Proposition. $\mathcal{C}$ *is the Cantor set,* $|\mathcal{C}| = |\mathbb{R}|$.

$\forall x \in \mathcal{C}$, *the ternary expansion of* $x = \sum_{i=1}^{\infty} \frac{a_i}{3^i}$, *where* $a_i's \in \{0,1,2\}$.

That is, the ternary representation of $x = 0.a_1a_2a_3\ldots_{base3}$, *and the ternary representation ending in infinitely consecutive (repeating)* $2's$ *is excluded so that the ternary representation is unique.*

For instance, $0.0222\ldots_{base3}$ *will be identified as* $0.1000\ldots_{base3}$.

Furthermore, I_{kj} *denotes the jth interval (from LHS to RHS) in the kth step of the construction of* $\mathcal{C}$, *where* $k \in \mathbb{N}, j \in \{1,2,3,\ldots,2^k\}$.

Moreover, x *is in some* $I_{kj} \wedge$ *all of its nonzero digits* $\neq 1 \Longleftrightarrow x$ *is the left endpoint of* I_{kj} *by induction.*

Claim : For the x just mentioned, $\exists i \in \mathbb{N}$ s.t. $a_i = 1 \Rightarrow a_m \neq 1$ for all $m \in \{1,2,3,\ldots,i-1\} \wedge$ $a_m = 0$ for all $m \in \mathbb{N}$ with $m \geq i+1$.

Proof : Suppose $\exists i \in \mathbb{N}$ s.t. $a_i = 1, i \in L := \{l \in \mathbb{N} \mid a_l = 1\} \neq \emptyset, n := minL \in L$ by well − ordering principle, $n \leq i \wedge a_m \neq 1$ for all $m \in \{1,2,3,\ldots,n-1\}$. If $n \neq i$, $0.a_1a_2a_3\ldots a_{n-1}1_{base3} < x < 0.a_1a_2a_3\ldots a_{n-1}2_{base3}$, note $0.a_1a_2a_3\ldots a_{n-1\,base3}$ is the left endpoint of $I_{(n-1)j}$ for some $j \in \{1,2,3,\ldots,2^{n-1}\}$, then

$0.a_1a_2a_3\ldots a_{n-1\,base3}$ is the left endpoint of $I_{n(2j-1)}$,

$0.a_1a_2a_3\ldots a_{n-1}1_{base3}$ is the right endpoint of $I_{n(2j-1)}$,

$0.a_1a_2a_3\ldots a_{n-1}2_{base3}$ is the left endpoint of $I_{n(2j)}$,

$\therefore x \notin \mathcal{C}$ (that is, x is deleted in the procedure of constructing $\mathcal{C}$), $C!$

$\therefore n = i, a_m \neq 1$ for all $m \in \{1,2,3,\ldots,i-1\}$.

Analogously, if $\exists m \in \mathbb{N}$ with $m \geq i+1$ s.t. $a_m \neq 0$, $0.a_1a_2a_3\ldots a_{i-1}1_{base3} < x < 0.a_1a_2a_3\ldots a_{i-1}2_{base3}$,

$\therefore x \notin \mathcal{C}, C!$ $\therefore a_m = 0$ for all $m \in \mathbb{N}$ with $m \geq i+1$.

Accordingly, by the construction process of $\mathcal{C}$ and induction, the ternary representation of x only contains a finite quantity of nonzero digits $\Longleftrightarrow x$ is an endpoint of some I_{kj}.

Note $\mathcal{C}$, thus, does not merely contain the endpoints but also points which are not the endpoints of $I_{kj}'s$.

Particularly, $0.202020\ldots_{base3} = \dfrac{2/3}{1-1/9} = \dfrac{3}{4}$.

Therefore, $\mathcal{C} \subseteq \mathcal{A} := \left\{ \sum_{i=1}^{\infty} \frac{a_i}{3^i} \,\middle|\, a_i's \in \{0,2\} \right\}$. Now $\forall x \in \mathcal{A}, n \in \mathbb{N}, x = \sum_{i=1}^{\infty} \frac{a_i}{3^i}$ for some $a_i's$ in $\{0,2\}$, $x_n := \sum_{i=1}^{n} \frac{a_i}{3^i} \in \mathcal{C}$ is closed, $\therefore x = \lim_{n\to\infty} x_n \in \mathcal{C}, \mathcal{A} \subseteq \mathcal{C}$. Consequently, $\mathcal{C} = \mathcal{A}$.

Define $f : [0,1) \longrightarrow \mathcal{C} : f(x) = \sum_{i=1}^{\infty} \frac{2b_i}{3^i}$, where $x = \sum_{i=1}^{\infty} \frac{b_i}{2^i}$ for some $b_i's$ in $\{0,1\}$.

Similarly, the binary representation ending in infinitely consecutive (repeating) $1's$ is excluded so that the binary representation is unique.

For example, $0.0111\ldots_{base2}$ will be identified as $0.1000\ldots_{base2}$.

$\therefore$ In $Im(f)$, the ternary representation ending in infinitely consecutive (repeating) $2's$ is also excluded s.t. the ternary representation is unique in $Im(f)$.

It follows that f, obviously, is well − defined.

And suppose $\sum_{i=1}^{\infty} \frac{b_i}{2^i} = x \neq t = \sum_{i=1}^{\infty} \frac{v_i}{2^i}$ for some $b_i's, v_i's$ in $\{0,1\}$

$\Rightarrow \emptyset \neq P := \{p \in \mathbb{N} \mid b_p \neq v_p\} \wedge q := minP\ exists\ in\ P \cap \mathbb{N}\ by\ well-ordering\ principle$

$\Rightarrow b_i = v_i\ for\ i = 1, 2, 3, \ldots, q-1 \wedge b_q \neq v_q\ (w.l.o.g.\ assume\ 1 = b_q > v_q = 0)$

$\Rightarrow 2b_i = 2v_i\ for\ i = 1, 2, 3, \ldots, q-1 \wedge 2 = 2b_q > 2v_q = 0$

$\Rightarrow f(x) \geq 0.(2b_1)(2b_2)(2b_3)\ldots(2b_{q-1})2 > 0.(2b_1)(2b_2)(2b_3)\ldots(2b_{q-1})1 = 0.(2v_1)(2v_2)(2v_3)\ldots(2v_{q-1})1$

$> 0.(2v_1)(2v_2)(2v_3)\ldots(2v_{q-1})0(2v_{q+1})(2v_{q+2})(2v_{q+3})\cdots = f(t) \Rightarrow f(x) \neq f(t) \Rightarrow f\ is\ injective.$

$Hence\ |\mathbb{N}| < |[0,1)| \leq |\mathcal{C}| \leq |\mathbb{R}|, and\ thus\ |\mathcal{C}| = |\mathbb{R}|\ by\ the\ Continuum\ Hypothesis.$