To begin with

LaTeX! $f(x,y) = \left(\frac{2|x|+2|y|}{2}\right)$

$$\cos a - \cos b = -2\sin(\frac{a+b}{2})\sin(\frac{a-b}{2})$$

$Q : (Aut\mathbb{Z}_{11}^*, \circ) \cong (\mathbb{Z}_4, +).$

$(In\ actuality, Aut\mathbb{Z} \cong \mathbb{Z}_2, Aut\mathbb{Z}_6 \cong \mathbb{Z}_2, Aut\mathbb{Z}_8 \cong \mathbb{Z}_2 \oplus \mathbb{Z}_2, Aut\mathbb{Z}_p \cong \mathbb{Z}_{p-1}\ for\ prime\ p.)$

$Proof:$

(i)

$2^0 = 1, 2^1 = 2, 2^2 = 4, 2^3 = 8, 2^4 = 5, 2^5 = 10, 2^6 = 9, 2^7 = 7, 2^8 = 3, 2^9 = 6, 2^{10} = 1, min\{s \in \mathbb{N} \mid 2^s = 1\} = 10.$

$\therefore (\mathbb{Z}_{11}^*, \cdot) = (<2>, \cdot)\ is\ cyclic, 1 \in B := \{s \in \mathbb{N} \cap [1,9] \mid (\mathbb{Z}_{11}^*, \cdot) = (<2^s>, \cdot)\} \neq \emptyset.$

$(In\ reality,\ p\ is\ prime \Rightarrow (\mathbb{Z}_p^*, \cdot)\ is\ cyclic.)$

(ii)

$s \in B \equiv \exists n \in \mathbb{Z}\ s.t.\ (2^s)^n = 2 \equiv 10|sn-1 \equiv \exists m \in \mathbb{Z}\ s.t.\ sn + 10m = 1 \equiv gcd(s,10) = 1 \equiv s \in \{1,3,7,9\};$

$that\ is, 2 = 2^1, 6 = 2^9, 7 = 2^7, 8 = 2^3\ are\ the\ generators\ of\ \mathbb{Z}_{11}^*.$

(iii)

$\forall \rho \in Aut\mathbb{Z}_{11}^*, <2> = Aut\mathbb{Z}_{11}^* = <\rho(2)> \equiv \rho(2) \in \{2,6,7,8\}.$

$Define\ \rho_{26} : \mathbb{Z}_{11}^* \longrightarrow \mathbb{Z}_{11}^* : \rho_{26}(2^i) = 6^i, where\ i \in \mathbb{Z} \cap [0,9]. \therefore \rho_{26}\ is\ surjetive\ immediately.$

$Claim\ \rho_{26} \in Aut\mathbb{Z}_{11}^*.$

$\forall i,j \in \mathbb{Z} \cap [0,9], 2^i = 2^j \Leftrightarrow 2^{i-j} = 1 \Leftrightarrow 10|i-j \Leftrightarrow 6^{i-j} = 1 \Leftrightarrow 6^i = 6^j,$

$where\ i-j = 10q + r\ for\ some\ q \in \mathbb{Z}, r \in \mathbb{Z} \cap [0,9] \Rightarrow r = 0; \rho_{26}\ is\ well-defined\ and\ injective.$

$\because <6> = Aut\mathbb{Z}_{11}^*, \rho_{26}(2^i \cdot 2^j) = \rho_{26}(2^{i+j}) = 6^{i+j} = 6^i \cdot 6^j = \rho_{26}(2^i) \cdot \rho_{26}(2^j), \therefore \rho_{26}\ is\ an\ epimorphism.$

(iv)

$Analogously, define\ \rho_{22}, \rho_{27}, \rho_{28} : \mathbb{Z}_{11}^* \longrightarrow \mathbb{Z}_{11}^* : \rho_{22}(2^i) = 2^i, \rho_{27}(2^i) = 7^i, \rho_{28}(2^i) = 8^i, where\ i \in \mathbb{Z} \cap [0,9].$

$Subsequently, similar\ to\ \rho_{26}, \rho_{22}(=\iota), \rho_{27}\ and\ \rho_{28}\ are\ distinct\ automorphisms, \therefore \rho \in \{\iota, \rho_{26}, \rho_{27}, \rho_{28}\}.$

$\therefore Aut\mathbb{Z}_{11}^* = \{\iota, \rho_{26}, \rho_{27}, \rho_{28}\}. Now\ note\ \rho_{27}^2(2) = \rho_{27}(7) = \rho_{27}(2^7) = 7^7 = 6 \neq 2, \therefore \rho_{27}^2 \neq \iota, |\rho_{27}| \neq 2.$

$Hence, Klein\ 4-group \ncong Aut\mathbb{Z}_{11}^* \cong \mathbb{Z}_4.$

$(Alternatively, define\ \lambda : Aut\mathbb{Z}_{11}^* \longrightarrow \mathbb{Z}_4 : \iota \mapsto 0, \rho_{26} \mapsto 2, \rho_{27} \mapsto 1, \rho_{28} \mapsto 3.$

$\therefore \lambda\ is\ bijective\ immediately.\ Claim\ \lambda\ is\ a\ homomorphism.$

$The\ operation\ preservation\ between\ \iota\ and\ \rho_{2k}'s\ is\ trivial.$

$\lambda(\rho_{26} \circ \rho_{27}) = \lambda(\rho_{28}) = 3 = 2 + 1 = \lambda(\rho_{26}) + \lambda(\rho_{27}), \lambda(\rho_{27} \circ \rho_{26}) = \lambda(\rho_{28}) = 3 = 1 + 2 = \lambda(\rho_{27}) + \lambda(\rho_{26}),$

$\lambda(\rho_{26} \circ \rho_{28}) = \lambda(\rho_{27}) = 1 = 2 + 3 = \lambda(\rho_{26}) + \lambda(\rho_{28}), \lambda(\rho_{28} \circ \rho_{26}) = \lambda(\rho_{27}) = 1 = 3 + 2 = \lambda(\rho_{28}) + \lambda(\rho_{26}),$

$\lambda(\rho_{27} \circ \rho_{28}) = \lambda(\iota) = 0 = 1 + 3 = \lambda(\rho_{27}) + \lambda(\rho_{28}), \lambda(\rho_{28} \circ \rho_{27}) = \lambda(\iota) = 0 = 3 + 1 = \lambda(\rho_{28}) + \lambda(\rho_{27}).$

$Thus, \lambda\ is\ an\ isomorphism, Aut\mathbb{Z}_{11}^* \cong \mathbb{Z}_4.)$