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Exponential model for option prices: Application to the Brazilian market

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HIGHLIGHTS

- Daily returns on the Ibovespa index follow an exponential distribution.
- Comparison is made between the Black-Scholes and the exponential models for option pricing.
- Near maturity, option prices are better described by the exponential model.
- Possible implications for investment strategies and risk management are briefly discussed.

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ABSTRACT

In this paper we report an empirical analysis of the Ibovespa index of the São Paulo Stock Exchange and its respective option contracts. We compare the empirical data on the Ibovespa options with two option pricing models, namely the standard Black–Scholes model and an empirical model that assumes that the returns are exponentially distributed. It is found that at times near the option expiration date the exponential model performs better than the Black–Scholes model, in the sense that it fits the empirical data better than does the latter model.

1. Introduction

Options are financial instruments that allow their holders the right of buying or selling the underlying asset for a given fixed price, also known as the strike price, on a pre-defined date called the expiration or maturity date. The central issue of modeling option prices is to determine the fair price, also called premium, to pay for a given option before the maturity date, taking into account the statistical distribution of the underlying asset.

The formulation of the Black, Scholes and Merton model in the 1970's established a landmark for option pricing [1]. This model assumes that asset prices in financial markets can be described by a geometric Brownian motion. This hypothesis is the backbone of the so-called *Efficient Market Hypothesis* (EMH), which asserts that the returns of a given stock follow an uncorrelated Gaussian process (white noise).

However, studies on high frequency financial data (e.g., on time scales of the order of a few minutes or less) have shown that price fluctuations behave as non-Gaussian processes [2–4], with the empirical probability distribution function (EDF) of

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returns exhibiting power law tails. On the other hand, on mesoscopic time scales (typically from hours to days) the central part of the EDFs are often better described by an exponential distribution [5–10]. At longer time lags, the EDFs tend to a Gaussian distribution as required by the central limit theorem. Given this scenario, it is natural to ask how option prices should be modeled when the distribution of returns of the underlying asset displays such a variability.

Several non-Gaussian option pricing models have been considered in the literature [11,12]. These include models based on Lévy processes [13,14] and on the so-called q-Gaussian distribution [15], in which cases the distribution of returns have power-law tails; stochastic volatility models [16,17], including the Hull–White and Heston models [18,19]; option pricing with the Edgeworth expansion [20,21]; etc. Of particular relevance to us here is the empirical model for option pricing introduced by McCauley and Gunaratne [22] which assumes an exponential distribution of returns. As mentioned above, financial markets often exhibit exponential distributions on time scales that are comparable to the lifetime of an option, and so the exponential model for option pricing is a natural candidate to describe options in such markets. In contradistinction, power-law distributions tend to be observed in high-frequency data (e.g., intraday quotes) and are thus expected to be less relevant for option pricing in such cases.

In the present paper we perform an empirical analysis of the prices of options on the Ibovespa index of the São Paulo Stock Exchange. First we show that, as in other financial markets [6–9], the central part of the distribution of the Ibovespa returns follows an exponential distribution at mesoscopic times scales. We then proceed to analyze the Ibovespa option market in light of two relevant option pricing models: (i) the standard Black–Scholes model [1] and (ii) the exponential model for option pricing [22] mentioned above. Both models yield an analytical solution for the price of an European call option, which can be easily compared with the quoted market prices. We find that the exponential model gives a better fit to the empirical data for times closer to the option expiration date, whereas for longer periods before expiration the Black–Scholes model offers a better description of the market prices. Our findings thus indicate that the market seems to implicitly take into account the fact that the returns of the Ibovespa (at mesoscopic time scales) follow an exponential distribution. Implications of this finding for possible investment strategies are briefly discussed.

2. Exponential model for option pricing

In this section we collect the main results concerning the exponential distribution for financial returns and its applications to option pricing.

2.1. Distribution of returns

Let us define the logarithmic returns at time lag τ by $x(t) = \ln[S(t+\tau)/S(t)]$, where $S(t)$ is the price of the relevant financial asset at time t and τ is the time lag. The exponential distribution, $f(x, t)$, of the log-returns x is defined by [22]

$$f(x,t) = \begin{cases} Ae^{\gamma(x-\delta)} & \text{if } x \le \delta \\ Be^{-\nu(x-\delta)} & \text{if } x > \delta, \end{cases} \tag{1}$$

where δ, γ and ν are parameters that characterize the distribution. From the normalization condition, $A/\gamma + B/\nu = 1$, and by imposing $\langle x\rangle = \delta$, one finds that $A/\gamma^2 = B/\nu^2$. One can also show that the variance of the exponential distribution is given by $2(\gamma\nu)^{-1}$; see Ref. [22] for details.

Let us now recall that the folded cumulative distribution, $G(x)$, associated with a probability density function $f(x)$ is defined by

$$G(x) = \begin{cases} F(x), & \text{if } F(x) \le \dfrac{1}{2} \\ 1 - F(x), & \text{otherwise} \end{cases} \tag{2}$$

where $F(x)$ is the cumulative distribution: $F(x) = \int_{-\infty}^{x} f(x)\mathrm{d}x$. Note that $G(x)$ for the exponential distribution given in (1) is also a bilateral exponential function. This means, in particular, that in a semi-log plot the function $G(x)$ for the exponential distribution has a tent-like shape, in contradistinction to a Gaussian distribution whose folded cumulative distribution has a downward concavity.

In Fig. 1(a) we plot in a semi-log scale the empirical folded cumulative distribution of the Ibovespa returns for $\tau = 1, 5$ and 20 days. These distributions were generated from the historical time series of the daily closing prices of the Ibovespa, from its inception in January 1968 up to February 2004, totaling 8889 data points. One sees from this figure that the empirical distributions (solid lines) deviate from a Gaussian (dashed line) and follow instead an exponential law, particularly in the central region of the EDF which displays the tent-like shape typical of an exponential distribution.

At shorter time scales, the distribution of returns of the Ibovespa becomes heavy-tailed as seen in Fig. 1(b), where we plot the folded cumulative distribution of intraday returns for $\tau = 15$, 60, and 180 min. The distributions in this figure were constructed from a time series of 19995 intraday quotes at every 15 min covering the years from 1998 to 2001. Note, in particular, that for $\tau = 15$ min the distribution has an upward concavity typical of power law distributions. However, as τ increases the empirical distribution evolves towards an exponential distribution, as seen in the case for $\tau = 180$ min

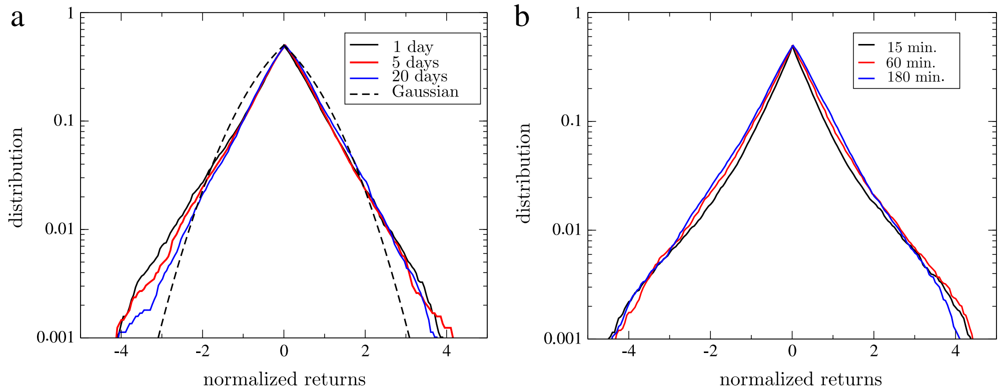

Fig. 1. Folded cumulative distribution of the Ibovespa returns. The returns were normalized by the standard deviation. (a) shows the distributions of returns with time lags of 1, 5 and 20 days, whereas (b) shows the distributions of intraday returns at time lags of 15, 60 and 180 min.

where the distribution already shows a tent-like shape. Analysis of Fig. 1 then reveals that the Ibovespa returns follow an exponential distribution for time lags varying from 3 h up to 20 days. This observation was the main motivation for our investigation of the Brazilian option market vis-a-vis the exponential model for option pricing.

2.2. Option pricing

We recall that one of the assumptions of the Black–Scholes model is that the price $S(t)$ of the underlying asset price follows a geometric Brownian motion, which implies that the distribution of the log-returns is a Gaussian. (Because of this, we shall often refer to the Black–Scholes model as the Gaussian model for option pricing.) Based on this assumption, the Black–Scholes formula [1] for option prices is given by

$$C_{\mathrm{BS}}(S, K, r, t; \sigma) = SN(d_1) - K\mathrm{e}^{-r(T-t)}N(d_2), \tag{3}$$

where the function $N(x)$ is the cumulative distribution of a normal random variable

$$N(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} \mathrm{e}^{-\frac{y^2}{2}} \mathrm{d}y, \tag{4}$$

and

$$d_1 = \frac{\ln(S/K) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}, \tag{5}$$

$$d_2 = \frac{\ln(S/K) + \left(r - \frac{1}{2}\sigma^2\right)(T-t)}{\sigma\sqrt{T-t}}. \tag{6}$$

Here T is the expiration date, K is the strike price, r is the daily interest rate, and σ is an unknown parameter called volatility.

In the empirical option pricing model introduced by McCauley and Gunaratne [22], it is assumed that the log-returns of the underlying asset follow an exponential distribution. For this reason we shall refer to this model as the *exponential model for option pricing*. The exponential model also yields an explicit formula for the fair price $C(S, K, r, t)$ of a European option [22], given by the following expression:

$$C\,\mathrm{e}^{r\Delta t} = \begin{cases} S\mathrm{e}^{\delta}\dfrac{\gamma^2(\nu-1)+\nu^2(\gamma+1)}{(\gamma+\nu)(\gamma+1)(\nu-1)} + \dfrac{K\gamma}{(\gamma+1)(\gamma+\nu)}\left(\dfrac{K}{S}\mathrm{e}^{-\delta}\right)^{\gamma} - K, & S > K\mathrm{e}^{-\delta} \\ \dfrac{K\nu}{(\nu-1)(\gamma+\nu)}\left(\dfrac{K}{S}\mathrm{e}^{-\delta}\right)^{-\nu}, & S < K\mathrm{e}^{-\delta}. \end{cases} \tag{7}$$

The parameters δ, γ and ν are related by the risk-neutral condition:

$$r = \frac{1}{\Delta t}\int_t^T \mu(s)\mathrm{d}s = \frac{1}{\Delta t}\left(\delta + \ln\left(\frac{\gamma\nu + (\nu-\gamma)}{(\gamma+1)(\nu-1)}\right)\right), \tag{8}$$

where μ is the theoretical expected return, which can be computed in terms of the parameters of the exponential distribution (1) leading to (8); see Ref. [22]. Since the interest rate r is assumed to be known one can solve this expression for δ, so that we end up with only two unknown parameters, namely, γ and ν.

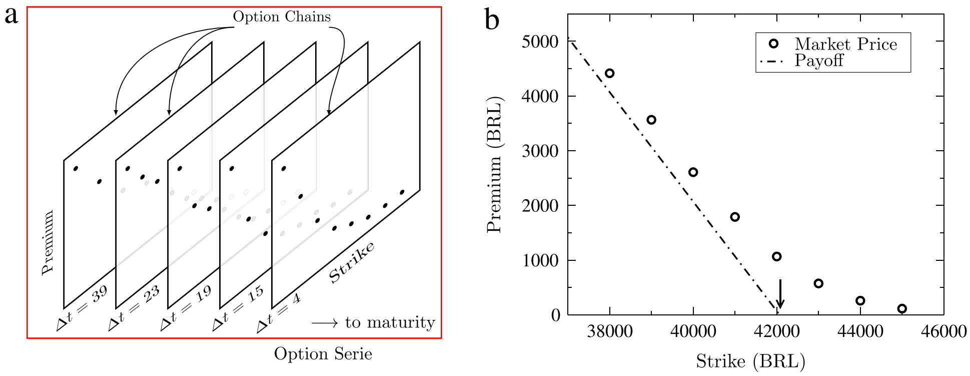

Fig. 2. (a) Schematics explaining the concept of *option chains*; (b) the option chain for the IBOVL option series at 15 days before expiration. The dash–dot line represents the intrinsic value of the option, corresponding to the difference between the current price (arrow) of the Ibovespa and the strike price.

3. Methodology

3.1. Option data

We study options whose underlying asset is the Ibovespa index, which is the main stock index of the São Paulo Stock Exchange. Analysis of the options on the Ibovespa thus offers a general idea about the behavior of the Brazilian option market. In the present study we analyzed the closing market prices of (European) call options on the Ibovespa traded daily in the years 2005 and 2006. Before we describe the data, let us first introduce some terminology about options on the Ibovespa.

A set of options that have the same date of expiration is called an *option series*. Option series on the Ibovespa are denoted by the symbol IBOV followed by a letter indicating the date of expiration according to the following convention: the letters from A to L indicate call options expiring on the months from January to December, respectively. Options on the Ibovespa index always expire on the Wednesday closest to the 15th day of the month, and so it suffices to specify the month to fix the expiration date. For instance, all options belonging to the series denoted by IBOVD expire on the Wednesday closest to April 15th, following the launch of the series. Once an option series is authorized by the Exchange, trading on the options of that series begins with investors taking short and long positions.

We shall denote by an *option chain* the set of options belonging to the same option series that are traded on a particular day. Thus, for each option chain we have a set of premiums (closing market prices) as a function of the strike price; see Fig. 2(a) for a schematic illustrating option chains. An example of a specific option chain for the option series IBOVL for the year 2006 at a time corresponding to 15 days before expiration is shown in Fig. 2(b).

Options on the Ibovespa expire on even months, hence there were 12 option series in the two-year period (2005–2006) analyzed here, with a total of 850 option chains. When a new option series is launched by the Exchange, typically 60 days before the expiration date, trading is authorized on several options with different strikes near the then-current market price of the Ibovespa. The number of strikes that are actually traded at each particular day varies considerably during the lifetime of an option series. This is illustrated in Fig. 3 where we show the average number (among our 850 options chains) of strikes traded as a function of the time to maturity. One sees that shortly after an option series is launched (i.e., for large Δt) trading is limited to a few strikes. As time passes (and Δt decreases), the number of strikes traded tends to grow and reaches an average number around five or more. Then, close to expiration (i.e., for small Δt) the number of strikes traded decreases considerably, as trading concentrates on those strikes that are closer to the current value of the Ibovespa.

In our empirical analysis described below, we used only option chains that had at least four strikes negotiated. Out of our total set of 850 option chains, 441 of them satisfied this criterion. Options chains with less than four traded strikes were discarded because the small number of data points renders the analysis less reliable. For each of these selected option chains we performed a least-square fit of the empirical data (market price *vs.* strike) by the two option-pricing formulas predicted by the Gaussian and the exponential models, respectively. Using a suitable figure of merit for the fits (see below), we then compared which model gives a better fit to the data as a function of time to maturity.

3.2. Empirical analysis

Our main object of study here is the set of option chains on the Ibovespa index. As explained above, each option chain consists of a list of premiums C_i with its respective strike prices K_i, all traded in a particular day t before the expiration date T.

The Black–Scholes model states that all options belonging to an option chain should have the same volatility σ, but options with different strike prices are negotiated separately and so their volatility may vary according to the investor valuations. Investors determine the volatility in two ways: (i) the historical method, which estimates statistically the volatility over the history of the underlying asset prices, and (ii) the "implied volatility" method that consists of finding the volatility σ_{imp} of the Black–Scholes formula that yields the quoted market price of an option, i.e., $C_{BS}(S, K, r, \Delta t; \sigma_{imp}) =$

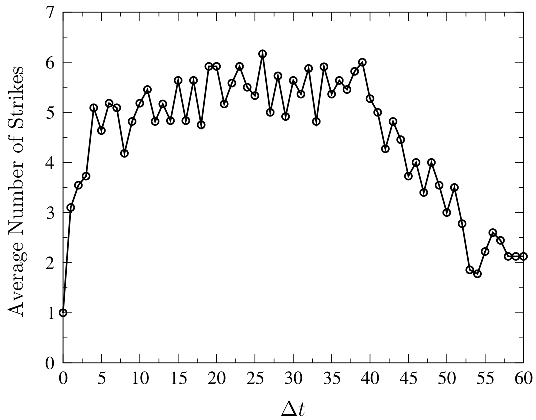

Fig. 3. Average number of strikes in an option chain as a function of the time to maturity.

C_{market}. In the first method (which relies only on the underlying asset prices) the volatility is the same for all options on that asset, whereas in the second method each option yields a value for the implied volatility which generally differs for different values of the strike price, giving rise to the so-called volatility smile effect.

Here we adopt a somewhat intermediate approach in that we shall assume that the volatility σ is the same for all options of a given option chain. We obtain σ by performing a least-square fitting of the formula $C_{\text{BS}}(S, K, r, \Delta t; \sigma)$ given in (3) to the empirical data corresponding to the option premium versus strike price. More specifically, for any given option chain we find the parameter σ that minimizes the residual sum of squares:

$$\frac{\partial}{\partial \sigma}[SS_{res}] = 0, \tag{9}$$

where

$$SS_{res} = \sum_{i=1}^{N} \left(C_i - C_{\text{BS}}(S, K_i, r, \Delta t; \sigma)\right)^2 . \tag{10}$$

Here N is the total number of strikes in the option chain, C_i is the "empirical" market price of the option, and K_i is the respective strike price. The other parameters are the time to maturity $\Delta t = T - t$, where T is the expiration date and t is the day when the option was negotiated, the asset price $S(t)$ at time t, and the daily interest rate r, which is assumed to be the reference rate DI (Interbank Deposit rate) extracted from Ref. [23]. The numerical method used here to compute the value σ that minimizes SS_{res} is the so-called golden section method [24].

A similar analysis can be performed with the exponential model for option pricing. In this case, one seeks to adjust the theoretical formula (7) predicted by the exponential model to the empirical data of the chain. This can easily be done by computing the unknown parameters μ and ν that minimize the corresponding residual sum of squares SS_{res}, where SS_{res} is as defined in (10) but with the difference that (7) now replaces the Black–Scholes formula. For this minimization procedure we have used the Downhill Simplex method [24] for two dimensions.

The fitting procedure described above has the advantage that it allows a direct comparison between the two models since we can easily compare which theoretical formula yields a better fit to the empirical data. For example, in Fig. 4 we show the respective fits given by the Gaussian model (solid curve) and the exponential model (dashed curve) for the option chain of the series IBOVL of year 2006 at 15 days before expiration. To estimate the error in the volatility σ for the Gaussian model and in the parameters γ and ν for the exponential model, we used the bootstrap method where we resampled the data (with repetition) 2000 times. For each bootstrap sample we performed the corresponding fitting procedures described above and then computed the average and standard deviation of the fitting parameters. For the data shown in Fig. 4 the Gaussian model yields $\sigma = 0.0135 \pm 0.0005$, whereas for the exponential model we found $\gamma = 20.2 \pm 1.4$ and $\nu = 30.7 \pm 1.6$. A visual inspection of Fig. 4 reveals that the theoretical curve predicted by the exponential model seems to adjust better this particular set of data.

This comparison can be made more quantitative by introducing the coefficient of determination R^2 defined as

$$R^2 = 1 - \frac{SS_{res}}{SS_{tot}}, \tag{11}$$

where SS_{tot} is the total sum of squared differences from the mean (proportional to the variance of the data):

$$SS_{tot} = \sum_{i=1}^{N} \left(C_i - \bar{C}\right)^2 , \tag{12}$$

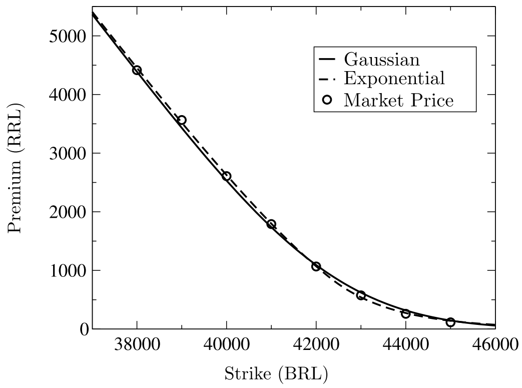

Fig. 4. Fits of the empirical option data shown in Fig. 2(b) by the Gaussian model (solid curve) and the exponential model (dashed curve). The parameters of the exponential distribution are $\gamma = 20.2 \pm 1.4$ and $\nu = 30.7 \pm 1.6$, whereas for the Gaussian model the volatility is $\sigma = 0.0135 \pm 0.0005$. The errors in the parameters were estimated using the bootstrap method; see text.

with $\bar{C}$ denoting the mean of the option premiums of the corresponding option chain. The coefficient of determination R^2 ranges from 0 to 1 and indicates how well the data is fitted by the model curve: the closer R^2 is to unity the better is the agreement between the theoretical curve and the empirical data. Strictly speaking, the coefficient of determination R^2 is not mathematically guaranteed to be in the range from 0 to 1 for nonlinear regression; nonetheless, it can still be used as a relevant measure of the goodness of the fit.

For example, in Fig. 4 we have $R_g^2 = 0.9982(3)$ for the Gaussian model and $R_e^2 = 0.99974(4)$ for the exponential model. Here the errors in R^2 were estimated via the bootstrap method described above. As $R_g^2 < R_e^2$, we conclude that the exponential model does indeed give a better description of the data, in agreement with our visual inspection of Fig. 4. It is interesting to note that both models describe relatively well the empirical data shown in Fig. 4 without the need of a volatility smile, i.e., using the same model parameters for the entire option chain. This agreement between the two models is reflected in the fact that both of them yield values of R^2 quite close to 1 (see above), although the exponential model performs slightly better in this case. For other option chains – particularly close to the expiration date – the two models differ more considerably. To investigate how the Black and Scholes and the exponential models behave as a function of the time to maturity we applied the fitting procedure described above to our entire set of selected option chains, as discussed in the next section.

4. Results and discussions

The 441 options chains selected for our analysis are spread over a wide range of times before expiration, varying from 59 days before expiration until the day before maturity. This allowed us to study which of the two models above gives a better fit to the empirical option data as a function of the time to maturity. To do this, we applied the fitting procedures described in Section 3.2 to each of the selected option chains and compared the resulting coefficients of determination, R_g^2 and R_e^2, for the Gaussian and exponential models, respectively.

Fig. 5(a) shows the difference $R_g^2 - R_e^2$ versus the time to maturity Δt. The quantity $R_g^2 - R_e^2$ leads to the following interpretation: a *negative* outcome means that the exponential model fits the data *better* than does the Gaussian model; conversely, a positive difference means that the Gaussian model performs better than the exponential model in the sense that it yields a better fit to the data. Thus, all points below the red line (horizontal axis) in Fig. 5(a) correspond to option chains for which the exponential model provided a better fit to the empirical data, whereas points above this line indicate the opposite, i.e., the Gaussian model gave a better fit.

One interesting fact in Fig. 5(a) is that the exponential seems to adjust better the empirical data near the expiration date, whereas the Gaussian model works better for longer times before maturity. To see this, notice that close to expiration, say, less than ten days to maturity, there are more points below zero; whereas the opposite is true for intermediate values of Δt, say, from 20 to 45 days before expiration, in which case the majority of points lie above zero.

To make this analysis more precise, we computed for each Δt the percentage of times that the exponential model gives a better fit to the data in comparison to the Gaussian model. This is indicated in Fig. 5(b) by the red squares. One sees from this figure that the exponential model gives a better fit to the empirical data in the majority of cases (i.e., above the 50% line) for each Δt up to 7 days prior to maturity. For longer times prior to maturity, Fig. 5(b) shows that the Gaussian model adjusts better the data in the majority of cases for most days up to 45 days before the expiration date. For $\Delta t > 45$ days there is no clear indication as to which model is best as the points (squares) in Fig. 5(b) fluctuate widely. This is in part due to the fact that at such long times before expiration, i.e., shortly after the option series is launched, there are few strikes traded at each particular day which makes the fitting procedure somewhat less reliable for these options chains.

To mitigate the fluctuations seen in the daily percentage measure shown in Fig. 5(b), we computed the cumulative frequency that gives the percentage of times that the exponential model performs better than the Gaussian model when

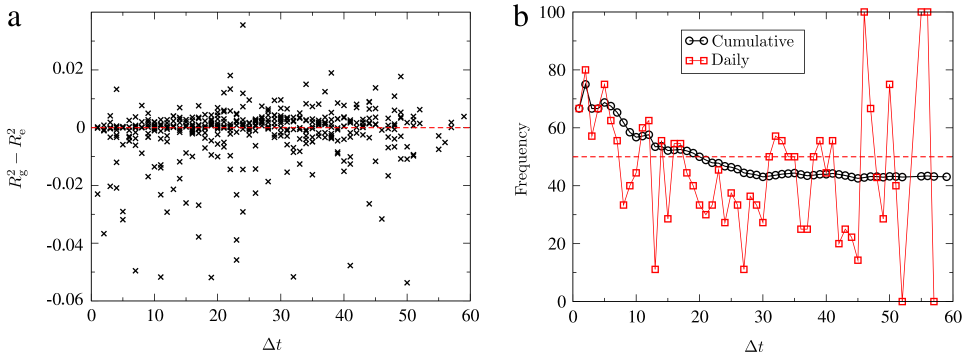

Fig. 5. (a) Difference between $R_g^2 - R_e^2$ as a function of the time Δt to maturity. Points below zero (horizontal dashed line) correspond to option chains for which the exponential model fits the data better than does the Gaussian model. Conversely, points above zero represent option chains where the Gaussian model performed better than the exponential model. (b) Percentage of option chains better fitted by the exponential model as a function of the time Δt to maturity. The red squares are the results for each Δt and the black circles are the cumulative frequency from 0 to Δt; see text. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

considering all option chains up to a time Δt. In other words, for a given Δt, we consider all options chains in the period from 0 to Δt and then compute the (relative) number of times that the exponential model gives a better fit to the data. This cumulative frequency is shown in Fig. 5(b) by open (black) circles. We see from this plot that for any period of time up to 20 days before the expiration date, the exponential model adjusts better the majority of option chains within this period.

Notice, in particular, that at around $\Delta t = 20$ the cumulative frequency crosses the 50% line (red dashed line). This means that for periods longer than 20 days the Gaussian model performs better than the exponential model in the sense that it gives a better fit to the data in the majority of cases within the period considered. In fact, it is noteworthy that 57% of all option chains studied here are best fitted by the Gaussian model. As most investors use the Black–Scholes model as a reference to the fair price of options, it is reasonable to expect that option market prices tend to follow this model in the overall majority of cases. However, near maturity the exponential model performs better than the Black–Scholes model, as discussed above.

The results shown in Fig. 5(b) reveal that up to 20 days prior to maturity the market option prices are better fitted by the exponential model. This agrees with the result of Section 2.1 where it was shown that the empirical distribution of the Ibovespa returns follows an exponential distribution for time lags up to 20 days. Our findings thus seem to indicate that the Brazilian option market implicitly takes into account the fact that the returns of Ibovespa follow an exponential distribution.

These findings suggest an influence of the non-Gaussian behavior of the underlying asset on the option price. So in a situation where traders are quite actively seeking to make profits from traded options at times near the expiration of the contract, it is quite plausible that the prices of these options do not follow a Gaussian-based model. This can be of significance to construct possible investment strategies.

5. Conclusions

We have analyzed the Brazilian option market in light of the Gaussian and exponential models for option pricing. By fitting the empirical data with the respective pricing formulas predicted by these models, we were able to compare which model performs better as a function of the time to maturity. We found that near maturity the majority of the option chains traded were better fitted by the exponential model. This result is in agreement with the fact that the distribution of daily returns of the Ibovespa follows an exponential distribution.

In other words, the exponential distribution for the Ibovespa returns does seem to have a direct impact on the Brazilian option market which is well captured by the exponential model for option pricing. In contrast, option models based on distributions with tails that go to zero slower than exponential, such as power-law distributions, are expected to be less relevant given that these heavy-tailed distributions occur in time scales (e.g., intraday quotes) much shorter than the typical lifetime of an option. It should also be noted that the exponential model describes relatively well the market prices in a given option chain without the need of volatility smiles.

The results obtained from our fitting procedure demand a more practical test, such as backtesting of historical data, to further compare the outcome of both models. In this context, the possibility of developing investment strategies based on the exponential model deserves to be investigated in detail.

The results reported here should also have implications for the risk management of derivative contracts, given that the exponential distribution provides a more conservative estimation of extreme events than the Gaussian distribution. Furthermore, the analytical formula for the option price provided by the exponential model should allow the computation of risk measures such as VaR and expected shortfall.

It is worth emphasizing that the Ibovespa index reflects the average behavior of stock prices in the Brazilian financial market. The results reported here – although based on options on the Ibovespa – should therefore reflect the generic behavior of the Brazilian option market. It would then be of interest to perform a similar analysis of options on individual stocks to verify whether the exponential model applies to these options as well. Furthermore, given that the exponential distribution has been observed in several other financial markets, it is expected that the exponential model for option pricing should also apply to these markets. It is thus hoped that the present work will stimulate further research on these directions.

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