$Proposition.\ f:\mathbb{R}\backslash[-1,0]\longrightarrow(0,\infty):f(x)=\left(1+\dfrac{1}{x}\right)^{x}, show\ that\ f\ strictly\ increases\wedge f(x)\to\begin{cases}e, as\ x\to-\infty;\\ \infty, as\ x\to-1^{-};\\ 1, as\ x\to 0^{+};\\ e, as\ x\to\infty.\end{cases}$

$Proof:$

(i)

$g:(-\infty,1)\longrightarrow\mathbb{R}:g(t)\coloneqq e^{-t}-(1-t).\ Claim\ \forall t\in(-\infty,1)\setminus\{0\},g(t)>0.$

$\forall t\in(0,1),g'(t)=-e^{-t}+1>-1+1=0,g(t)=g(t)-g(0)=g'(c)(t-0)>0\ for\ some\ c\in(0,t)\ by\ Mean\ Value\ Thm;$

$analogously,\forall t\in(-\infty,0),g'(t)=-e^{-t}+1<-1+1=0,g(t)=g(t)-g(0)=g'(c)(t-0)>0\ for\ some\ c\in(t,0).$

(ii)

$h:\mathbb{R}\setminus[-1,0]\longrightarrow\mathbb{R}:h(x)\coloneqq\ln(1+\dfrac{1}{x})-\dfrac{1}{x+1}.\ Claim\ h(x)>0\ on\ \mathbb{R}\setminus[-1,0].$

$\forall x\in\mathbb{R}\setminus[-1,0],t\coloneqq\dfrac{1}{x+1}\in(-\infty,1)\setminus\{0\},g(t)>0\ by\ (i)\equiv e^{-t}>1-t\equiv\exp(\dfrac{-1}{x+1})>(1+\dfrac{1}{x})^{-1}\big(where\ 1+\dfrac{1}{x}>0\ on\ \mathbb{R}\setminus[-1,0]\big)\equiv 1+\dfrac{1}{x}>\exp(\dfrac{1}{x+1})\equiv\ln(1+\dfrac{1}{x})>\dfrac{1}{x+1}\equiv h(x)>0.$

(iii)

$\forall x\in\mathbb{R}\setminus[-1,0],f'(x)=\left(\left(1+\dfrac{1}{x}\right)^{x\ln e}\right)'=\left(\exp\left(x\ln\left(1+\dfrac{1}{x}\right)\right)\right)'=\exp\left(x\ln\left(1+\dfrac{1}{x}\right)\right)\cdot\left(x\ln\left(1+\dfrac{1}{x}\right)\right)'=f(x)h(x)>0\ by\ (ii),\therefore f\ is\ stricyly\ increasing.$

(iv)

$$\lim_{x\to-1^{-}}f(x)=\lim_{x\to-1^{-}}\exp\left(x\ln\left(1+\frac{1}{x}\right)\right)=\exp\left(\lim_{x\to-1^{-}}x\ln\left(1+\frac{1}{x}\right)\right)=\infty;analogously,$$

$$\lim_{x\to\infty}f(x)=\exp\left(\lim_{x\to\infty}x\ln\left(1+\frac{1}{x}\right)\right)=\exp\left(\lim_{x\to\infty}\frac{\ln\left(1+\frac{1}{x}\right)}{\frac{1}{x}}\right)=\exp\left(\lim_{x\to\infty}\frac{\left(\ln\left(1+\frac{1}{x}\right)\right)'}{\left(\frac{1}{x}\right)'}\right)$$

$$=\exp\left(\lim_{x\to\infty}\frac{\left(1+\frac{1}{x}\right)^{-1}\left(1+\frac{1}{x}\right)'}{\left(\frac{1}{x}\right)'}\right)=\exp\left(\lim_{x\to\infty}\frac{1}{\left(1+\frac{1}{x}\right)}\right)=e\ by\ L'H\hat{o}pital's\ rule.$$

$Similarly,\lim_{x\to-\infty}f(x)=e,\lim_{x\to0^{+}}f(x)=1.$