c} = \chi^{\frac{-1}{\sigma}} \left[ r^{\frac{\eta+\alpha}{1-\alpha}} \left( \frac{1}{z\alpha} \right)^{\frac{1+\eta}{1-\alpha}} \left( \frac{\alpha}{1-\alpha} \right) \right]^{\frac{-1}{\sigma}} k^{\frac{-\eta}{\sigma}} \quad (5)$$

Subbing equation 5 into the market clearing condition gives:

$$\bar{k} = \left[ \chi^{\frac{-1}{\sigma}} D^{\frac{-1}{\sigma}} \left( \frac{\bar{r}}{\alpha} - \delta \right)^{-1} \right]^{\frac{\sigma}{\sigma+\eta}} \quad (6)$$

Where  $D = r^{\frac{\eta+\alpha}{1-\alpha}} \left( \frac{1}{z\alpha} \right)^{\frac{1+\eta}{1-\alpha}} \left( \frac{\alpha}{1-\alpha} \right)$

### Problem 3

The planners problem is:

$$\begin{aligned} & \max_{c_t, k_t, l_t} \sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \\ \text{s.t. } & F(k_t, l_t) = c_t + k_{t+1} - (1-\delta)k_t \\ & c_t \geq 0, \quad k_t \geq 0, \quad 0 \leq l_t \leq 1 \forall t \\ & k_0 \text{ given} \\ \text{TVC: } & \lim_{t \rightarrow \infty} \beta^t u'(F(k_t, l_t) - k_{t+1}) F_k(k_t, l_t) k_t = 0 \end{aligned}$$

### Problem 4

I use  $\chi = 57$  which gives  $\bar{l} = 0.3993$ . (I did this by trial and error)

### Problem 5

Part A: 8.18s, 215 iterations