$Proposition.\ Given\ n \in \mathbb{N} \cap [2,\infty).\ \forall\ prime\ p \in [2,\sqrt{n}], p \nmid n.\ Show\ that\ n\ is\ prime.$

$Proof:$

$Suppose\ n\ is\ composite, \exists\ prime\ a\ s.t.\ a|n \Rightarrow a \notin [2,\sqrt{n}]\ by\ hypothesis; otherwise, prime\ a \in [2,\sqrt{n}], a \nmid n, C!$

$\therefore n-1 \geq a > \sqrt{n}, 2 \leq b \coloneqq \dfrac{n}{a} < \sqrt{n}, where\ a|n \Rightarrow b \in \mathbb{N}.$

$That\ is, b \in [2,\sqrt{n}] \wedge b|n, accordingly\ b\ is\ composite\ by\ hypothesis\ again.$

$Hence, \exists\ prime\ q \in \mathbb{N}\ s.t.\ q|b, then\ 2 \leq q < b, \therefore q \in [2,\sqrt{n}], thus\ q \nmid n\ by\ hypothesis.$

$Nonetheless, q|b \wedge b|n \Rightarrow q|n, C!$

$Proposition.\ \forall m,n \in \mathbb{Z}, gcd(m,n) = 1 \Rightarrow gcd(m^i n^j, m^k \pm n^l) = 1\ for\ all\ i,\ j,\ k,\ l \in \mathbb{N}.$

$Proof:$

(i)

$If\ d \coloneqq gcd(m^i n^j, m^k \pm n^l) \neq 1.\ \exists\ prime\ p \in \mathbb{N}\ s.t.\ p|d, \therefore p|m^i n^j,$

$p|m \vee p|n\ by\ Arithmetic\ Fundamental\ Thm.;\ w.l.o.g.\ let\ p|m.$

(ii)

$\because p|d, \therefore p|m^k \pm n^l, thus\ p|n\ by\ (i)\ and\ Arithmetic\ Fundamental\ Thm., \therefore p|gcd(m,n) = 1,\ C!$