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Casimir-Polder Force on a Nanoparticle Rotating above a Nonreciprocal Medium

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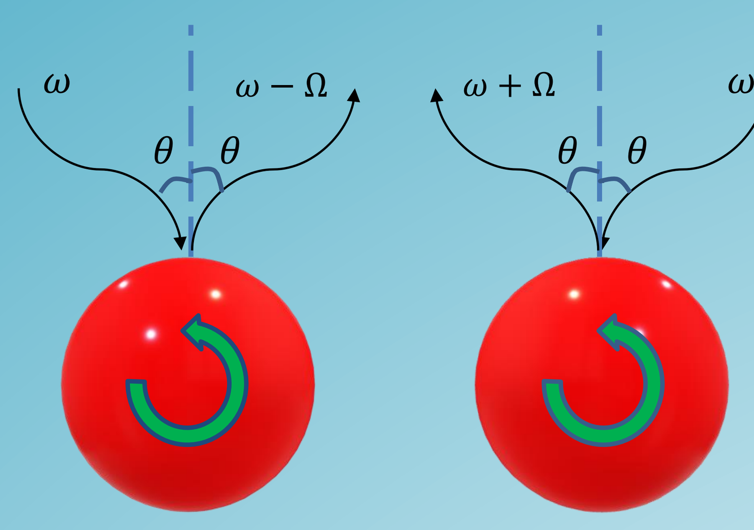
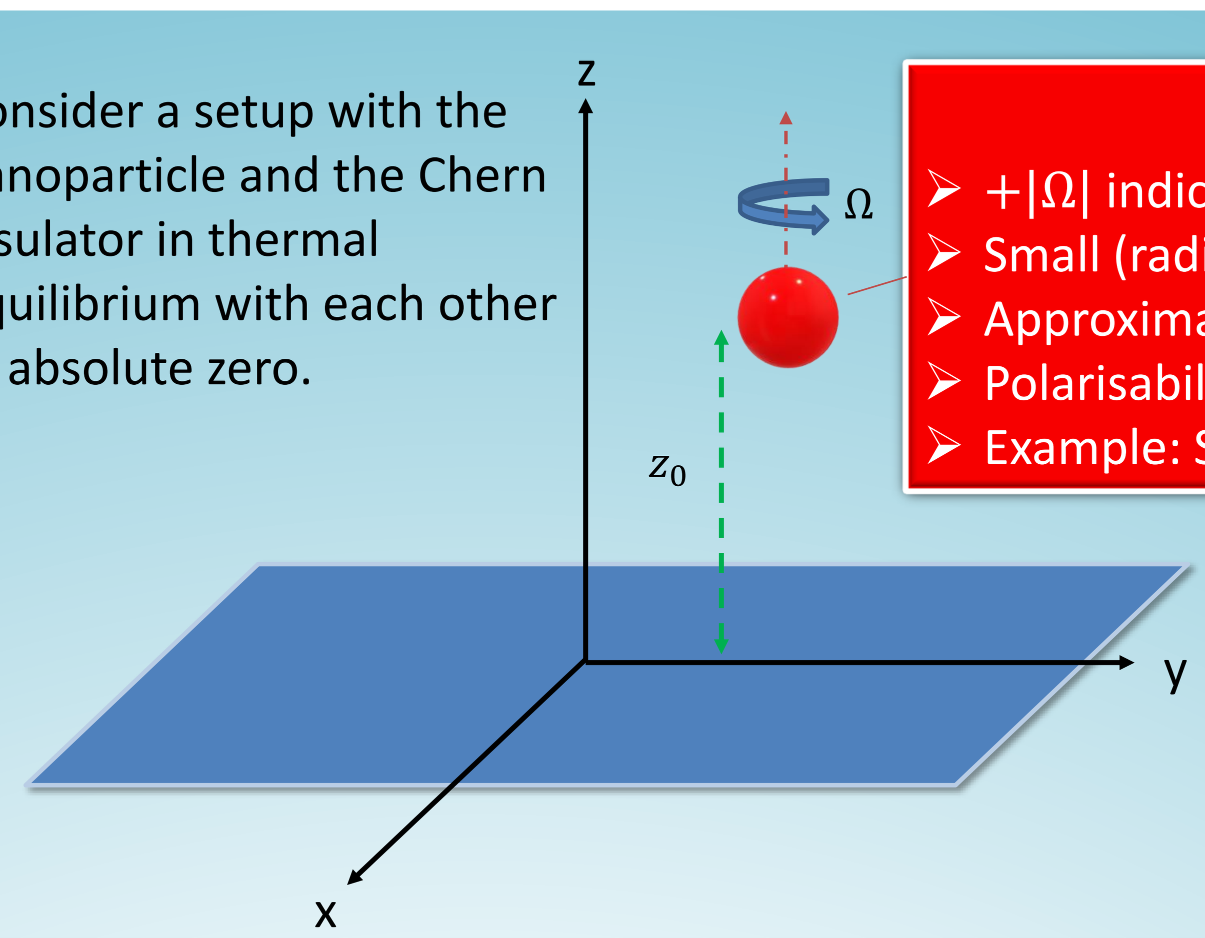
Introduction



Top View of the Particle

- A photon of frequency ω reflects off the particle in the direction of rotation gains frequency Ω, and vice versa.
- The particle undergoes a net change in both linear and angular momentum, thus experiences a force and a torque.
- Here, we derive an expression for the force involved, known as Casimir-Polder (perpendicular) force, starting from $f_z(t) = \langle p_a(t)\,\nabla E_a(\mathbf{r_0}, t)\rangle$.

Consider a setup with the nanoparticle and the Chern insulator in thermal equilibrium with each other at absolute zero.



Rotating Nanoparticle

- $+|\Omega|$ indicates anticlockwise rotation, $-|\Omega|$ clockwise.
- Small (radius on the order of ~100nm).
- Approximated as a dipole.
- Polarisability is isotropic (in corotating frame).
- Example: Silicon carbide (SiC) or indium antimonide (InSb).

Surrounded by vacuum

- True vacuum? No. Virtual and photons permeates the spacetime as a result of quantum fluctuations even at zero temperature.

Nonreciprocal Surface (Chern insulator)

- A surface in which Hall current flows in a definite orientation, thus breaking the time reversal symmetry.
- Can take on Chern number, C = 1 or -1, which discerns the direction of Hall current.

Concepts Involved

Fluctuation Sources and Green Tensor

- Two fluctuation sources: Nanoparticle and Chern insulator.
- Dipole fluctuation $\mathbf{p}^{\mathrm{NP}}$ will induce a fluctuating electric field $\mathbf{E}^{\mathrm{NP}}$ via the Green tensor G.
- Fluctuating electric field on the insulator $\mathbf{E}^{\mathrm{CI}}$ contributes additional $\mathbf{p}^{\mathrm{CI}}$ to the nanoparticle.
- Hence, $\mathbf{p} = \mathbf{p}^{\mathrm{NP}} + \mathbf{p}^{\mathrm{CI}}$ and $\mathbf{E} = \mathbf{E}^{\mathrm{NP}} + \mathbf{E}^{\mathrm{CI}}$.
- The Green tensor is characterized by the Chern insulator's reflection coefficients.

Fluctuation-dissipation Theorem (FDT)

- To describe and average the statistics of blackbody radiation.
- The fluctuating electric fields in the Chern insulator and dipole fluctuations at the position of the nanoparticle in the corotating frame are given by:

$$\langle \mathrm{E}_a^{\mathrm{CI}}(\mathbf{r},\omega)\mathrm{E}_b^{\mathrm{CI}}(\mathbf{r},\omega)\rangle = -2\pi i\hbar\delta(\omega+\omega')\left(G_{ab}(\mathbf{r},\mathbf{r}';\omega) - G_{ba}^*(\mathbf{r}',\mathbf{r};\omega)\right)\left(n(T_{\mathrm{CI}},\omega)+\frac{1}{2}\right)$$

$$\langle \mathrm{p}_a^{\mathrm{NP}}(\omega)\mathrm{p}_b^{\mathrm{NP}}(\omega')\rangle = -2\pi i\hbar\delta(\omega+\omega')\left(\alpha_{ab}(\omega) - \alpha_{ba}^*(\omega)\right)\left(n(T_{\mathrm{NP}},\omega)+\frac{1}{2}\right)$$

where $n(T,\omega) = [exp(\beta\hbar\omega) - 1]^{-1}$ is the Bose-Einstein distribution.

Results and Discussions

Perpendicular Force Expression

$$\langle f_z\rangle = \frac{h}{\pi}\int_0^\infty d\xi\left[2\,\mathrm{Re}\,\alpha(i\xi+|\Omega|)\partial_z G_{xx}(\mathbf{r}_0,\mathbf{r}_0;i\xi) \pm 2\,\mathrm{Im}\,\alpha(i\xi+|\Omega|)\partial_z G_{xy}(\mathbf{r}_0,\mathbf{r}_0;i\xi) + \alpha(i\xi)\partial_z G_{zz}(\mathbf{r}_0,\mathbf{r}_0;i\xi)\right]$$
$$+\frac{Nh}{\omega_i}\Theta(|\Omega|-\omega_i)\left[\partial_z\,\mathrm{Re}\,G_{xx}(\mathbf{r}_0,\mathbf{r}_0;|\Omega|-\omega_i) \mp \partial_z\,\mathrm{Im}\,G_{xy}(\mathbf{r}_0,\mathbf{r}_0;|\Omega|-\omega_i)\right]$$

- Derived for $\pm|\Omega|$.
- The second term contributes only when $|\Omega|$ exceeds some resonant frequency ω_i.
- For SiC, $\omega_i = 1.752\times 10^{14} s^{-1}$.

Non-Dispersive (Conductivity tensors of the Chern insulator independent of frequency)

$0 < |\Omega| < \omega_i$

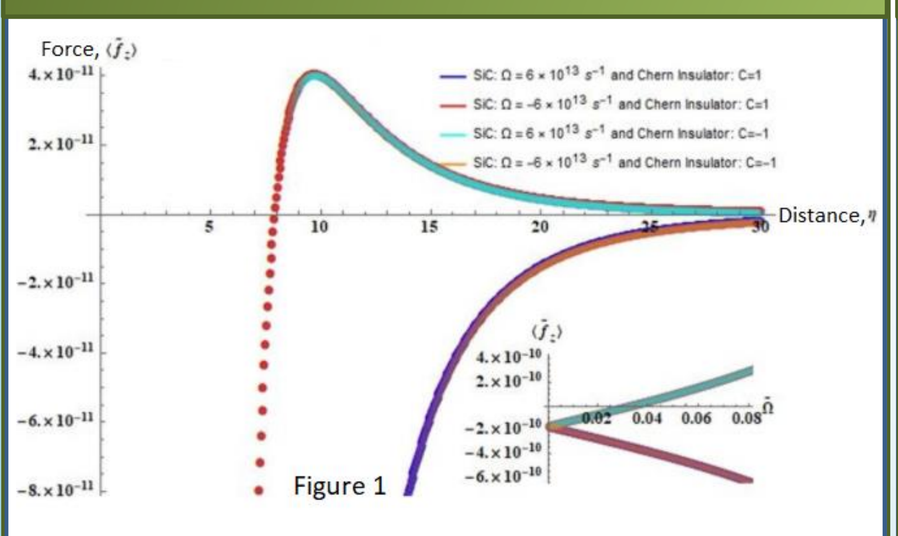


Figure 1

$|\Omega| > \omega_i$

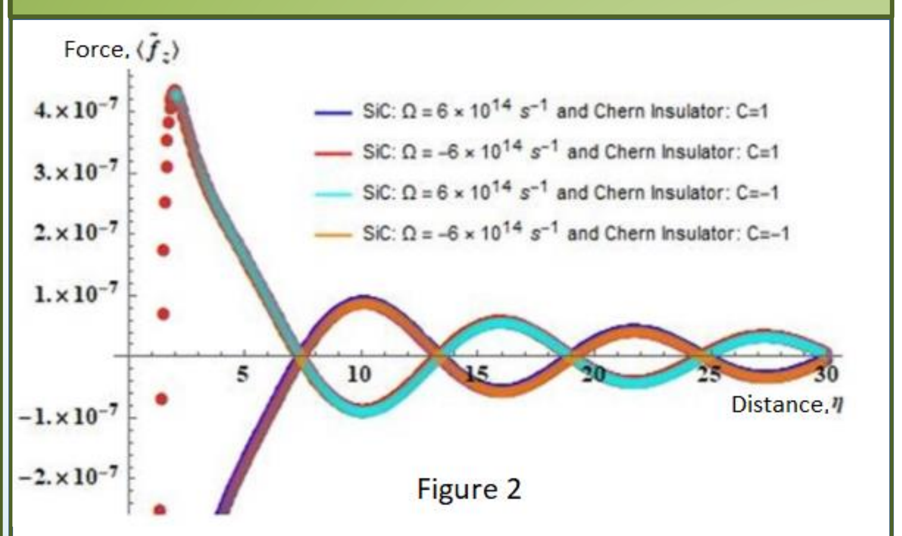


Figure 2

- **Figure 1:**
The force is **always attractive (+ve)** when the rotational direction aligns with the Hall current direction in the Chern insulator (i.e. C and Ω have the same signs). Meanwhile, if the rotational direction opposes the Hall current (opposite signs), the force becomes **repulsive (-ve)** in the range $8 \le \eta \le 109$. The inset highlights an **asymmetry** of force behaviour at $\eta = 10$, in which the attractive force only turns repulsive when the rotational frequency is increased in a particular direction.

- **Figure 2:**
The force becomes **oscillatory** with respect to the distance. The configuration with C and Ω having same signs oscillates antiphasally to that with opposite signs starting from $\eta \approx 3$.

Dispersive (Full frequency dispersion of conductivity tensor is accounted for using Kubo formula)

$0 < |\Omega| < \omega_i$

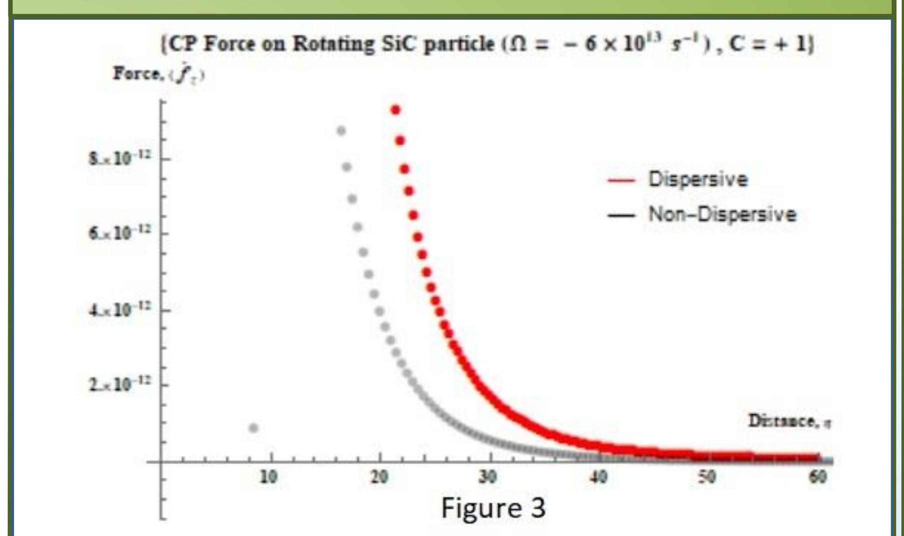


Figure 3

$|\Omega| > \omega_i$

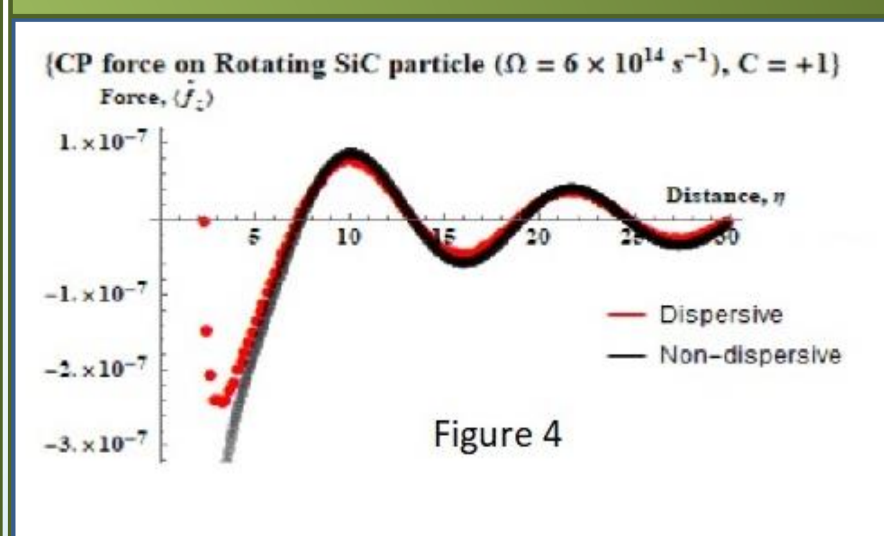


Figure 4

- **Figure 3:**
The force is **more repulsive** in the dispersive case. The dispersive plot converges with the non-dispersive plot in the far field regime.

- **Figure 4:**
The force is again oscillatory with respect to distance in the dispersive case, in phase with that of non-dispersive case, albeit having **smaller amplitudes**. The plots become out of phase starting from $\eta \approx 3.5$, in which the attractive force turns repulsive in the dispersive case.

Forthcoming Research: To study the effect of temperature difference on the perpendicular force.

Conclusion

1. Repulsive Casimir-Polder forces are observed in configurations where the rotational direction of the nanoparticle opposes the Hall current direction in the Chern insulator.
2. Perpendicular forces become oscillatory with respect with distance when $|\Omega| > \omega_i$ due to resonant contributions.

ACKNOWLEDGEMENTS

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