Linear Regression with Metropolis-Hastings and Stan

We compare a simple Metropolis-Hastings algorithm with Stan for a real simple linear regression problem.

a) Loading of data. You can load the data via:

```r
df = read.csv("https://raw.githubusercontent.com/tensorchiefs/data/main/data/sbp.csv")
```

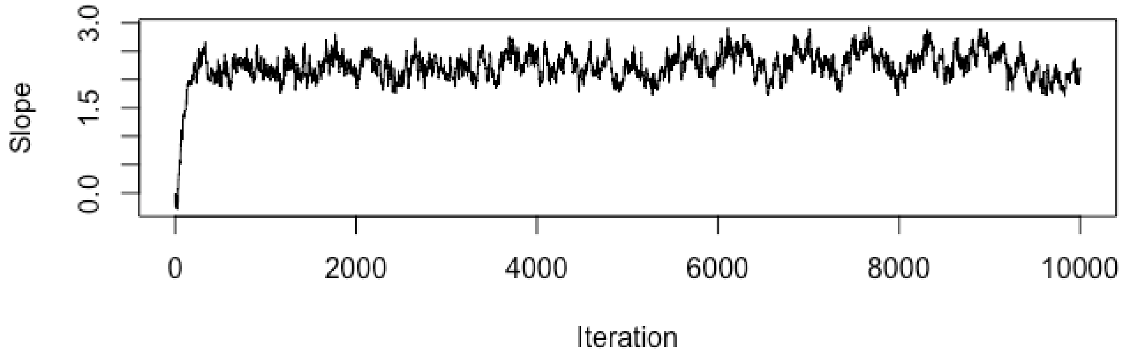
b) The code for the Metropolis-Hastings algorithm can be found at https://github.com/oduerr/da/blob/maste
you can use via

```r
#source('https://raw.githubusercontent.com/oduerr/da/master/code/MCMC_MH.R') #Load the Met
source('~/Documents/GitHub/da/code/MCMC_MH.R') #Load the Metropolis-Hastings code
proposal_sd <- c(0.1, 5, 5)
data = list(N=33, x=df$x, y=df$y, K=1)
n_iter = 10000
initial_values <- c(0, 0, 1)
# Run the Metropolis-Hastings algorithm
samples_hm <- metropolis_hastings(log_posterior, initial_values, data, n_iter, proposal_sd
```

What does the code do, what is the meaning of the variables `proposal_sd`,`n_iter`, `initial_values` and `samples`? Make a trace plot of the slope from the samples.

```r
#The code runs a Metropolis-Hastings algorithm to sample from the posterior distribution of
```

```r
plot(samples_hm[,1], type = "l", xlab = "Iteration", ylab = "Slope")
```



c) Formulate the same problem in Stan, have a lock at the code to come up with the complete code including the priors. Sample one chain from the posterior.

```
data{
  int<lower=0> N;
  vector[N] y;
  vector[N] x;
}

parameters{
  real a;
  real b;
  real<lower=0> sigma;
}

model{
  //y ~ normal(mu, sigma);
  y ~ normal(a * x + b, sigma);
  a ~ normal(0, 10);
  b ~ normal(0, 10);
  sigma ~ uniform(0,100);
}
```

```
library(cmdstanr)
```

```
This is cmdstanr version 0.8.0
```

```
- CmdStanR documentation and vignettes: mc-stan.org/cmdstanr


- CmdStan path: /Users/oli/.cmdstan/cmdstan-2.34.1


- CmdStan version: 2.34.1
```

```r
#Reading the model definition
m_rcmdstan <- cmdstan_model('~/Documents/GitHub/da/lab/lr_1_MH_vs_Stan/linear_regression.stan
samples_stan = m_rcmdstan$sample(data=data, chains=1, iter_sampling = 10000)
```

```
Running MCMC with 1 chain...

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Chain 1 finished in 0.2 seconds.
```
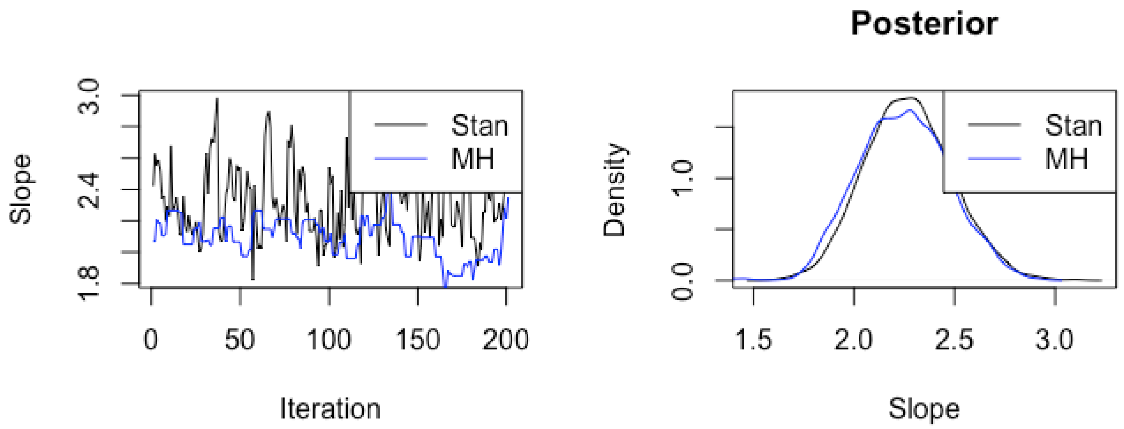

```
samples_stan = samples_stan$draws(format = 'df')
```

Compare the trace plots of the slope from the Metropolis-Hastings algorithm and Stan. Also compare the posterior plots. Which of the two algorithms has a larger n_eff?

```
  par(mfrow=c(1,2))
  plot(samples_stan$a[1000:1200], type = "l", xlab = "Iteration", ylab = "Slope")
  lines(samples_hm[1000:1200,1], col='blue')
  legend("topright", legend=c("Stan", "MH"), col=c("black", "blue"), lty=1)

  plot(density(samples_stan$a), main = "Posterior", xlab = "Slope", ylab = "Density")
  lines(density(samples_hm[,1]), col='blue')
  legend("topright", legend=c("Stan", "MH"), col=c("black", "blue"), lty=1)
```


Slope
Stan
MH
Iteration
Posterior
Density
Stan
MH
Slope