Lemma. $x > \log_2 x$ *on* $(0,\ \infty)$.

Proof : (*i*)

Define $f : (0,\ \infty) \longrightarrow \mathbb{R} : f(x) = x - \log_2 x,\ f$ *is continuous and differentiable on* $(0,\ \infty),\ f'(x) = 1 - \dfrac{1}{x \ln 2} = \dfrac{x - \log_2 e}{x} \begin{cases} > 0, & if\ x \in (\log_2 e,\ \infty) \\ = 0, & if\ x = \log_2 e \\ < 0, & if\ x \in (0,\ \log_2 e) \end{cases}$. *Note that* $\log_2 x$ *is strictly increasing on* $(0,\ \infty),\ f(\log_2 e) = \log_2 e - \log_2(\log_2 e) > \log_2 e - \log_2(\log_2 2.8) > \log_2 e - \log_2(\log_2 2\sqrt{2}) = \log_2 e - \log_2 1.5 > 0.$ $\therefore$ *By Mean Value Theorem* : $\forall x \in (\log_2 e,\ \infty),\ \exists c_x \in (\log_2 e,\ x)\ s.t.\ f(x) - f(\log_2 e) = f'(c_x)(x - \log_2 e) > 0,\ f(x) > f(\log_2 e) > 0.$

(*ii*)

Similarly, by Mean Value Theorem : $\forall x \in (0,\ \log_2 e),\ \exists t_x \in (x,\ \log_2 e)\ s.t.\ f(x) - f(\log_2 e) = f'(t_x)(x - \log_2 e) > 0,\ f(x) > f(\log_2 e) > 0.$

Accordingly, by(*i*)(*ii*) : $f > 0$ *on* $(0,\ \infty)$, *i.e.* $x > \log_2 x$ *on* $(0,\ \infty)$.

Proposition. $n > \log_2 n,\ \forall n \in \mathbb{N}$, *where* $\mathbb{N}$ *denotes the set of all positive numbers.*

Proof 1 : *By the above lemma, done.*

Proof 2 : $\log_2 x$ *is strictly increasing on* $(0,\ \infty)$.

$$\forall n \in \mathbb{N},\ n \geq 1,\ 2^n = (1+1)^n = \sum_{k=0}^{n} \binom{n}{k} \geq \binom{n}{0} + \binom{n}{1} = 1 + n > n \iff n = \log_2(2^n) > \log_2 n.$$