# Total expected reward equals the weakest pre-expectation

Oliver Emil Bøving

The slides and souce-code are located online here:

https://oembo-sse.github.io/02913-Advanced-Analysis-Techniques-Jun-24/
https://github.com/oembo-sse/02913-Advanced-Analysis-Techniques-Jun-24/

- The language and theory is based on a combination of Kaminski (2019) and the unpublished PhD thesis of Kevin Batz.
- The stucture of the proof takes great inspiration from an unreleased paper by Kevin Batz, Benjamin Lucien Kaminski, Christoph Matheja, Tobias Winkler.
- The paper Gretz, Katoen, and McIver (2014) formalizes the same result as we are showing, but using a different approach, which some details missing that we attempt to formalize foundationally.
- The book Baier and Katoen (2008) is the go-to book for model checking and contains chapters on MDP's. One downside is that it mostly, if not only, concerns it self with finite models. This book formalizes notions of expected rewards in terms of paths and cylinder sets.
- The book Puterman (1994) contains much formalism of MDP's in-general. One interesting aspect, is that this book mostly concerns it self with *states* while paths are mostly implicit. Additionally, its formalizations are mostly *backwards*, which is to say that it expresses expected reward using Bellman-operators.
- My fomalization is writting in Lean, see Moura and Ullrich (2021), and heavily uses the Mathlib4 library which has formalized many aspects of mathematics, see Community (2020).

## References

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