AMC12/AIME 2022 Practice Series, P 1. Start and Bars

**Important: We are practicing and learning, not taking exam. If you get stuck for any of these questions for more than 20 minutes, check the answer quickly. You can find all problem solutions at here:**

**https://artofproblemsolving.com/wiki/index.php/AMC\_Problems\_and\_Solutions**

The **ball-and-urn** technique, also known as **stars-and-bars**, is a commonly used technique in [combinatorics](https://artofproblemsolving.com/wiki/index.php/Combinatorics).

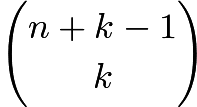
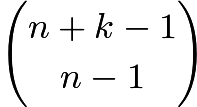
It is used to solve problems of the form: how many ways can one distribute $k$ indistinguishable objects into $n$ distinguishable bins? We can imagine this as finding the number of ways to drop $k$ balls into $n$ urns, or equivalently to arrange $k$ balls and $n-1$ dividers. For example,

\[****||\]

\[***|*|\]

\[*|**|*\]



represent the ways to put $k=4$ objects in $n=3$ bins. The number of ways to do such is , or .

Video Explanation: <https://www.youtube.com/watch?v=PGDaKxFVS9Q>

# Ex. 1 2016 AMC10A Problem 20

For some particular value of $N$, when $(a+b+c+d+1)^N$ is expanded and like terms are combined, the resulting expression contains exactly $1001$ terms that include all four variables $a, b,c,$ and $d$, each to some positive power. What is $N$?

$\textbf{(A) }9 \qquad \textbf{(B) } 14 \qquad \textbf{(C) } 16 \qquad \textbf{(D) } 17 \qquad \textbf{(E) } 19$

Hint:

All the desired terms are in the form $a^xb^yc^zd^w1^t$, where $x + y + z + w + t = N$

# Ex. 2 2018 AMC8 Problem 25

Alice has $24$ apples. In how many ways can she share them with Becky and Chris so that each of the three people has at least two apples?

$\textbf{(A) }105\qquad\textbf{(B) }114\qquad\textbf{(C) }190\qquad\textbf{(D) }210\qquad\textbf{(E) }380$

# Ex. 3 2020 AMC10B Problem 25

Let $D(n)$ denote the number of ways of writing the positive integer $n$ as a product\[n = f_1\cdot f_2\cdots f_k,\]

where $k\ge1$, the $f_i$ are integers strictly greater than $1$, and the order in which the factors are listed matters (that is, two representations that differ only in the order of the factors are counted as distinct). For example, the number $6$ can be written as $6$, $2\cdot 3$, and $3\cdot2$, so $D(6) = 3$. What is $D(96)$?

$\textbf{(A) } 112 \qquad\textbf{(B) } 128 \qquad\textbf{(C) } 144 \qquad\textbf{(D) } 172 \qquad\textbf{(E) } 184$

# Ex. 4 AoPS Mock AIME 3 Pre 2005 Problem 2

Let $N$ denote the number of $7$ digit positive integers have the property that their digits are in increasing order. Determine the remainder obtained when $N$ is divided by $1000$. (Repeated digits are allowed.)

Hint: A $7$ digit increasing integer is determined once we select a set of $7$ digits

Answer: 435

# Ex. 5 2007 AIME I, Problem 10

In a 6 x 4 grid (6 rows, 4 columns), 12 of the 24 squares are to be shaded so that there are two shaded squares in each row and three shaded squares in each column. Let $N$ be the number of shadings with this property. Find the remainder when $N$ is divided by 1000.

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |
|  |  |  |  |

Warn: Multiple solutions, don’t stop here.

Solutions: <https://artofproblemsolving.com/wiki/index.php/2007_AIME_I_Problems/Problem_10>