## Introduction

### K-Means Clustering

K-means clustering is an iterative algorithm that tries to divide the dataset into predefined K different clusters where each data point belongs to only one cluster. It tries to make the inter-cluster data points as close as possible while keeping the clusters as far as possible. Data points are assigned to a clusters using the sum of the squared distance between the data points using centroids. The less cluster variation means, the more homogeneous the data points are in the same cluster.

#### Elbow Method

Elbow method applies k-means clustering over loop and output the total sum of square over number of clusters graph where it is possible to observe changes.

#### Silhouette Method

First for the data point we find the silhouette coefficient which is calculated using the mean intra-cluster distance and the mean nearest-cluster distance, then we take the mean of it and based on the graph which also shows the number of clusters, we decide on the optimal cluster size.

#### **PCA**

Principle component analysis allows us to extract principle components which explains the variation in the dataset without having to use all of the other components.

### Agglomerative Clustering

Agglomerative clustering is a bottom-up strategy where each observation begins in its own cluster and when one goes up the hierarchy, pairs of clusters are combined. It has quadratic time and space complexity which makes it a bad choice if we work with high amount of data. It outputs a dendrogram which then be used for clustering based on observation.

### **Dataset**

TASK1 - Dataset in this assignment contains comprehensive information about student evaluations of teachers from Gazi University. It involves data regarding to instructor identifier, course code which represented as class attribute, nb.repeat as the students course repetition counter, attendance as the level of attendance and level of difficulty student perceived.

# student\_eval... 5820 obs. of 33 variables

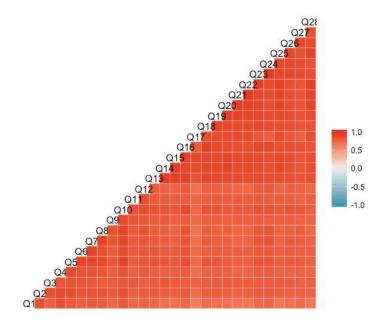
We have total of 5820 observations.

There are total of 28 questions with 5 levels which are all Likert-type. Since they are likert-type we can threat them as numeric for extracting certain metrics as-well, that's why I have used the as numeric data for the rest of the assignment.

```
: Factor w/ 5 levels
$ Q1
                                                 : Factor w/ 5 levels
                                     $ 015
$ 02
             : Factor w/ 5 levels
                                    $ 016
                                                 : Factor w/ 5 levels
$ Q3
             : Factor w/ 5 levels
                                                 : Factor w/ 5 levels
                                    $ 017
$ 04
             : Factor w/ 5 levels
                                    $ Q18
                                                 : Factor w/ 5 levels
            : Factor w/ 5 levels
$ Q5
                                    $ 019
                                                 : Factor w/ 5 levels
$ Q6
             : Factor w/ 5 levels
                                    $ 020
                                                 : Factor w/ 5 levels
$ Q7
             : Factor w/ 5 levels
                                    $ 021
                                                 : Factor w/ 5 levels
$ Q8
             : Factor w/ 5 levels
                                    $ 022
                                                 : Factor w/ 5 levels
$ 09
             : Factor w/ 5 levels
                                    $ 023
                                                 : Factor w/ 5 levels
$ 010
             : Factor w/ 5 levels
                                    $ 024
                                                 : Factor w/ 5 levels
$ 011
             : Factor w/ 5 levels
                                    $ 025
                                                 : Factor w/ 5 levels
$ 012
             : Factor w/ 5 levels
                                    $ Q26
                                                 : Factor w/ 5 levels
$ 013
             : Factor w/ 5 levels
                                    $ 027
                                                 : Factor w/ 5 levels
             : Factor w/ 5 levels
$ Q14
                                                 : Factor w/ 5 levels
                                    $ Q28
```

- Q1: The semester course content, teaching methods and evaluation system were provided at the start.
- Q2: The course aims, and objectives were clearly stated at the beginning of the period.
- Q3: The course was worth the amount of credit assigned to it.
- Q4: The course was taught according to the syllabus announced on the first day of class.
- Q5: The class discussions, homework assignments, applications and studies were satisfactory.
- Q6: The textbook and other courses resources were sufficient and up to date.
- Q7: The course allowed field work, applications, laboratory, discussion and other studies.
- Q8: Quizzes, assignments, projects and exams contributed to helping the learning.
- Q9: I greatly enjoyed the class and was eager to participate during the lectures actively.
- Q10: My initial expectations about the course were met at the end of the period or year.
- Q11: The course was relevant and beneficial to my professional development.
- Q12: The course helped me look at life and the world with a new perspective.
- Q13: The Instructor's knowledge was relevant and up to date.
- Q14: The Instructor came prepared for classes.

- Q15: The Instructor taught in accordance with the announced lesson plan.
- Q16: The Instructor was committed to the course and was understandable.
- Q17: The Instructor arrived on time for classes.
- Q18: The Instructor has a smooth and easy to follow delivery/speech.
- Q19: The Instructor made effective use of class hours.
- Q20: The Instructor explained the course and was eager to be helpful to students.
- Q21: The Instructor demonstrated a positive approach to students.
- Q22: The Instructor was open and respectful of the views of students about the course.
- Q23: The Instructor encouraged participation in the course.
- Q24: The Instructor gave relevant homework assignments/projects and helped/guided students.
- Q25: The Instructor responded to questions about the course inside and outside of the course.
- Q26: The Instructor's evaluation system (midterm and final questions, projects, assignments, etc.) effectively measured the course objectives.
- Q27: The Instructor provided solutions to exams and discussed them with students.
- Q28: The Instructor treated all students in a right and objective manner.



Running the below line results in correlation matrix as displayed in the above figure.

# ggcorr(dataset)

Which indicates there is medium to strong positive correlation between the features. Below figure gives insight about features distribution, all of them do have 3 likert-type value for their respected mean.

Q1	Q2	Q3	Q4	Q5	Q6
Min. :1.00	Min. :1.000				
1st Qu.:2.00	1st Qu.:2.000				
Median :3.00	Median :3.000				
Mean :2.93	Mean :3.074	Mean :3.179	Mean :3.082	Mean :3.106	Mean :3.107
3rd Qu.:4.00	3rd Qu.:4.000				
Max. :5.00	Max. :5.000				
Q7	Q8	Q9	Q10	Q11	Q12
Min. :1.000					
1st Qu.:2.000					
Median :3.000					
Mean :3.066	Mean :3.042	Mean :3.166	Mean :3.091	Mean :3.184	Mean :3.036
3rd Qu.:4.000					
Max. :5.000					
Q13	Q14	Q15	Q16	Q17	Q18
Min. :1.000	Min. :1.000	Min. :1.000	Min. :1.00	Min. :1.000	Min. :1.000
1st Qu.:2.000	1st Qu.:3.000	1st Qu.:3.000	1st Qu.:2.00	1st Qu.:3.000	1st Qu.:2.000
Median :3.000	Median :3.000	Median :3.000	Median :3.00	Median :4.000	Median :3.000
Mean :3.243	Mean :3.291	Mean :3.287	Mean :3.17	Mean :3.398	Mean :3.223
3rd Qu.:4.000	3rd Qu.:4.000	3rd Qu.:4.000	3rd Qu.:4.00	3rd Qu.:4.000	3rd Qu.:4.000
Max. :5.000	Max. :5.000	Max. :5.000	Max. :5.00	Max. :5.000	Max. :5.000
Q19	Q20	Q21	Q22	Q23	Q2 <del>4</del>
Min. :1.000					
1st Qu.:3.000	1st Qu.:3.000	1st Qu.:3.000	1st Qu.:3.000	1st Qu.:2.000	1st Qu.:2.000
Median :3.000					
Mean :3.262	Mean :3.285	Mean :3.307	Mean :3.318	Mean :3.202	Mean :3.167
3rd Qu.:4.000					
Max. :5.000					
Q25	Q26	Q27	Q28		
Min. :1.000	Min. :1.000	Min. :1.000	Min. :1.000		
1st Qu.:3.000	1st Qu.:2.000	1st Qu.:2.000	1st Qu.:3.000		
Median :3.000	Median :3.000	Median :3.000	Median :3.000		
Mean :3.313	Mean :3.222	Mean :3.155	Mean :3.308		
3rd Qu.:4.000	3rd Qu.:4.000	3rd Qu.:4.000	3rd Qu.:4.000		

# **Preprocessing**

TASK 2 - In order to apply clustering, I have selected the features (Q1 to Q28) which are Likert-type.

dataset = student\_evaluation[,c(6:33)]

## **Results**

### 1) K-Means Clustering

When we run K-Means Clustering with different cluster sizes on the data set, results were as follows.

#### Cluster Size 3

```
K-means clustering with 3 clusters of sizes 2358, 2223, 1239
Cluster means:
        Q1
                 Q2
                          Q3
                                   Q4
                                            Q5
                                                     06
                                                              07
1 2.631891 2.831637 3.003817 2.849449 2.868533 2.898643 2.814249 2.786684 2.966497 2.836302 3.000848
2 4.100765 4.261808 4.294647 4.247413 4.313540 4.275753 4.264507 4.239316 4.304543 4.320288 4.334683
3 1.396287 1.403551 1.509282 1.435835 1.390638 1.408394 1.396287 1.379338 1.502825 1.368846 1.467312
       Q12
                Q13
                         Q14
                                  Q15
                                           016
                                                    Q17
                                                             Q18
                                                                       Q19
                                                                                Q20
1 2.784563 3.116200 3.200170 3.188295 2.983036 3.371077 3.080153 3.141645 3.194656 3.223494 3.227311
2 4.224022 4.393162 4.413855 4.406658 4.377418 4.417454 4.389114 4.409357 4.418803 4.424202 4.436347
3 1.380952 1.419693 1.448749 1.467312 1.357546 1.622276 1.400323 1.430993 1.424536 1.463277 1.481840
       023
                024
                         Q25
                                  Q26
                                           Q27
                                                    028
1 3.031807 2.979220 3.223919 3.074215 2.982188 3.223070
2 4.390463 4.358075 4.419253 4.377418 4.311741 4.409807
3 1.393059 1.386602 1.495561 1.430993 1.407587 1.493140
```

#### Cluster Size 5

```
K-means clustering with 5 clusters of sizes 1915, 729, 1608, 719, 849
Cluster means:
        01
                                                                                                  011
1 2.699217 2.860574 3.024543 2.894517 2.904961 2.933681 2.869452 2.831854 2.966580 2.872585 3.012010
2 1.116598 1.093278 1.115226 1.091907 1.057613 1.052126 1.061728 1.056241 1.131687 1.027435 1.098765
3 3.568408 3.801617 3.853856 3.773010 3.854478 3.797264 3.769900 3.730100 3.858831 3.851990 3.910448
4 1.824757 1.930459 2.173853 1.990264 1.931850 1.994437 1.901252 1.910987 2.186370 1.913769 2.105702
5 4.733804 4.845701 4.870436 4.832744 4.893993 4.899882 4.885748 4.875147 4.879859 4.909305 4.898704
       012
                013
                         014
                                  015
                                           016
                                                    017
                                                             018
                                                                      019
                                                                               020
                                                                                        021
                                                                                                 022
1 2.824543 3.101828 3.175979 3.151958 2.984334 3.310183 3.058486 3.120627 3.156136 3.185901 3.182245
 1.058985 1.049383 1.038409 1.042524 1.019204 1.111111 1.034294 1.042524 1.024691 1.048011 1.053498
3 3.733831 4.018035 4.054104 4.049129 3.988184 4.079602 4.026741 4.060323 4.078358 4.088930 4.103856
4 1.908206 2.100139 2.212796 2.257302 1.934631 2.603616 2.066759 2.129346 2.184979 2.242003 2.282337
5 4.840989 4.943463 4.951708 4.949352 4.929329 4.944641 4.926973 4.931684 4.948174 4.943463 4.954064
       Q23
                Q24
                         Q25
                                  Q26
                                           Q27
                                                    Q28
1 3.012010 2.965535 3.177023 3.039687 2.964491 3.175979
2 1.026063 1.023320 1.049383 1.034294 1.028807 1.043896
3 4.016169 3.970771 4.074005 4.010572 3.923507 4.069652
4 2.038943 2.016690 2.333797 2.168289 2.068150 2.340751
5 4.941107 4.912839 4.948174 4.911661 4.873969 4.926973
```

#### Cluster size 7

```
K-means clustering with 7 clusters of sizes 840, 388, 579, 724, 459, 1329, 1501
Cluster means:
        01
                 Q2
                          Q3
                                   Q4
                                            Q5
                                                     Q6
                                                              07
                                                                       Q8
                                                                                Q9
1 4.720238 4.839286 4.863095 4.817857 4.897619 4.901190 4.890476 4.873810 4.882143 4.913095 4.900000
2 1.590206 1.927835 2.631443 2.018041 2.051546 2.213918 1.958763 1.943299 2.590206 1.976804 2.646907
3 1.889465 1.967185 2.160622 2.032815 1.968912 2.006908 1.970639 1.960276 2.117444 1.965458 2.075993
4 1.110497 1.085635 1.104972 1.087017 1.049724 1.044199 1.055249 1.048343 1.127072 1.022099 1.089779
5 2.307190 2.960784 3.274510 2.838780 2.997821 2.967320 2.784314 2.636166 3.126362 3.067538 3.263617
6 3.823928 3.951843 3.970655 3.967645 3.997743 3.940557 3.940557 3.937547 3.977427 3.975922 4.000000
7 2.952032 3.025316 3.039973 3.044637 3.049300 3.059294 3.036642 3.010660 3.035976 3.013991 3.052632
                                                                                        Q21
       012
                Q13
                         Q14
                                  Q15
                                           016
                                                    Q17
                                                            Q18
                                                                      019
                                                                               Q20
1 4.839286 4.953571 4.960714 4.960714 4.940476 4.957143 4.938095 4.944048 4.960714 4.955952 4.967857
2 1.981959 2.981959 3.275773 3.250000 2.458763 3.979381 2.873711 3.054124 3.280928 3.474227 3.463918
3 1.948187 1.991364 2.062176 2.094991 1.867012 2.336788 1.960276 2.006908 2.006908 2.034542 2.077720
4 1.045580 1.046961 1.037293 1.041436 1.017956 1.104972 1.033149 1.042818 1.024862 1.048343 1.053867
5 2.793028 3.952070 4.095861 4.080610 3.840959 4.311547 4.050109 4.119826 4.257081 4.270153 4.278867
6 3.891648 4.002257 4.020316 4.015801 3.984951 4.018059 3.990971 4.015801 4.018059 4.031603 4.042889
7 2.994004 3.005330 3.029314 3.015989 2.975350 3.063957 2.962025 2.998001 2.986676 2.986676 2.990007
       023
                024
                         025
                                 026
                                           027
                                                    028
1 4.955952 4.923810 4.958333 4.920238 4.882143 4.933333
2 2.693299 2.551546 3.422680 2.907216 2.621134 3.481959
3 1.929188 1.939551 2.131261 2.013817 1.948187 2.119171
4 1.026243 1.023481 1.049724 1.034530 1.029006 1.041436
5 3.928105 3.769063 4.202614 3.949891 3.697168 4.180828
6 3.995485 3.970655 4.025583 3.993228 3.938299 4.023326
7 2.967355 2.954031 3.006662 2.969354 2.957362 3.005330
```

#### Cluster Size 9

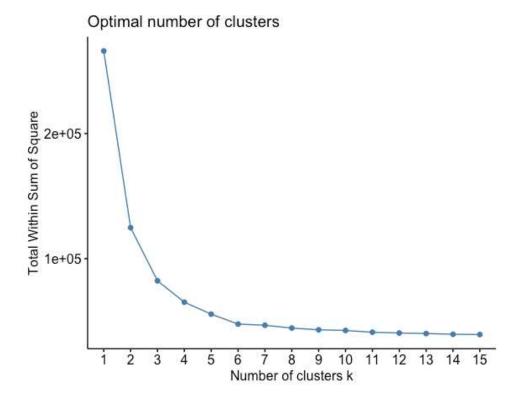
```
K-means clustering with 9 clusters of sizes 210, 1463, 306, 749, 720, 1170, 255, 525, 422
Cluster means:
                 Q2
                                   Q4
                          Q3
                                            Q5
                                                     06
                                                              07
                                                                                09
                                                                       08
1 3.023810 3.776190 3.880952 3.490476 3.952381 3.795238 3.719048 3.566667 4.047619 4.033333 4.028571
2 2.941217 3.001367 3.028708 3.025974 3.037594 3.051948 3.036227 3.004785 3.023240 2.995899 3.038278
3 1.555556 1.833333 2.431373 1.941176 1.895425 2.039216 1.741830 1.803922 2.421569 1.856209 2.323529
4 4.877170 4.937250 4.954606 4.961282 4.966622 4.955941 4.962617 4.950601 4.942590 4.967957 4.962617
5 1.106944 1.083333 1.102778 1.083333 1.045833 1.043056 1.050000 1.044444 1.125000 1.019444 1.079167
6 3.928205 4.002564 4.007692 4.031624 4.024786 4.007692 3.991453 3.998291 3.999145 4.011966 4.017094
7 1.776471 2.286275 2.976471 2.349020 2.290196 2.392157 2.180392 2.105882 2.647059 2.262745 2.807843
8 1.918095 1.982857 2.129524 2.041905 2.001905 2.034286 2.017143 2.005714 2.110476 1.998095 2.099048
9 2.682464 3.222749 3.409953 3.085308 3.329384 3.244076 3.156398 3.030806 3.407583 3.353081 3.592417
       012
                013
                         014
                                  015
                                           016
                                                    Q17
                                                             018
                                                                      019
                                                                               020
1 3.628571 4.709524 4.780952 4.752381 4.676190 4.880952 4.776190 4.847619 4.909524 4.880952 4.923810
2 2.991798 2.994532 3.014354 3.008886 2.967874 3.053999 2.959672 2.982912 2.976077 2.967874 2.970608
3 1.800654 2.653595 2.980392 2.934641 2.196078 3.709150 2.500000 2.709150 2.846405 3.058824 3.107843
4 4.915888 4.969292 4.967957 4.973298 4.951936 4.969292 4.951936 4.953271 4.961282 4.966622 4.971963
5 1.037500 1.038889 1.036111 1.038889 1.015278 1.108333 1.031944 1.041667 1.026389 1.045833 1.051389
6 3.952991 3.996581 4.018803 4.013675 3.987179 3.987179 3.980342 3.997436 3.995726 4.005983 4.017094
7 2.160784 3.635294 3.882353 3.886275 3.145098 4.450980 3.690196 3.792157 4.109804 4.215686 4.192157
8 2.000000 1.965714 1.980952 2.043810 1.881905 2.184762 1.940952 1.963810 1.954286 1.973333 2.009524
9 3.132701 3.758294 3.857820 3.793839 3.687204 3.990521 3.763033 3.879147 3.936019 3.969194 3.962085
                024
                         025
                                  Q26
                                           Q27
1 4.738095 4.514286 4.861905 4.704762 4.409524 4.900000
2 2.954887 2.953520 2.989747 2.961039 2.944634 2.978811
3 2.418301 2.307190 3.163399 2.705882 2.450980 3.241830
4 4.965287 4.939920 4.966622 4.935915 4.915888 4.941255
5 1.025000 1.020833 1.045833 1.034722 1.026389 1.041667
6 3.989744 3.986325 3.995726 3.979487 3.945299 3.994017
7 3.439216 3.172549 4.015686 3.450980 3.141176 4.019608
8 1.918095 1.935238 2.047619 1.975238 1.931429 2.030476
9 3.715640 3.630332 3.954976 3.767773 3.613744 3.931280
```

#### TASK 3 - Elbow Method

I have specified number of cluster size range using the below figure.

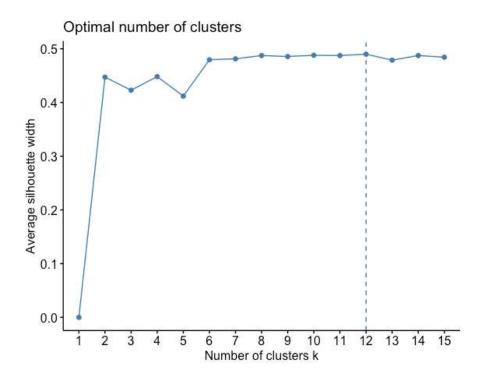
```
clusterRange = c(1:15)
maxCluster = max(clusterRange)
```

I have obtained the following graph by running the below line. Drastic change occurs when cluster size is incremented to 2 and 3. Since its instructed to choose odd number as the cluster size, I have chosen 3.



#### TASK 4 - Silhouette Method

fviz\_nbclust(dataset, kmeans , method = "silhouette",k.max = maxCluster)



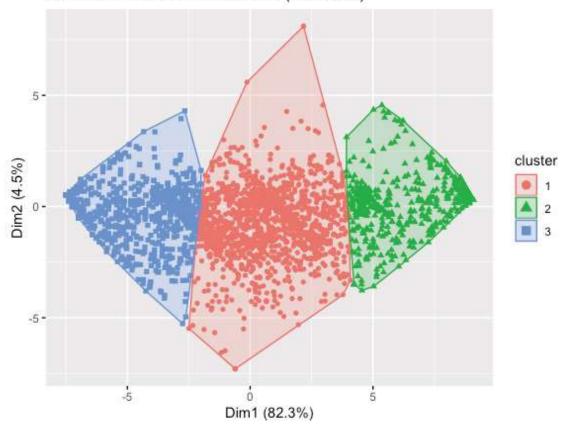
When I have applied the silhouette method and again, value 3 was the candidate for optimal cluster size.

Later, I have applied k-means clustering using cluster size as 3. Below figure represents the corresponding code block.

```
kMeansResult = kmeans(datasetPCA, centers = 3, nstart = 20)
kMeansResult # K-means clustering with 3 clusters of sizes 1239, 2358, 2223
```

K-means clustering with 3 clusters of sizes 2358, 1239, 2223

### Clusters of Student Evaluation (K-Means)



#### TASK 5 - PCA

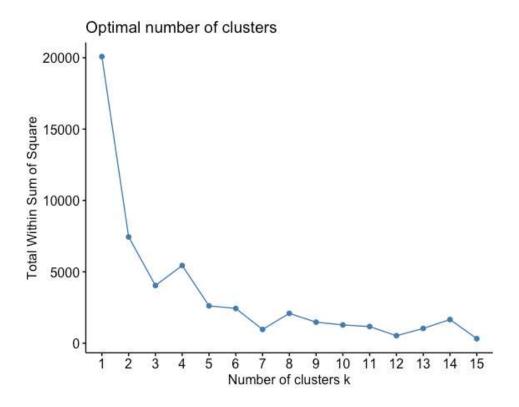
I have executed the "prcomp" function which is responsible from finding the principle components in the dataset.

datasetPCA = prcomp(dataset, center = TRUE)
summary(datasetPCA)

```
PC<sub>1</sub>
                                         PC3
                                                 PC4
                                                         PC5
                                                                 PC6
                                                                        PC7
                                                                                PC8
                                                                                         PC9
                       6.129 1.43666 0.8169 0.76634 0.68817 0.65281 0.5777 0.54607 0.52703 0.4827 0.47764
Standard deviation
Proportion of Variance 0.822 0.04516 0.0146 0.01285 0.01036 0.00932 0.0073 0.00652 0.00608 0.0051 0.00499
Cumulative Proportion 0.822 0.86714 0.8817 0.89459 0.90495 0.91427 0.9216 0.92810 0.93417 0.9393 0.94426
                                  PC13
                                           PC14
                                                  PC15
                                                          PC16
                                                                                  PC19
                          PC12
                                                                  PC17
                                                                          PC18
Standard deviation
                       0.47149 0.44491 0.43642 0.4328 0.42369 0.41829 0.40532 0.39378 0.38956 0.37073
Proportion of Variance 0.00486 0.00433 0.00417 0.0041 0.00393 0.00383 0.00359 0.00339 0.00332 0.00301
Cumulative Proportion 0.94913 0.95346 0.95763 0.9617 0.96565 0.96948 0.97307 0.97647 0.97979 0.98279
                          PC22
                                  PC23
                                           PC24
                                                         PC26
Standard deviation
                       0.36744 0.36181 0.35278 0.3379 0.3313 0.29799 0.28881
Proportion of Variance 0.00295 0.00286 0.00272 0.0025 0.0024 0.00194 0.00182
Cumulative Proportion 0.98575 0.98861 0.99133 0.9938 0.9962 0.99818 1.00000
```

Output was as follows, PC1 - Question 1 and PC2 - Question 2 combined, explains 86.71% variance of the dataset.

Then I have reduced the dataset to size in which only principle components are available and then applied elbow method to figure out the optimal cluster size.

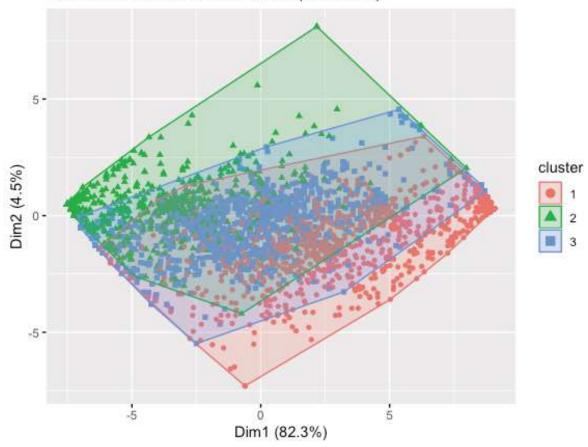


As previous results, again 3 was the optimal cluster size, so I have ran "kmeans" using dataset having only principle components and 3 as the number of clusters.

```
kMeansResult = kmeans(datasetPCA, centers = 3, nstart = 20)
kMeansResult # K-means clustering with 3 clusters of sizes 1767, 2043, 2010
```

Below figure also illustrates the result of the clustering using the dataset with only having principle components.





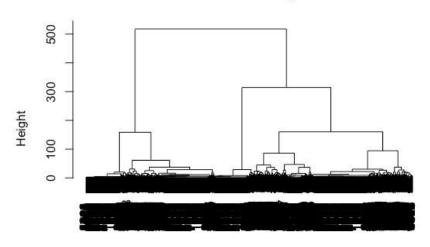
### 3) Agglomerative Clustering

#### TASK 6 -

First I have created a distance matrix named as "d" and then I have applied the hierarchical clustering.

```
d = dist(dataset)
hc = hclust(d,method="ward.D2")
plot(hc)
```

#### Cluster Dendrogram



d hclust (\*, "ward.D2")

It resulted as in the above dendrogram. I have cut the tree to 2 using the below code.

```
hc_2_cluster = cutree(hc,k=2)
plot(hc)
rect.hclust(hc, k=2, border="red")
visualiseAgglomerative(hc_2_cluster)
```

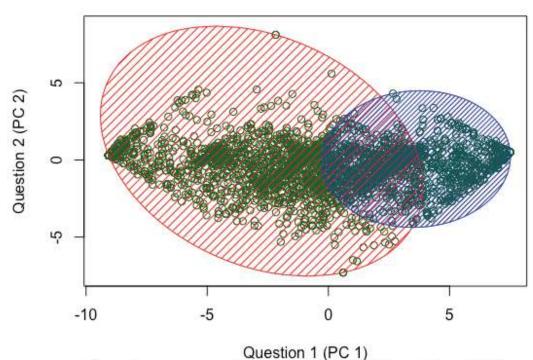
### Cluster Dendrogram



d hclust (\*, "ward.D2")

And then, I have visualized the result using the "visualiseAgglomerative" function. It resulted in the below figure.

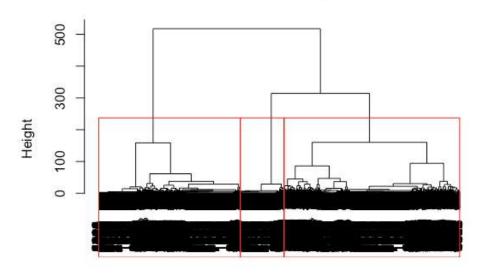
## Clusters of Student Evaluation (Agglomerative)



These two components explain 86.76 % of the point variability.

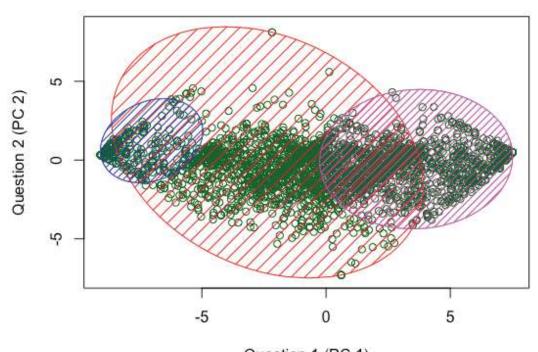
I have applied the same routine but this time I have cut the tree to 3 parts. Results were as follows.





d hclust (\*, "ward.D2")

## Clusters of Student Evaluation (Agglomerative)



Question 1 (PC 1)
These two components explain 86.76 % of the point variability.

## **Shortcut for Task Answers**

TASK 1 - I have shown the descriptive summary, statistics and correlation matrix with figures and I have written brief explanations about my findings.

TASK 2 - Since questions are likert-type, I have just separated the questions from the data set for pre-processing.

TASK 3 - Running elbow method resulted in optimal cluster size as 3.

TASK 4 - Yes, using silhouette method I had 3 as the optimal cluster size as-well.

TASK 5 - After applying PCA, dataset with only PC resulted in cluster size 3 (using elbow method)

TASK 6 - I have found 2 clusters using ward's method.

#### TASK 7 -

One of the biggest difference between hierarchical clustering and k-means clustering is the amount of data they can handle in a given time, hierarchical clustering have quadratic complexity whereas it's linear in k-means. Also, k-means might generate different results for different cluster sizes, however, in hierarchical clustering we have the fixed dendrogram. In k-means we need some information about the cluster size before the clustering to obtain a good result, elbow method and silhouette methods do help in this process whereas we do not require such analysis in the hierarchical clustering. K-means is sensitive to scaling whereas hierarchical is sensitive to large data, k-means can change the sample's cluster, however when hierarchical assigns a branch to the sample it cannot change it. Finally, since hierarchical clustering generates a dendrogram it is easier to interpret the results.

# **Conclusion**

In this assignment, I have learnt the fundamental workflow that is required to be followed when applying clustering. I have used methods for determining the optimal cluster size for k-means clustering and interpreting dendrogram which was generated by applying the hierarchical clustering. Also, I have applied PCA on the dataset and re-run k-means clustering and observed the differences. Finally I have applied hierarchical clustering and I have visualized my findings for each task.