

*Corollary. Given  $G$  is a group with  $|G| = 4$ ,  $G \cong \mathbb{Z}_4 \vee G \cong \text{Klein } 4 - \text{group } V$ .*

*Proof :*

(i)

$G := \{e, a, b, c\}$ , if  $\forall x \in G^* := G \setminus \{e\}, |x| = 2$ . Claim  $ab = c$ . Otherwise,  $ab = e, b = eb = a^2b = a(ab) = a, C!$ . Analogously,  $ba = c, ac = b = ca, bc = a = cb$ , hence  $G \cong V$ .

(ii)

If  $\exists x \in G^*$  s.t.  $|x| \neq 2$ .  $\therefore |x| \mid |G| = 4, \therefore |x| = 4, G = \langle x \rangle \cong \mathbb{Z}_4$ .