Universidad Torcuato Di Tella 2023
Maestría en Economía y Econometría

El trabajo es individual y debe ser subido al campus antes del 3 de julio. No se contestarán preguntas sobre el contenido del trabajo.
El trabajo tiene 2 secciones. La sección 1 debe ser contestada por todos los alumnos del curso. La sección 2 solo debe ser contestada por aquellos alumnos que esten interesados en una carta de recomendación para continuar con sus estudios.

Sección 1

1. Use the Eviews data file named Findata.wf1

   Consider the series retgoog

a) Find the ARMA(p,q) that characterizes the series. Test whether the series has ARCH effects.

b) Estimate an ARCH(1) and comment on the results. Plot the conditional variances of the extimated model.

c) Do you think the results on (a) and (b) are consistent ? And, if not, to what could you attribute the difference?

2. Use the Eviews data file named Findata.wf1

   Consider de the variables retgoog, retko and retmcd:

a) Find the order of a VAR(p) which contains those variables

b) Estimate a diagonal Multivariate GARCH model for those variables

c) Compare for the results obtained in exercise 1 and 2 for retgoog.

3. Use the Eviews data file argentinaactual.wf1

   The file contains the following variables pbireal (for real GDP) and 30 and 60 days interest rates.

a) Find the order of a VAR(p) using the growth of pbreal and the spread between the 60 and the 30 days interest rates.

b) Establish whether the spread is a valid predictor of the growth in pbreal.

c) Find the impulse response functions for shocks to both the spread and growth in pbreal. Compare your results with those obtained doing local projections. Discuss the differences and similarities between these two methods

4. Use the Eviews data file named uncerindex.wf1

The data file contains the variables *ersf* : the excess (with respect to the risk-free rate) return inclusive of dividend yields of stock prices, *pd* : the Price-Dividend ratio, *vrp3*: a volatility index, *exret*: ex- dividends returns, and *deltasent12*: a variable that captures consumer confidence.

a) Estimate a regression of *ersf* on *vrp3* and *deltasent12* to determine if those variables (endogeneity issues aside), can be considered determinants of the excess return (the Risk Premium).

b) Estimate a regression of *ersf* (the Risk Premium) on *pd*, *vrp3*, *exret* and *deltasent12* **past values** to determine if those variables have predictive power for future Risk Premia.

c) Estimate a *Threshold* regression of *ersf* on *pd*, *vrp3*, exret **past values** to determine if those variables have predictable power for future Risk Premia. Use **past** deltasent12 as your threshold and analyze whether your results differ between regimes.

d) Estimate a Markov Switching regression of ersf on past pd, vrp3, exret, and deltasent12 values to determine if those variables have predictive power for future Risk Premia. Compare the results and the separation of the regimes with those obtained in (c).

5. Use the Eviews data file named AnnLee.wf1

The data file contains the variables *EX3MHOLD12* EX3MHOLD24 EX3MHOLD60 EX3MHOLD120 representing the excess (with respect to the 3 months rate) realized returns of holding a 3 months bond of maturity 12, 24, 60 and 120 months, respectively.

a) Use the Kalman filter to extract common factor that explains the movements of those returns. Store the common factor.

b) Add the slope to the measurement equation of your previous specification in (a). Compare the predicted states of both specifications.

6 Use the Eviews data file named Money.wf1

The data file contains the variables m, y, i and p representing M1, industrial production, a 3 months interest rate and prices, respectively.

Consider the following equation:

$$\log(m_t) - \log(p_t) = \alpha_0 + \alpha_1 i_t + \alpha_2 \log(y_t) + \varepsilon_t$$

a) Estimate for the period 1962m1-2019m11 the above equation using Markov Chain Monte Carlo techniques.

b) Compare the dispersion of α_1 and of α_2 for the sub-samples 1962m1-1979m1, 1979m1-1982m12 and 1983m1- 2019m11.

c) How do the results that use a non-informative prior compare with those using OLS?

Sección 2

1. Consider the bivariate VAR process defined by

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} 0.2 & -0.3 \\ \alpha & 0 \end{bmatrix} \begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} u_{1t} \\ u_{2t} \end{bmatrix},$$

   where $\{u_t = (u_{1t}, u_{2t})'\}$ are independent and identically distributed random vectors with zero mean and variance–covariance matrix equal to the identity matrix.

   (a) Under what condition(s) on α is the VAR stationary?

   (b) Show that x_t has a univariate AR(2) representation and y_t has a univariate ARMA(2, 1) representation.

   (c) Compare the stationarity conditions in (a) and (b).

   (d) Derive the impulse response functions Ψ_s of the VAR for $s = 1, 2, 3, 4$. Discuss whether the innovations of the above VAR need to be orthogonalized. Explain how to obtain information for $\dfrac{\partial x_{t+s}}{\partial u_{2t}}$.

   (e) Find an expression for $E_t y_{t+n}$, where E_t denotes expectation conditional on information available at time t.

2. Suppose that the bivariate time series of real stock prices (P_t) and real dividends (D_t) satisfies the system

$$\begin{aligned} P_t + \beta D_t &= u_{1t}, \qquad u_{1t} = c_1 + \phi u_{1,t-1} + \varepsilon_{1t}, \\ D_t &= u_{2t}, \qquad u_{2t} = c_2 + u_{2,t-1} + \varepsilon_{2t}, \end{aligned}$$

   where $\beta \neq 0$, $-1 < \phi < 1$, and $\{(\varepsilon_{1t}, \varepsilon_{2t})'\}$ are independent and identically distributed random vectors with zero mean and variance–covariance matrix equal to the identity matrix.

   (a) Determine the order of integration of $\{P_t\}$ and $\{D_t\}$.

   (b) Are $\{P_t\}$ and $\{D_t\}$ cointegrated? Explain.

   (c) Derive the error-correction representation of the system.

   (d) Assume that stock prices are the discounted value of the future dividends stream, i.e.,

$$P_t = E_t \left[\sum_{s=1}^{\infty} \left(\frac{1}{1+r} \right)^s D_{t+s} \right] + \varepsilon_{1t},$$

   where r is a fixed discounting rate, ε_{1t} is a measurement error, and E_t denotes expectation conditional on information available at time t.

(i) Derive the stochastic process for P_t assuming that D_t follows the process given above. (Note that $\sum_{s=1}^{\infty} a^s = a\sum_{s=0}^{\infty} a^s = \frac{a}{1-a}$ and $\sum_{s=1}^{\infty} a^s s = \frac{a}{(1-a)^2}$ for $|a| < 1$). [7 marks]

(ii) What value of ϕ ensures that the long-run relationship $P_t + \beta D_t$ will be identical to the present-value solution of the model? Explain. [3 marks]

3. Consider the model

$$y_t = \mu + \varepsilon_t, \qquad \varepsilon_t = e_t\sigma_t,$$

where $\{e_t\}$ are independent $\mathsf{N}(0,1)$ random variables and

$$\text{(i)} \quad \sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2,$$

with $\omega > 0$, $\alpha \geq 0$, $\beta \geq 0$, and $\alpha + \beta < 1$.

(a) Establish whether the following representations for ε_t^2 and σ_t^2 are valid:

$$\begin{aligned}
\text{(ii)} \quad \varepsilon_t^2 &= \omega + (\alpha+\beta)\varepsilon_{t-1}^2 - \beta\nu_{t-1} + \nu_t, \\
\text{(iii)} \quad \sigma_t^2 &= \omega + (\alpha+\beta)\sigma_{t-1}^2 + \alpha\nu_{t-1}, \\
\text{(iv)} \quad \sigma_t^2 &= \frac{\omega}{1-\beta} + \alpha\sum_{i=0}^{\infty}\beta^i\varepsilon_{t-i-1}^2,
\end{aligned}$$

where $\{v_t\}$ is white noise.

(b) Discuss which of the above expressions for the conditional variance σ_t^2 should be included in the likelihood function when estimating $(\mu, \omega, \alpha, \beta)$.

(c) Show that the n-periods-ahead forecast of the conditional variance of ε_t can be written as

$$E_{t-1}\sigma_{t+n}^2 = \frac{\omega}{1-(\alpha+\beta)} + \left(\sigma_t^2 - \frac{\omega}{1-(\alpha+\beta)}\right)(\alpha+\beta)^n,$$

where E_{t-1} denotes conditional expectation given information available at time $t-1$.

(d) Show that

$$Q_t = \sum_{i=1}^{n} E_{t-1}\sigma_{t+i}^2 = \frac{n\omega}{1-(\alpha+\beta)} + \frac{(\alpha+\beta)[1-(\alpha+\beta)^n]}{1-(\alpha+\beta)}\left(\sigma_t^2 - \frac{\omega}{1-(\alpha+\beta)}\right).$$

(e) Consider the estimator of the average forecast W_t defined as $W_t = n^{-1}Q_t$. Show that $E(W_t) = \lim_{n\to\infty} W_t$.

4. Consider the following VAR model

$$\begin{bmatrix} y_t' \\ x_t' \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} y_{t-1}' \\ x_{t-1}' \end{bmatrix} + \begin{bmatrix} \varepsilon_t \\ u_t \end{bmatrix},$$

where $\{\varepsilon_t\}$ and $\{u_t\}$ are white noise processes, y_t' and x_t' are given by

$$y_t' = y_t - \alpha_0 - \alpha_1 S_t,$$

$$x_t' = x_t - \alpha_0 - \alpha_2 S_t,$$

and $\{S_t\}$ follows a two-state Markov process with transition probabilities

$$p = P(S_t = 1 | S_{t-1} = 1),$$

$$q = P(S_t = 0 | S_{t-1} = 0).$$

(a) Derive $E(S_t|S_{t-1})$ and $\text{var}(S_t|S_{t-1})$.

(b) Derive $E(S_t)$ and $\text{var}(S_t)$, and state the conditions under which they exist.

(c) Derive the expected value of the state n periods ahead conditional on the information about the state at time t, $E(S_{t+n}|S_t)$.

(d) Suppose y_t is the first difference of the spot exchange rate (e_t), $y_t = e_t - e_{t-1}$, and x_t is the spread between the one period forward rate (f_t) and the spot exchange rate, $x_t = f_t - e_t$. A version of the hypothesis of the umbiasedness of the forward rate as a predictor of the future spot rate may be expressed as

$$x_t = E_t y_{t+1},$$

where E_t denotes expectation conditional on information available at time t. Which testable restrictions does this hypothesis impose on the parameters of the VAR and which on the switching parameters?