Cheat Sheet for EE463

Performance Parameters

$$I_{rms} = \sqrt{\frac{1}{T}\int i^2 dt}$$

$$CrestFactor = \frac{V_{peak}}{V_{rms}}$$

$$DistortionFactor = \frac{I_{1rms}}{I_{rms}} = \frac{1}{\sqrt{1+THD^2}}$$

ϕ : phase difference between fundamentals of current and voltage

$$DisplacementPowerFactor = \cos(\phi)$$

$$TruePowerFactor = \frac{P}{S} = DPF\frac{I_{1,RMS}}{I_{RMS}}$$

$$THD = \sqrt{(\frac{I_{rms}}{I_{1rms}})^2 - 1}$$

Single Phase Diode Rectifier

$$V_{av} = \frac{2\sqrt{2}V_s}{\pi}(Fullwave), V_{av} = \frac{\sqrt{2}V_s}{\pi}(Halfwave)$$

u: commutation period

$$\cos(u) = 1 - \frac{2\omega L_s I_d}{\sqrt{2}V_s} \qquad \text{(Full wave)}$$

$$\cos(u) = 1 - \frac{\omega L_s I_d}{\sqrt{2}V_s} \qquad \text{(Half wave)}$$

$$A_u = \int_0^u V_s\sqrt{2}sin(\omega t)d\omega t = \omega L_s I_d$$

$$\text{Commutation Loss}: \frac{2\omega L_s I_d}{2\pi} \qquad \text{(Full wave)}$$

$$\frac{\omega L_s I_d}{2\pi} \qquad \text{(Half wave)}$$

$$I_{d,avg} = \frac{\int_b^f i(\theta)d\theta}{\pi}$$

$$I_{d,shortcircuit} = \frac{V_s}{\omega L_s}$$

For Flyback

$$\hat{I}_{sw peak} = \frac{1}{(1-D)}\frac{N_2}{N_1}I_o + \frac{N_1}{N_2}\frac{(1-D)T_s}{2L_m}V_o$$

$$\hat{V}_{sw peak} = V_d + \frac{N_1}{N_2}V_o = \frac{V_d}{(1-D)}$$

Three Phase Rectifier

- Half Wave

$$V_{av} = \frac{3\sqrt{6}V_s}{2\pi} = \frac{3\sqrt{2}V_{ll}}{2\pi}$$

- Full Wave

  Full Bridge Rectifier Average Output V_s:rms value of source voltage

$$V_{av} = \frac{3\sqrt{6}V_s}{\pi} - \frac{3wL_sI_d}{\pi}$$

Comparison of Rectifiers

Type	Vout	ΔVout	f_{ripple}
Single Phase	$\frac{2\sqrt{2}}{\pi}V_{ph}$ = 207 V	$\sqrt{2}V_{ph} = 325V$	100 Hz
3-phase Half Bridge	$\frac{3\sqrt{2}}{2\pi}V_{l-l}$ = 270 V	$\frac{\sqrt{2}}{2}V_{ph} = 162.5V$	150 Hz
3-phase Full Bridge	$\frac{3\sqrt{2}}{\pi}V_{l-l}$ = 540 V	$(1-\frac{\sqrt{3}}{2})\sqrt{2}V_{l-l} = 75.8V$	300 Hz

Pros / Cons of Ćuk:

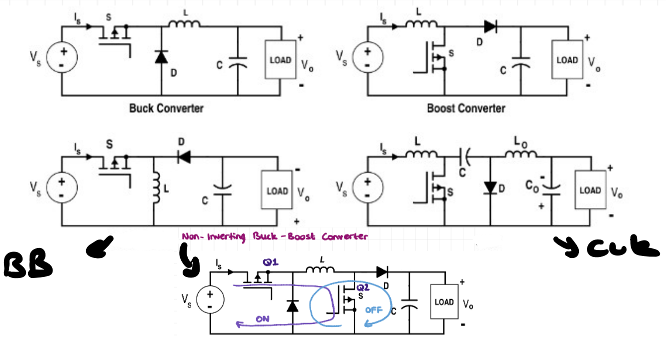
- ✓ input-output currents ripple free (no sharp edges)
- ✓ lower filtering requirements
- ✓ ≈ Constant source current
- ✗ C1 bulky, needs large ripple current rating
- ✗ Complex circuit

Voltage Ripple of Buck Converters

$$\Delta Q = \frac{\frac{\Delta I_L}{2}\cdot\frac{T_s}{2}}{2} = \frac{T_s \Delta I_L}{8} \qquad \Delta V_o = \frac{\Delta Q}{C} \Rightarrow \Delta V_o = \frac{T_s \Delta I_L}{8C}$$

using off-time $(1-D)$ & $I_L = \frac{1}{L}\int V_L dt$ $\Rightarrow \Delta I_L = \frac{V_o(1-D)T_s}{L} \Rightarrow \Delta V_o = \frac{T_s V_o (1-D) T_s}{8LC} \Rightarrow \boxed{\frac{\Delta V_o}{V_o} = \frac{(1-D)T_s^2}{8LC}}$

$T_s = \frac{1}{f_s}$, $f_c = \frac{1}{2\pi\sqrt{LC}} \Rightarrow LC = \frac{1}{4\pi^2 f_c^2} \rightarrow \frac{\Delta V_o}{V_o} = \frac{(1-D)\cdot 4\pi^2 f_c^2}{8 f_s^2} \Rightarrow \boxed{\frac{\Delta V_o}{V_o} = \frac{(1-D)\pi^2}{2}\left(\frac{f_c}{f_s}\right)^2}$



Buck Converter (Step-Down)

Gain: $V_o = DV_d$

$V_L = V_d - V_o$ (On), $V_L = -V_o$ (Off)

Inductor Ripple: $\Delta I_L = \frac{V_o(1-D)}{Lf_s}$

Voltage Ripple: $\frac{\Delta V_o}{V_o} = \frac{1-D}{8LCf_s^2}$

CCM/DCM Boundary: $I_{LB} = \frac{\Delta I_L}{2} = \frac{DT_s(V_d)(1-D)}{2L}$

CCM/DCM Boundary: $L_{min} = \frac{DT_s(V_d - V_o)}{2I_{LB}} = \frac{RT_s(1-D)}{2}$

Boost Converter (Step-Up)

Gain: $V_o = \frac{V_d}{1-D}$

$V_L = V_d$ (On), $V_L = V_d - V_o$ (Off)

Inductor Ripple: $\Delta I_L = \frac{V_d D}{Lf_s}$

Voltage Ripple: $\frac{\Delta V_o}{V_o} = \frac{D}{RCf_s}$

CCM/DCM Boundary: $I_{LB} = \frac{\Delta I_L}{2} = \frac{V_d DT_s}{2L}$

Min Inductance: $L_{min} = \frac{D(1-D)^2 R}{2f_s}$

Buck-Boost Converter

Gain: $V_o = V_d\frac{D}{1-D}$

$V_L = V_d$ (On), $V_L = -V_o$(Off)

Inductor Ripple: $\Delta I_L = \frac{V_d D}{Lf_s}$

Voltage Ripple: $\frac{\Delta V_o}{V_o} = \frac{D}{RCf_s}$

CCM/DCM Boundary: $I_{LB} = \frac{\Delta I_L}{2} = \frac{V_d DT_s}{2L}$

Min Inductance: $L_{min} = \frac{(1-D)^2 R}{2f_s}$

Cuk Converter

Gain: $V_o = -V_d\frac{D}{1-D}$

$V_{L1} = V_d$ (On), $V_{L1} = V_d - V_{C1}$ (Off)

Inductor Ripple (L_1): $\Delta I_{L1} = \frac{V_d D}{L_1 f_s}$

Inductor Ripple (L_2): $\Delta I_{L2} = \frac{V_d D}{L_2 f_s}$

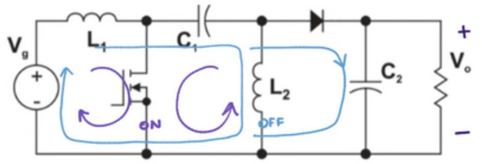
Voltage Ripple (C_o): $\frac{\Delta V_o}{V_o} = \frac{1-D}{8L_2C_2f_s^2}$

CCM/DCM Boundary: $I_{LB1} = \frac{V_d DT_s}{2L_1}$ (for L_1)

Min Inductance (L_1): $L_{1,min} = \frac{(1-D)^2 R}{2Df_s}$

Min Inductance (L_2): $L_{2,min} = \frac{(1-D)R}{2f_s}$

Cuk Feature: Input and Output currents are continuous (low ripple) due to inductors on both sides.



Can be used both as a buck or a boost converter

Popular in battery powered systems

Can be shut down completely

SEPIC CONVERTER

(single ended primary inductor conv.)

ON State

$V_{L_1} = V_s$, $V_{L_2} = V_{C_1} = V_s$

$-V_s + V_{L1} + V_{C1} - V_{L2} = 0$

in steady state: $V_{L1} = V_{L2} = 0$

assume C_1 large enough $\Rightarrow V_{C1}$ const

$\Rightarrow V_{C1} = V_s$

OFF state

$-V_s + V_{L1} + V_{C1} + V_o = 0$

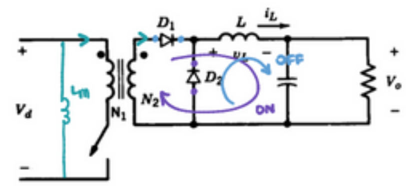
$V_{C1} \cong V_s \Rightarrow V_{L1} = -V_o$

$V_s \cdot D \cdot T_s + (-V_o)(1-D)T_s = 0 \Rightarrow \boxed{V_o = V_s \cdot \frac{D}{1-D}}$

Pros / Cons of SEPIC:

- ✓ Functions as non-inverting buck-boost
- ✓ Efficient as can be completely turned off
- ✓ Inductors can be combined to a single core
- ✓ Smaller inductance requirement
- ✗ Pulsating output current
- ✗ Large capacitance & ripple current rating required
- ✗ 4th order TF: difficult to control

FORWARD CONVERTER

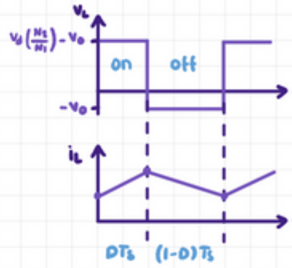


ON State

$$V_L = V_d \left(\frac{N_2}{N_1}\right) - V_o$$

OFF State

$$V_L = -V_o$$



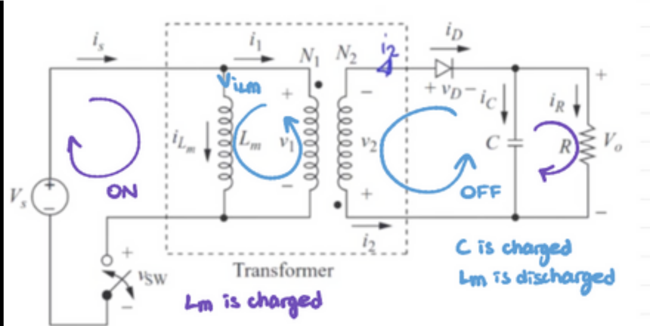
$$\left(V_d\left(\frac{N_2}{N_1}\right) - V_o\right) D = V_o(1-D)$$

$$V_d \cdot \frac{N_2}{N_1} \cdot D - V_o D = V_o - V_o D$$

$$\boxed{V_o = V_d \frac{N_2}{N_1} D}$$

Like a buck converter with added turns ratio

The energy stored in L_m, if discharged through the switch, could destroy the MOSFET. To prevent this, a snubber circuitry could be added but would cause important losses.
A practical solution is to add an extra winding:



FLYBACK CONVERTER

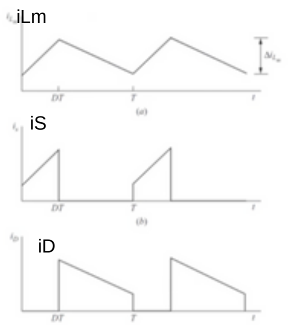
ON State

Diode is reverse biased, hence OFF
⇒ $i_2 = 0$ ⇒ $i_4 = 0$
⇒ transformer acts as a mere inductor
L_m is charging (as in buck-boost)

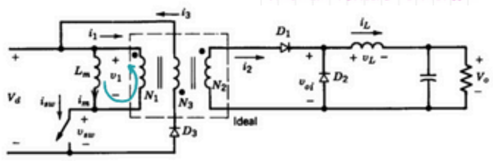
$$V_{Lm} = V_s$$

OFF State

$$V_{SW} = V_s + V_o\left(\frac{N_1}{N_2}\right)$$



Practical Forward Converter:



With this implementation, the energy stored in L_m is discharged to the source.

$$i_1 = -i_{Lm}$$

$$N_1 i_1 = N_2 i_2 - N_3 i_3$$

$$N_1 i_1 = -N_3 i_3$$

$$(L_m)_{min} = \frac{(1-D)^2 R}{2f} \left(\frac{N_1}{N_2}\right)^2$$

ON State

$V_1 = V_s$

$V_2 = V_1 \frac{N_2}{N_1} = V_s \frac{N_2}{N_1}$

$V_3 = V_1 \frac{N_3}{N_1} = V_s \frac{N_3}{N_1}$

$V_{D3} = -V_s - V_3 < 0$ ⇒ D_3 is off

$V_{Lm} = V_s$ $\Delta i_{Lm} = \frac{V_s D T_s}{L_m}$

$i_{sw} = i_1 + i_{Lm}$

OFF State

$i_{Lm} = -i_1$

$V_3 = -V_s$

$V_1 = V_3 \cdot \frac{N_1}{N_3} = -V_s \frac{N_1}{N_3}$

$V_2 = V_3 \cdot \frac{N_2}{N_3} = -V_s \cdot \frac{N_2}{N_3}$

$N_1 i_1 = N_2 i_2 - N_3 i_3$ $N_1 i_1 = -N_3 i_3$

$i_3 = \frac{-N_1}{N_3} i_1$

Transformer Reset: $t_m < (1-D)T_s$

Max Duty Cycle: $D_{max} = \dfrac{1}{1+(N_3/N_1)}$

$$V_{Lm} = V_1 = -V_s \frac{N_1}{N_3} = L_m \frac{di_{Lm}}{dt}$$

$$\frac{di_{Lm}}{dt} = \frac{-V_s}{L_m} \cdot \frac{N_1}{N_3}$$

There are multiple ways to calculate the voltage relations of a flyback. It can be done using:

- transformer flux
- steady state current
- voltage-seconds balance

FLYBACK CONVERTER

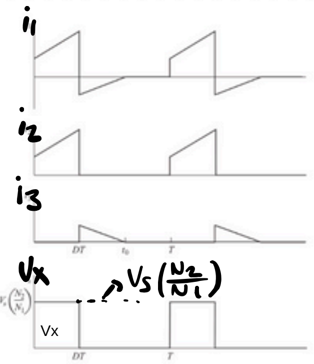
Gain: $V_o = V_d \dfrac{N_2}{N_1} \dfrac{D}{1-D}$

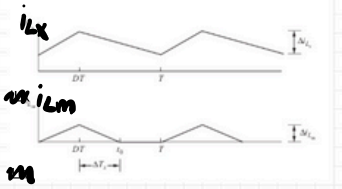
Inductor Voltage: $V_L = Vd$ (On), $V_L = -\dfrac{N_1}{N_2} V_o$ (Off)

Inductor Ripple: $\Delta I_L = \dfrac{N_1}{L_m N_2}(1-D)V_o$

Voltage Ripple: $\dfrac{\Delta V_o}{V_o} = \dfrac{N_1}{8 f_s C L_m N_2}(1-D)$

CCM/DCM Boundary: $I_{LB} = \dfrac{\Delta I_L}{2} = \dfrac{(1-D)V_o T_s N_1}{2 L_m N_2}$





$\Delta T_x < (1-D)T_s$

$$i_{Lm(max)} = \frac{V_s D T_s}{L_m} = \frac{V_s \cdot D \cdot T_s}{L_m}$$

$$\Delta T_x = \frac{\frac{V_s D T_s}{L_m}}{\frac{V_s}{L_m} \cdot \frac{N_1}{N_3}} = D \cdot T_s \cdot \frac{N_3}{N_1}$$

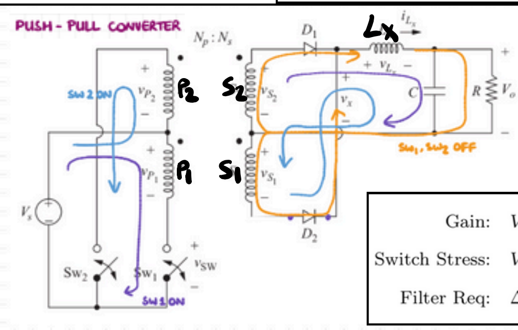
$$\Delta T_x = D T_s \frac{N_3}{N_1} < (1-D) T_s$$

$$D\left(1 + \frac{N_3}{N_1}\right) < 1$$

usually, $N_1 = N_3$

⇒ $\boxed{D < 0{,}5}$

PUSH-PULL CONVERTER



The design uses a center-tap transformer (turns ratio can be adjusted)

The converter has 3 operating regions:

Gain: $V_o = 2V_d \dfrac{N_2}{N_1} D$

Switch Stress: $V_{sw} = 2V_d$

Filter Req: $\Delta I_L = \dfrac{V_o(0.5-D)}{Lf_s}$ (for $D < 0.5$)

SW1 ON, SW2 OFF

D_1 conducts, D_2 reverse biased

$V_L = V_x - V_o = \frac{N_2}{N_1} V_s - V_o$

i_L increases linearly

SW2 ON, SW1 OFF

Symmetrical operation

$V_L = V_x - V_o = \frac{N_2}{N_1} V_s - V_o$

i_L increases linearly

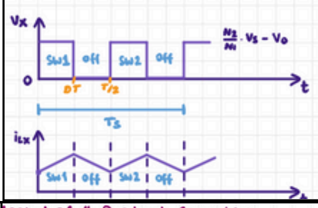
Both Switches OFF

For a period Δ, D_1 & D_2 conduct

$I_{D_1} = I_{D_2} = 0{,}5 I_L$

$V_x = 0$ ⇒ $V_L = -V_o$

inductor feeds the load



Waveform is repeating every $T_s/2$

$$DT_s + \Delta = T_s/2$$

$$\Delta = \frac{(1-2D)}{2} T_s$$

$$\boxed{\frac{V_o}{V_d} = 2\frac{N_2}{N_1} D}$$

Advantages/Disadvantages:

✓ Better usage of the core compared to Forward conv.
✓ Better B-H curve: uses both +/- regions
✗ Need to control 2 switches

Advantages/Disadvantages over Flyback

✓ Better use of transformer (direct & higher power transfer)
✓ Gapless core can be used (higher L_m → less ripple)
✓ Output inductor + diode ensure continuous output current

Practical Forward Converter:

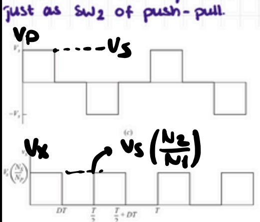
✗ Possibly increased cost
✗ Gain changes a lot in DCM & may require closed-loop control
✗ Higher voltage requirement for MOSFET

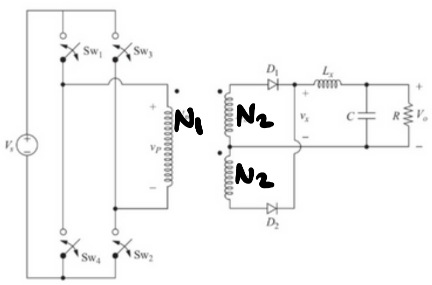
FULL BRIDGE ISOLATING CONVERTER

Operation very similar to push-pull conv.

Switches 1 & 2 are operated together just as SW_1 of push-pull.

Switches 3 & 4 are operated together just as SW_2 of push-pull.

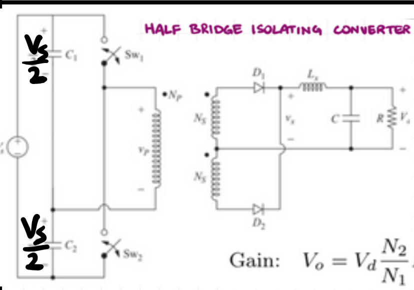




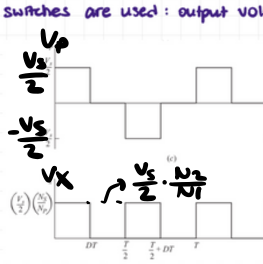
$$\boxed{V_o = 2 V_s \frac{N_2}{N_1} D}$$

$D < 0{,}5$

HALF BRIDGE ISOLATING CONVERTER

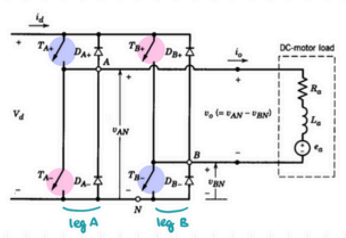


Very similar to full-wave converter.
Less switches are used: output voltage halved



Gain: $V_o = V_d \dfrac{N_2}{N_1} D$

Control of 4 Quadrant Converters



Bipolar Voltage Switching

T_{A+} & T_{B-} are ON or OFF together

T_{A-} & T_{B+} are complimentary to T_{A+}, T_{B-}

Output can be $+V_d$ or $-V_d$

Average output according to switch durations

Uni-Polar Voltage Switching

T_{A+} & T_{B+} controlled seperately

T_{A-} & T_{B-} complimentary of T_{A+}, T_{B+}

Output can be $+V_d$, 0, $-V_d$

$V_o = 0$ if two (+) or two (-) are ON

The output relation is the same in both cases:

$$V_o = V_d \frac{V_{control}}{\hat{V}_{tri}}$$

However, the ripple is lower & efficiency is higher in the uni-polar case as the voltage level does not have to be negative to decrease the average; instead it can be zero (0).

MAGNETIC DESIGN

For the same flux density, more energy can be stored with a material of lower permeability (μ).
When designing a core, air gaps can intentionally be added to increase the energy storage capacity.

Ferrite vs Powder Cores

Ferrite cores have sharp, powder cores have soft saturation
Powder cores are usually smaller
Powder cores' magnetic properties are less dependent on temperature
Powder cores have higher inductance tolerances

$$R = \frac{\ell}{\mu A}$$

$$L = N^2/R$$