Proposition. $\lim_{n\to\infty} \frac{2^n}{n^{\frac{n}{2}}} = 0.$

Proof : $\forall n \in \mathbb{N},\ a_n \coloneqq \frac{2^n}{n^{\frac{n}{2}}},\ \ \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\dfrac{2^{n+1}}{(n+1)^{\frac{n+1}{2}}}}{\dfrac{2^n}{n^{\frac{n}{2}}}} = \lim_{n\to\infty} \frac{2}{\sqrt{n+1}} \cdot \frac{1}{\sqrt{(1+\frac{1}{n})^n}} = 0 \cdot \frac{1}{\sqrt{e}} = 0.$

Accordingly (*actually* $\sum_{n=1}^{\infty} a_n$ *converges absolutely by Ratio Test*), $\therefore$ *for* $\frac{1}{2}$, $\exists n_0 \in \mathbb{N}$ *s.t.* $\frac{a_{n+1}}{a_n} < \frac{1}{2},\ \forall n \geq n_0$. *Thus*, $\forall n \geq n_0,\ a_n < \frac{1}{2}a_{n-1} < \left(\frac{1}{2}\right)^2 a_{n-2} < \ldots < \left(\frac{1}{2}\right)^{n-n_0} a_{n_0} \longrightarrow 0$ *as* $n \longrightarrow \infty$, *i.e.* $\lim_{n\to\infty} \frac{2^n}{n^{\frac{n}{2}}} = 0.$

Proposition. $\lim_{n\to\infty} \frac{n^{\frac{n}{2}}}{n!} = 0.$

Proof : $\forall n \in \mathbb{N},\ a_n \coloneqq \frac{n^{\frac{n}{2}}}{n!},\ \ \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{\dfrac{(n+1)^{\frac{n+1}{2}}}{(n+1)!}}{\dfrac{n^{\frac{n}{2}}}{n!}} = \lim_{n\to\infty} \frac{1}{\sqrt{n+1}} \cdot \sqrt{(1+\frac{1}{n})^n} = 0 \cdot \sqrt{e} = 0.$

Accordingly (*actually* $\sum_{n=1}^{\infty} a_n$ *converges absolutely by Ratio Test*), $\therefore$ *for* $\frac{1}{2}$, $\exists n_0 \in \mathbb{N}$ *s.t.* $\frac{a_{n+1}}{a_n} < \frac{1}{2},\ \forall n \geq n_0$. *Thus*, $\forall n \geq n_0,\ a_n < \frac{1}{2}a_{n-1} < \left(\frac{1}{2}\right)^2 a_{n-2} < \ldots < \left(\frac{1}{2}\right)^{n-n_0} a_{n_0} \longrightarrow 0$ *as* $n \longrightarrow \infty$, *i.e.* $\lim_{n\to\infty} \frac{n^{\frac{n}{2}}}{n!} = 0.$