

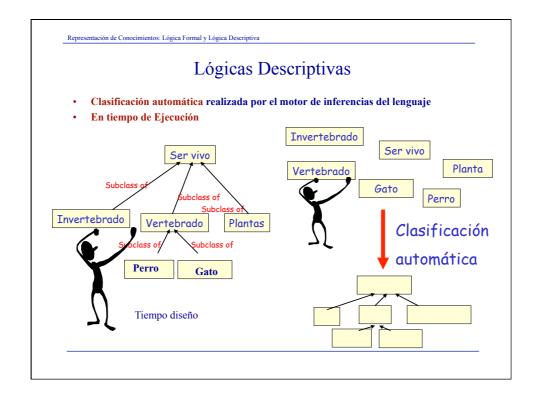
# Representación de Conocimientos: Lógica descriptiva

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# What is Description Logic?

- A family of logic based Knowledge Representation formalisms
  - Descendants of semantic networks and KL-ONE
  - Describe domain in terms of concepts (classes), roles (relationships) and individuals
    - Specific languages characterised by the constructors and axioms used to assert knowledge about classes, roles and individuals.
    - Example: ALC (the least expressive language in DL that is propositionally closed)
      - Constructors: boolean (and, or, not)
      - Role restrictions
- · Distinguished by:
  - Formal semantics (typically model theoretic)
    - · Decidable fragments of FOL
    - · Closely related to Propositional Modal & Dynamic Logics
  - Provision of inference services
    - · Sound and complete decision procedures for key problems
    - Implemented systems (highly optimised)

Representación de Conocimientos: Lógica Formal y Lógica Descriptiva

# Motivation. Why Description Logic?

- DL and Semantic Networks / Frames
  - Semantic networks and frames allow describing knowledge in terms of concepts, properties and instances, and organising it in hierarchies
  - However, they lack from a formal support
    - · Hence reasoning is not always understood in the same way
    - Especially important in multiple classification and exception handling
- DL and First Order Logic
  - First-order logic is undecidable
  - First-order logic is not focused on the definition of terminological knowledge bases (concepts, properties and instances)

Tíutlo del tema

# Structure of DL Ontologies

- A DL ontology can be divided into two parts:
  - Tbox (Terminological KB): a set of axioms that describe the structure of a domain:
    - Doctor ⊆ Person
    - Person ⊆ Man ∪ Woman
    - HappyFather ⊆ Man ∩ ∀hasDescendant.(Doctor ∪ ∀hasDescendant.Doctor)
  - Abox (Assertional KB): a set of axioms that describe a specific situation:
    - John  $\in$  HappyFather
    - hasDescendant (John, Mary)

#### Representación de Conocimientos: Lógica Formal y Lógica Descriptiva Construct Syntax Language Concept Role name R $FL_0$ Intersection Value restriction ∀R.C ALLimited existential quantification ∃R Top or Universal Bottom Atomic negation $\neg A$ Negation<sup>15</sup> $\neg C$ Union $C \cup D$ U Existential restriction ∃ R.C Е Number restrictions $(\geq n\;R)\;\;(\leq n\;R)$ N {Colombia, Argentina, México, ...} → MercoSur countries 0 $\{a_1 \dots a_n\}$ Role hierarchy Н Inverse role → ≤2 hasChild.Female, ≥1 hasParent.Male (≥ n R.C) (≤ n R.C) 12 Names previously used for Description Logies were: terminological knowledge representation languages, concept languages, term subsumption languages, and KL-ONE-based knowledge representation languages. 13 In this table, we use A to refer to atomic concepts (concepts that are the basis for building other concepts), C and D to any concept definition, R to atomic roles and S to role definitions. FL is used for structural DL languages and AL for attributive languages (Baader et al., 2003). <sup>14</sup> S is the name used for the language ALC<sub>R+</sub>, which is composed of ALC plus transitive roles. <sup>15</sup> ALC and ALCUE are equivalent languages, since union (U) and existential restriction (E) can be represented using negation (C).

### Most common constructors in class definitions

Intersection:  $C_1 \cap ... \cap C_n$ Human ∩ Male Union:  $C_1 \cup ... \cup C_n$ Doctor ∪ Lawyer Negation: ¬C ¬Male Nominals:  $\{x_1\} \cup ... \cup \{x_n\}$ {john} ∪ ... ∪ {mary} Universal restriction: ∀P.C ∀hasChild.Doctor Existential restriction: ∃P.C ∃hasChild.Lawyer ≤3hasChild Maximum cardinality: ≤nP Minimum cardinality: ≥nP ≥1hasChild

• Specific Value:  $\exists P.\{x\}$   $\exists hasColleague.\{Matthew\}$ 

- Nesting of constructors can be arbitrarily complex
  - Person ∩ ∀hasChild.(Doctor ∪ ∃hasChild.Doctor)
- · Lots of redundancy
  - A∪B is equivalent to  $\neg(\neg A \cap \neg B)$
  - ∃P.C is equivalent to  $\neg \forall P$ .  $\neg C$

Representación de Conocimientos: Lógica Formal y Lógica Descriptiva

# Most common axioms in class, property and individual definitions

 - Subclass
 C1 ⊆ C2
 Human ⊆ Animal ∩ Biped

 - Equivalence
 C1 = C2
 Man = Human ∩ Male

 - Disjointness
 C1 ∩ C2 ⊆ ⊥
 Male ∩ Female ⊆ ⊥

Proportion (roles)

Properties/roles

Classes

 $hasDaughter \subseteq hasChild$ P1 ⊆ P2 Subproperty Equivalence P1 = P2cost ≡ price Inverse P1 = P2hasChild = hasParent  $P+\subseteq P$ ancestor+ ⊆ ancestor Transitive  $T \subseteq \leq 1$  has Mother - Functional  $T \subseteq \leq 1P$  InverseFunctional  $T \subseteq \leq 1P^{-}$ T ⊆ ≤1hasPassportID

• Individuals

 $\begin{array}{lll} - & Equivalence & \{x1\} = \{x2\} & \{oeg:OscarCorcho\} = \{img:Oscar\} \\ - & Different & \{x1\} = \neg \{x2\} & \{john\} = \neg \{peter\} \end{array}$ 

- Most axioms are reducible to inclusion (∪)
  - $\quad C = D \text{ iff both } C \subseteq D \text{ and } D \subseteq C$
  - $\quad C \text{ disjoint } D \text{ iff } C \subseteq \neg D$



### Understand the meaning of universal and existential restrictions

- Decide which is the set that we are defining with different expressions, taking into account Open and Close World Assumptions  $\,$ 

Do we understand these constructors?

- ShasColleague.Lecturer
- VhasColleague.Lecturer
- BhasColleague. {Oscar}

- MasColleague. {Oscar}

- MasColleague

# Formalisation. Some basic DL modelling guidelines

•	X must be Y, X is an Y that	$\rightarrow$ X $\subseteq$ Y	
•	X is exactly Y, X is the Y that	$\rightarrow X \equiv Y$	
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• X is not Y (not the same as X is whatever it is not Y)  $\rightarrow X \subseteq \neg Y$ 

• X and Y are disjoint  $\rightarrow X \cap Y \subseteq \bot$ 

• X is Y or Z  $\rightarrow X \subseteq Y \cup Z$ 

 X is Y for which property P has only instances of Z as values
 → X ⊆ Y ∩ (∀P.Z)

• X is Y for which property P has at least an instance of Z as a value  $\Rightarrow$  X  $\subseteq$  Y  $\cap$  ( $\exists$ P.Z)

- X is Y for which property P has at most 2 values  $\Rightarrow X \subseteq Y \cap (\leq 2.P)$ 

Individual X is a Y  $\rightarrow$  X $\in$ Y

Representación de Conocimientos: Lógica Formal y Lógica Descriptiva

# Chunk 1. Formalize in DL, and then in OWL DL

#### 1. Concept definitions:

Grass and trees must be plants. Leaves are parts of a tree but there are other parts of a tree that are not leaves. A dog must eat bones, at least. A sheep is an animal that must only eat grass. A giraffe is an animal that must only eat leaves. A mad cow is a cow that eats brains that can be part of a sheep.

#### 2 Restrictions

Animals or part of animals are disjoint with plants or parts of plants.

#### 3. Properties:

Eats is applied to animals. Its inverse is eaten\_by.

#### 4. Individuals:

Tom.

Flossie is a cow.

Rex is a dog and is a pet of Mick.

Fido is a dog.

Tibbs is a cat.

# Chunk 2. Formalize in DL, and then in OWL DL

#### 1. Concept definitions:

Bicycles, buses, cars, lorries, trucks and vans are vehicles. There are several types of companies: bus companies and haulage companies.

An elderly person must be adult. A kid is (exactly) a person who is young. A man is a person who is male and is adult. A woman is a person who is female and is adult. A grown up is a person who is an adult. And old lady is a person who is elderly and female. Old ladies must have some animal as pets and all their pets are cats.

#### Restrictions:

Youngs are not adults, and adults are not youngs.

#### 3. Properties

Has mother and has father are subproperties of has parent.

### 4. Individuals:

Kevin is a person.

Fred is a person who has a pet called Tibbs.

Joe is a person who has at most one pet. He has a pet called Fido.

Minnie is a female, elderly, who has a pet called Tom.

Representación de Conocimientos: Lógica Formal y Lógica Descriptiva

# Chunk 3. Formalize in DL, and then in OWL DL

#### 1. Concept definitions:

A magazine is a publication. Broadsheets and tabloids are newspapers. A quality broadsheet is a type of broadsheet. A red top is a type of tabloid. A newspaper is a publication that must be either a broadsheet or a tabloid.

White van mans must read only tabloids.

#### 2. Restrictions:

Tabloids are not broadsheets, and broadsheets are not tabloids.

#### 3. Properties

The only things that can be read are publications.

#### 4. Individuals:

Daily Mirror

The Guardian and The Times are broadsheets

The Sun is a tabloid

## Chunk 4. Formalize in DL, and then in OWL DL

#### 1. Concept definitions:

A pet is a pet of something. An animal must eat something. A vegetarian is an animal that does not eat animals nor parts of animals. Ducks, cats and tigers are animals. An animal lover is a person who has at least three pets. A pet owner is a person who has animal pets. A cat liker is a person who likes cats. A cat owner is a person who has cat pets. A dog liker is a person who likes dogs. A dog owner is a person who has dog pets.

#### 2. Restrictions:

Dogs are not cats, and cats are not dogs.

#### 3. Properties

Has pet is defined between persons and animals. Its inverse is is\_pet\_of.

#### 4. Individuals:

Dewey, Huey, and Louie are ducks.

Fluffy is a tiger.

Walt is a person who has pets called Huey, Louie and Dewey.

Representación de Conocimientos: Lógica Formal y Lógica Descriptiva

## Chunk 5. Formalize in DL, and then in OWL DL

### 1. Concept definitions

A driver must be adult. A driver is a person who drives vehicles. A lorry driver is a person who drives lorries. A haulage worker is who works for a haulage company or for part of a haulage company. A haulage truck driver is a person who drives trucks ans works for part of a haulage company. A van driver is a person who drives vans. A bus driver is a person who drives buses. A white van man is a man who drives white things and vans.

#### 2. Restrictions:

-

#### 3. Properties:

The service number is an integer property with no restricted domain

#### 4. Individuals:

Q123ABC is a van and a white thing.

The 42 is a bus whose service number is 42.

Mick is a male who read Daily Mirror and drives Q123ABC.

## Chunk 1. Formalisation in DL

```
grass \subseteq plant
tree \subseteq plant
leaf \subseteq \exists partOf.tree
dog \subseteq \exists eats.bone
sheep \subseteq animal \cap \forall eats.grass
giraffe \subseteq animal \cap \forall eats.leaf
madCow = cow \cap \exists eats.(brain \cap \exists partOf.sheep)
(animal \cup \exists partOf.animal) \cap (plant \cup \exists partOf.plant) \subseteq \bot
```

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 $hasFather \subseteq hasParent$ 

## Chunk 2. Formalisation in DL

```
bicycle \subseteq vehicle; bus \subseteq vehicle; car \subseteq vehicle; lorry \subseteq vehicle; truck \subseteq vehicle \\ busCompany \subseteq company; haulageCompany \subseteq company \\ elderly \subseteq person \cap adult \\ kid = person \cap young \\ man = person \cap male \cap adult \\ woman = person \cap female \cap adult \\ grownUp = person \cap adult \\ oldLady = person \cap female \cap elderly \\ oldLady \subseteq \exists hasPet.animal \cap \forall hasPet.cat \\ young \cap adult \subseteq \bot \\ hasMother \subseteq hasParent
```

## Chunk 3. Formalisation in DL

```
magazine \subseteq publication
broadsheet \subseteq newspaper
tabloid \subseteq newspaper
qualityBroadsheet \subseteq broadsheet
redTop \subseteq tabloid
newspaper \subseteq publication \cap (broadsheet \cup tabloid)
whiteVanMan \subseteq \forall reads.tabloid
```

 $tabloid \cap broadsheet \subseteq \perp$ 

Representación de Conocimientos: Lógica Formal y Lógica Descriptiva

## Chunk 4. Formalisation in DL

```
pet = \exists isPetOf. T
animal \subseteq \exists eats. T
vegetarian = animal \cap \forall eats. \neg animal \cap \forall eats. \neg (\exists partOf. animal)
duck \subseteq animal; cat \subseteq animal; tiger \subseteq animal
animalLover = person \cap (\geq 3hasPet)
petOwner = person \cap \exists hasPet.animal
catLike = person \cap \exists likes.cat; catOwner = person \cap \exists hasPet.cat
dogLike = person \cap \exists likes.dog; dogOwner = person \cap \exists hasPet.dog
dog \cap cat \subseteq \bot
```

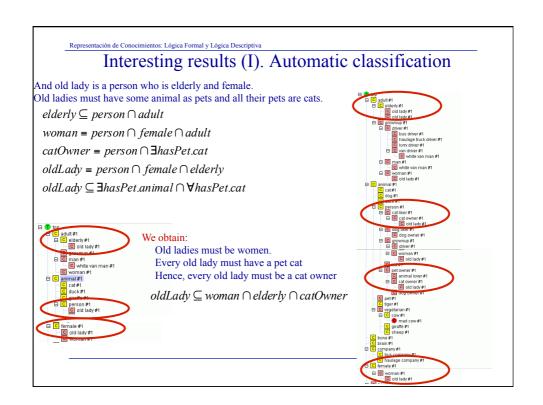
### Chunk 5. Formalisation in DL

```
driver \subseteq adult
driver = person \cap \exists drives.vehicle
lorryDriver = person \cap \exists drives.lorry
haulageWorke = \exists worksFor.(haulageCompany \cup \exists partOf.haulageCompany)
haulageTruckDriver = person \cap \exists drives.truck \cap
\exists worksFor.(\exists partOf.haulageCompany)
vanDriver = person \cap \exists drives.van
busDriver = person \cap \exists drives.bus
whiteVanMan = man \cap \exists drives.(whiteThing \cap van)
```

Representación de Conocimientos: Lógica Formal y Lógica Descriptiva

### Inference Basic Inference Tasks

- Subsumption check knowledge is correct (captures intuitions)
  - Does C subsume D w.r.t. ontology O? (in *every* model I of O, C<sup>I</sup>  $\subseteq$  D<sup>I</sup>)
- Equivalence check knowledge is minimally redundant (no unintended synonyms)
  - Is C equivalent to D w.r.t. O? (in every model | of O, C! = D!)
- Consistency check knowledge is meaningful (classes can have instances)
  - Is C satisfiable w.r.t. O? (there exists *some* model I of O s.t.  $C^{I} \neq \emptyset$ )
- Instantiation and querying
  - Is x an instance of C w.r.t. O? (in every model I of O,  $x^I \in C^I$ )
  - Is (x,y) an instance of R w.r.t. O? (in every model I of O,  $(x^I,y^I) \in R^I$ )
- All reducible to KB satisfiability or concept satisfiability w.r.t. a KB
- Can be decided using highly optimised tableaux reasoners



# Interesting results (II). Instance classification

A pet owner is a person who has animal pets
Old ladies must have some animal as pets and all their pets are cats.
Has pet has domain person and range animal
Minnio is a female alderly, who has a not called Tom.

Minnie is a female, elderly, who has a pet called Tom.  $petOwner = person \cap \exists hasPet.animal$ 

 $oldLady \subseteq \exists hasPet.animal \cap \forall hasPet.cat$  $hasPet \subseteq (person, animal)$ 

 $Minnie \in female \cap elderly$ 

hasPet(Minnie,Tom)

### We obtain:

Minnie is a person

Hence, Minnie is an old lady

Hence, Tom is a cat

 $Minnie \in person; Tom \in animal$ 

Minnie∈ petOwner

 $Minnie \in oldLady$ 

*Tom*∈*cat* 

# Interesting results (III). Instance classification and redundancy detection

An animal lover is a person who has at least three pets Walt is a person who has pets called Huey, Louie and Dewey.

 $animalLover = person \cap (\ge 3hasPet)$ 

 $Walt \in person$ 

hasPet(Walt, Huey)

hasPet(Walt,Louie)

hasPet(Walt, Dewey)

We obtain:

Walt is an animal lover Walt is a person is redundant

Walt ∈ animalLover

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# Interesting results (IV). Instance classification

A van is a type of vehicle

A driver must be adult

A driver is a person who drives vehicles

A white van man is a man who drives vans and white things

White van mans must read only tabloids

Q123ABC is a white thing and a van

Mick is a male who reads Daily Mirror and drives Q123ABC

 $van \subseteq vehicle$ 

 $driver \subseteq adult$ 

 $driver = person \cap \exists drives.vehicle$ 

 $whiteVanMan = man \cap \exists drives.(van \cap whiteThing)$ 

 $\textit{whiteVanMan} \subseteq \forall \textit{reads.tabloid}$ 

 $Q123ABC \in whiteThing \cap van$ 

 $Mick \in male$ 

reads(Mick, DailyMirror)

drives(Mick,Q123ABC)

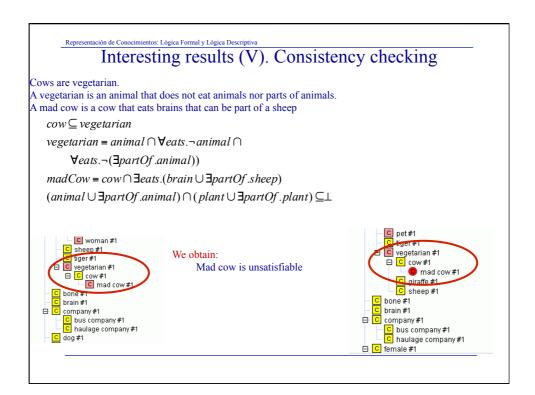
We obtain:

Mick is an adult Mick is a white van man Daily Mirror is a tabloid

 $Mick \in adult$ 

 $\mathit{Mick} \in \mathit{whiteVanMan}$ 

 $DailyMirror \in tabloid$ 



# Tableaux Algorithms

- Try to prove satisfiability of a knowledge base
- · How do they work
  - They try to build a model of input concept C
    - · Tree model property
      - If there is a model, then there is a tree shaped model
    - If no tree model can be found, then input concept unsatisfiable
  - Decompose C syntactically
    - Work on concepts in negation normal form (De Morgan's laws)
    - Use of tableaux expansion rules
    - If non-deterministic rules are applied, then there is search
  - Stop (and backtrack) if clash
    - E.g. A(x),  $\neg A(x)$
  - Blocking (cycle check) ensures termination for more expressive logics
- The algorithm finishes when no more rules can be applied or a conflict is detected

$x \bullet \{C_1 \sqcap C_2, \ldots\}$	$\rightarrow_{\sqcap}$	$x \bullet \{C_1 \sqcap C_2, C_1, C_2, \ldots\}$
$x \bullet \{C_1 \sqcup C_2, \ldots\}$	$\rightarrow_{\sqcup}$	$x \bullet \{C_1 \sqcup C_2, \textcolor{red}{C}, \ldots\}$ for $C \in \{C_1, C_2\}$
$x \bullet \{\exists R.C, \ldots\}$	→∃	$x \bullet \{\exists R.C, \ldots\}$ $R$ $y \bullet \{C\}$
$\begin{bmatrix} x \bullet \{ \forall R.C, \ldots \} \\ R \\ y \bullet \{ \ldots \} \end{bmatrix}$	$\rightarrow \forall$	$x \bullet \{ \forall R.C, \ldots \}$ $R$ $y \bullet \{C, \ldots \}$
$\begin{bmatrix} x \bullet \{ \forall R.C, \ldots \} \\ R \\ y \bullet \{ \ldots \} \end{bmatrix}$	$\rightarrow$ $\forall$ +	$x \bullet \{ \forall R.C, \ldots \}$ $R$ $y \bullet \{ \forall R.C, \ldots \}$

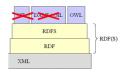
# Tableaux examples and exercises

- Example
  - $-\quad \exists S.C \land \ \forall S.(\neg C \lor \neg D) \land \exists R.C \land \ \forall R.(\exists R.C)$
- Exercise 1
  - $\quad \exists R. (\exists R.D) \land \exists S. \neg D \land \forall S. (\exists R.D)$
- Exercise 2
  - $\exists R.(C \lor D) \land \forall R. \neg C \land \neg \exists R.D$

# **OWL**

Web Ontology Language

Built on top of RDF(S) and renaming DAML+OIL primitives



### 3 layers:

- OWL Lite
  - A small subset of primitives
  - Easier for frame-based tools to transition to
- OWL DL

  - Description logic
     Decidable reasoning
- OWL Full
  - RDF extension, allows metaclasses

### Several syntaxes:

- Abstract syntax
- Manchester syntax
- RDF/XML

Representación de Conocimientos: Lógica Formal y Lógica Descriptiva Class taxonomy of the OWL KR ontology rdfs:Resource owl:Thing rdf:Property rdfs:Class owl:Nothing owl:Ontology owl:Class owl:DeprecatedClass owl:DataRang owl:AllDifferent owl:Restriction owl:TransitiveProperty owl:SymmetricProperty

# Representación de Conocimientos: Lógica Formal y Lógica Descriptiva Property list of the OWL KR ontology

Property name	domain	range
owl:intersectionOf	owl:Class	rdf:List
owl:unionOf	owl:Class	rdf:List
owl:complementOf	owl:Class	owl:Class
owl:oneOf	owl:Class	rdf:List
owl:onProperty	owl:Restriction	rdf:Property
owl:allValuesFrom	owl:Restriction	rdfs:Class
owl:hasValue	owl:Restriction	not specified
owl:someValuesFrom	owl:Restriction	rdfs:Class
owl:minCardinality	owl:Restriction	xsd:nonNegativeInteger OWL Lite: {0,1} OWL DL/Full: {0,,N}
owl:maxCardinality	owl:Restriction	xsd:nonNegativeInteger OWL Lite: {0,1} OWL DL/Full: {0,,N}
owl:cardinality	owl:Restriction	xsd:nonNegativeInteger OWL Lite: {0,1} OWL DL/Full: {0,N}
owl:inverseOf	owl:ObjectProperty	owl:ObjectProperty
owl:sameAs	owl:Thing	owl:Thing
owl:equivalentClass	owl:Class	owl:Class
owl:equivalentProperty	rdf:Property	rdf:Property
owl:sameIndividualAs	owl:Thing	owl:Thing
owl:differentFrom	owl:Thing	owl:Thing
owl:disjointWith	owl:Class	owl:Class
owl:distinctMembers	owl:AllDifferent	rdf:List
owl:versionInfo	not specified	not specified
owl:priorVersion	owl:Ontology	owl:Ontology
owl:incompatibleWith	owl:Ontology	owl:Ontology
owl:backwardCompatibleWith	owl:Ontology	owl:Ontology
owl:imports	owl:Ontology	owl:Ontology

# Representación de Conocimientos: Lógica Formal y Lógica Descriptiva OWL: Most common constructors in class definitions and axioms for classes, properties and individuals

Intersection:	$C_1 \cap \cap C_n$	intersectionOf	Human ∩ Male
Union:	$C_1 \cup \cup C_n$	unionOf	Doctor ∪ Lawyer
Negation:	¬C	complementOf	¬Male
Nominals:	$\{x_1\} \cup \cup \{x_n\}$	oneOf	{john} ∪ ∪ {mary}
Universal restriction:	∀P.C	allValuesFrom	∀hasChild.Doctor
Existential restriction:	∃P.C	someValuesFrom	∃hasChild.Lawyer
Maximum cardinality:	≤nP	maxCardinality	≤3hasChild
Minimum cardinality:	≥nP	minCardinality	≥1hasChild
Specific Value:	$\exists P.\{x\}$	hasValue	∃hasColleague.{Matthew}
Subclass	C1 ⊆ C2	subClassOf	Human ⊆ Animal ∩ Biped
Equivalence	C1 = C2	equivalentClass	Man ≡ Human ∩ Male
Disjointness	C1 ∩ C2 ⊆ ⊥	disjointWith	Male $\cap$ Female $\subseteq$ ⊥
Subproperty	P1 ⊆ P2	subPropertyOf	hasDaughter ⊆ hasChild
Equivalence	P1 = P2	equivalentProperty	cost ≡ price
Inverse	P1 = P2-	inverseOf	hasChild = hasParent-
Transitive	$P+\subseteq P$	TransitiveProperty	ancestor+ ⊆ ancestor
Functional	$T \subseteq \leq 1P$	FunctionalProperty	$T \subseteq \leq 1$ has Mother
InverseFunctional	$T \subseteq \leq 1P$ -	InverseFunctionalProperty	$T \subseteq \leq 1$ has Passport ID-
Equivalence	$\{x1\} = \{x2\}$	sameIndividualAs	{oeg:OscarCorcho}={img:Oscar
Different	$\{x1\} \equiv \neg \{x2\}$	differentFrom, AllDifferent	$\{john\} = \neg \{peter\}$

