

Cmpe 492 Final Project
Final Report

Player Rating Estimation
via
Bayesian Inference



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Contents

1	Introduction & Motivation	4
2	State of the Art	5
2.1	Elo	5
2.2	Glicko & TrueSkill	5
3	Methods	7
3.1	Model	7
3.2	Inference	12
3.2.1	Markov Chain Monte Carlo	12
3.2.2	Expectation Propagation	15
4	Results	17
5	Conclusion & Discussion	62
6	Future Work	62
7	All Tests	63
8	Appendices	63

1 Introduction & Motivation

Player rating systems are essential nearly to any game in situated in different contexts (video games, sports etc). This need arises from the fact that an *unbalanced* match is not enjoyable to any of it's competitors. In an *unbalanced* match, there is a high variation among the skills of the competitors, therefore a remarkable challenge is not achieved and inadequate amount of fun is generated.

Assuming the fact that the skill levels of the players in a match is the dominant factor affecting the result of that match, if we could figure out skill levels of players of a particular game then we could arrange *balanced* matches that are entertaining for both highly and poorly skilled players. In additions to this, estimating skills of players with respected to a particular model enables us to predict results of future matches. Intuitively a more complex model would possess less of bias and would yield more accurate predictions.

All in all, rating systems attempt to measure goodness of players, for instance, in most national football leagues, wins worth 3 points, draws worth 1 point and losses worth 0 point. At the end of the season, we obtain an estimate of skills/goodness of competing teams in that league according to this underlying model. Some can argue that it ignores many details therefore it is too simple and inaccurate. Another disadvantage of this model is that it does not specify rules to compute probabilities for possible outcomes of a future match.

To attack this problem, many context-free rating systems are proposed but Elo, Glicko, TrueSkill are the most prominent ones [?]. In this project we assume a particular model (resembling to the one proposed by TrueSkill algorithm) governing winning/losing probabilities in reality. Afterwards, we attempt to estimate skills of players by training on their past data, i.e. match results. Our model necessitates that we perform Bayesian inference to figure out skills of players, therefore we implemented three different methods to achieve inference in this project. We run the algorithms both on synthetic and real data and interpret the results.

In the next section, we tell briefly about the significant works in this field as well as the state of the art methods. After that, in section 3, we explain the model we assumed in depth and overview different practices to perform Bayesian inference. We explain the algorithms (MCMC, Expectation Propagation) we have used in this project in detail in the following subsections. In section 4, we visualize the results we have obtained with various tables and plots and interpret them in section 5. We conclude by discussing how this project can be extended in section 6 and we include the full source code in the last section.

2 State of the Art

2.1 Elo

Arpad Elo constructed a rating system for the board game Chess in 1959. According to his modeling [1], rating of a player is a unit-less positive number. The probabilities for different outcomes of a match is a function of skills, s_i , of competing players. Players exhibit a *performance* according to their skills. Performance $p_i \sim \mathcal{N}(p_i; s_i, \beta^2)$, in a sense, is a noisy version of a player's skill. The probability that player 1 wins is calculated as follows, where ϕ is the cumulative density function of zero-mean, unit-variance Gaussian:

$$P(p_1 > p_2 | s_1, s_2) = \phi\left(\frac{s_1 - s_2}{\sqrt{2}\beta}\right)$$

When the actual game outcome of the game is observed skills are updated so that their sum remains the same. Let the actual outcome $y = 1$ if player 1 wins, $y = -1$ if player 2 wins and $y = 0$ if it's a stalemate. Elo update is calculated as follows: $s_1 \leftarrow s_1 + \delta y$ and $s_2 \leftarrow s_2 + \delta y$

$$\delta = \alpha\beta\sqrt{\pi} \left(\frac{y+1}{2} - \phi\left(\frac{s_1 - s_2}{\sqrt{2}\beta}\right) \right)$$

Here α is between 0 and 1 and represents the significance of new observations compared to past. This rating system is still in use on many platforms although it has a few drawbacks.

- Competitions has to be one player versus one player. *Elo* does not cover the case when multiple players compete in a game and the outcome is a permutation of the players to signify ranks.
- A new player's skill is provisional until he plays a decent number of matches.

2.2 Glicko & TrueSkill

Glicko [2] and *TrueSkill* [3] attempts to tackle these problems and generalize underlying idea proposed in *Elo* rating system. *Glicko* suggests modeling of a player's skill as a belief distribution (in particular a Gaussian distribution) rather than representing it with a single number. *TrueSkill* extends this idea to adapt it to matches with multiple teams and players.

Let k teams compete in a match. Team assignments A_j are non-overlapping, in other words, $A_j \cap A_i = \emptyset$ if $i \neq j$. The outcome $r = (r_1, \dots, r_k) \in \{1, \dots, k\}$. Probability of the outcome is modeled as a function of skills of players and team assignments

$P(r|s, A)$. Through Bayes' rule we get the following posterior distribution for skills of players.

$$p(s|r, A) = \frac{P(r|s, A)p(s)}{P(r|A)}$$

Skill is represented as a random variable $s_i \sim \mathcal{N}(s_i; \mu_i, \sigma_i^2)$ where μ_i is the point estimate of it and σ_i is the uncertainty associated with it. Similarly, as in the *Elo* rating system, it is assumed that players generate a performance based on their skills $p_i \sim \mathcal{N}(p_i; s_i, \beta^2)$. Furthermore, teams performances are computed by $t_j = \sum_{i \in A_j} p_i$. This model can be represented with a factor graph compactly. See Figure 1 for an example factor graph representation.

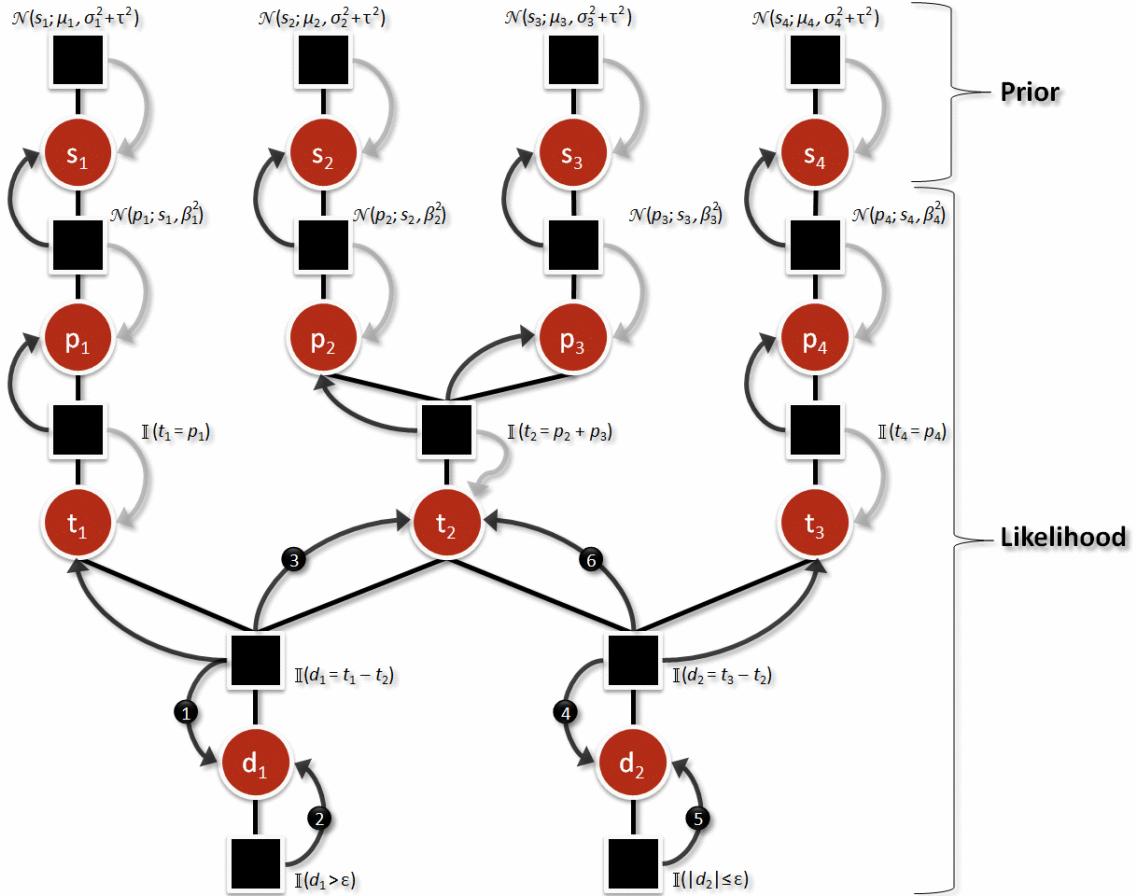
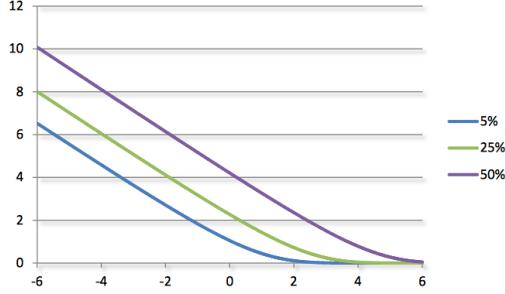


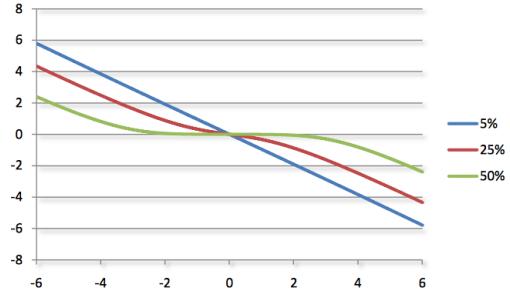
Figure 1: Factor graph representation of *TrueSkill* model for a particular match where $k = 3$, $A_1 = \{1\}$, $A_2 = \{2, 3\}$, $A_3 = \{4\}$, $r = (1, 2, 2)$ i.e. team 1 won, team 2 and 3 draw.

The see Figure 2 and 3 for interpretation of (comparison/result) factors on the bottom of the graph.

In principle *TrueSkill* assumes this model and uses *Expectation Propagation* algorithm to infer skill distributions. EP is a deterministic approximation algorithm that can be considered a generalization of a well-known exact inference algorithm used in factor graphs known as *Belief Propagation/Sum-Product algorithm* [4].

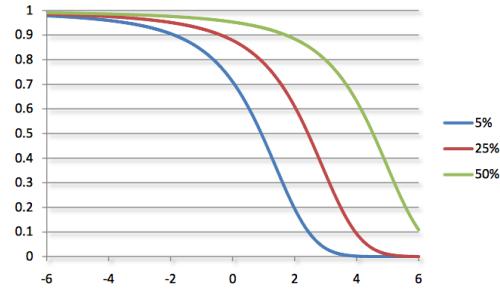


(a) $\mathbb{I}(\cdot > \epsilon)$ i.e. *win*. Notice this indicates a major update in means if an upsetting result is observed.

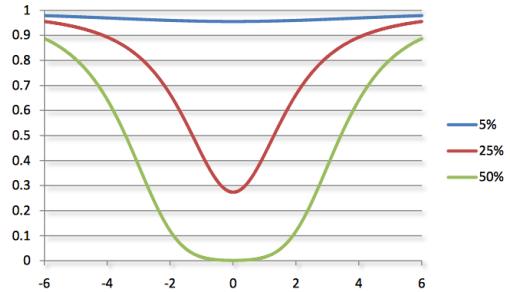


(b) $\mathbb{I}(\cdot \leq |\epsilon|)$ i.e. *draw*. Notice this indicates a major update in means only when difference is far from zero.

Figure 2: Function V i.e. *mean update*. x-axis represents the skill difference between winning team and losing team.



(a) $\mathbb{I}(\cdot > \epsilon)$ i.e. *win*. Notice this indicates a major reduction in uncertainties if an upsetting result is observed.



(b) $\mathbb{I}(\cdot \leq |\epsilon|)$ i.e. *draw*. Notice this indicates a major update in uncertainties only when difference is far from zero

Figure 3: Function W i.e. *uncertainty update*. x-axis represents the skill difference between winning team and losing team.

3 Methods

In this project we aim to demonstrate Bayesian skill inference (as done in *TrueSkill*) via three different methods both on synthetic data that we generate and on real-world data such as Tennis, football, basketball match results from 2015-2016 season. First two algorithms, namely Metropolis-Hastings and Gibbs sampler, are two different instances of a method known as Markov Chain Monte Carlo sampling. Here, we perform inference by stochastic approximation. In the third method we implement *TrueSkill*'s EP algorithm [5] to perform deterministic approximate inference according to the model described above.

3.1 Model

Models exist to capture and explain the real world dynamics of various incidents. For instance, Newton's equations is a model to explain the physical motions of the

objects situated in the space. Similarly laws of thermodynamics define a model to capture relation between heat, temperature and energy, work. Models can be accurate or inaccurate with respect to some criteria and this depends on the complexity of the model.

In this project for our Markov Chain Monte Carlo (MCMC from now on) algorithms we propose a particular model. When it comes to Expectation Propagation (EP from now on) algorithm we assume *TrueSkill*'s model. Following are the rules of our model.

- Let a game G is played by n different players, i.e. $p_i, i \in \{1, 2, \dots, n\}$ where each p_i denotes a player.
- A match of the game G takes place between a number of teams $k \in \{2, 3, \dots\}$
- In a match, each team $t_j, j \in \{1, 2, \dots, k\}$ is a set of players with size $n_j \in \{1, 2, \dots\}$
- In a match, intersection of any two different team is an empty set.
- Each player p_i has a fixed unit-less skill value $s_i \in (0, 50)$ associated with itself.
- The skill value of a team w_i is the sum of skill values of it's members, i.e. $w_i = \sum_{p_j \in t_i} s_j$
- Matches with more than 2 competitors/teams are collapsed down to it's components that are atomic matches between any two team competed in the match. See Tables 1 and 2

Name	Rank	Name	W/L/D	Name	W/L/D	Name	W/L/D
Team A	1	Team A	W	Team A	W	Team B	W
Team B	2	Team B	L	Team C	L	Team C	L
Team C	3						

Table 1: An example how matches with more than 2 competitors/teams is collapsed down to it's components.

Name	Rank	Name	W/L/D	Name	W/L/D	Name	W/L/D
Team A	1	Team A	W	Team A	W	Team B	D
Team B	2	Team B	L	Team C	L	Team C	D
Team C	2						

Table 2: An example how matches with more than 2 competitors/teams is collapsed down to it's components.

- Outcome, r , of atomic matches is a function of skills w_i, w_j of two teams t_i and t_j .

$$p(r|w_1, w_2) = \begin{cases} \exp(|w_1 - w_2|, c_1, c_2) & \text{if } r = 0 \\ (1 - \exp(|w_1 - w_2|, c_1, c_2)) * \sigma(w_1 - w_2, c_3) & \text{if } r = 1 \\ (1 - \exp(|w_1 - w_2|, c_1, c_2)) * \sigma(w_2 - w_1, c_3) & \text{if } r = -1 \end{cases}$$

Figure 4: Definition of the win/lose/draw model

- In an atomic match teams are named as team 1 and team 2.
- Outcome $r \in \{1, -1, 0\}$ where 1 indicates win of team 1, -1 indicates win of team 2 and 0 indicates a draw.
- Decay parameter of exponential function is set $c_2 = 0.05$
- Steepness parameter of sigmoid function is set $c_3 = 0.06$
- Draw factor parameter is set $c_1 = 0.33$, is the y-intercept of exponential function. It regulates draw probabilities together with decay parameter. For example, when skills of teams are equal to each other this model estimates the chance of draw as 33%. If draws are more likely in a particular type of game it can be set higher. See Figures 5, 6, 7
- We assume that initially all skills $s_i \sim \mathcal{U}(s_i; 0, 50)$
- We can represent the model we described using a Bayesian network. See an example in Figure 8 and 9
- We get the joint probability distribution $P(s_1, s_2, \dots, s_n, w_1, w_2, \dots, w_k, r_1, r_2, \dots)$

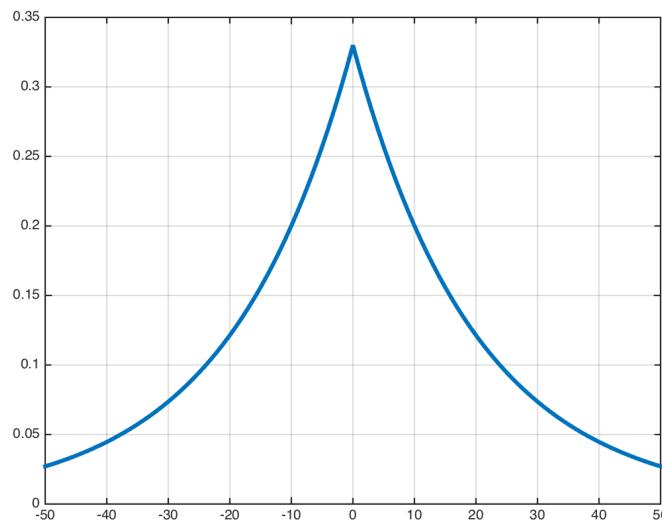


Figure 5: $p(r = 0|w_1, w_2)$ with parameters, $c_1 = 0.33$, $c_2 = 0.05$

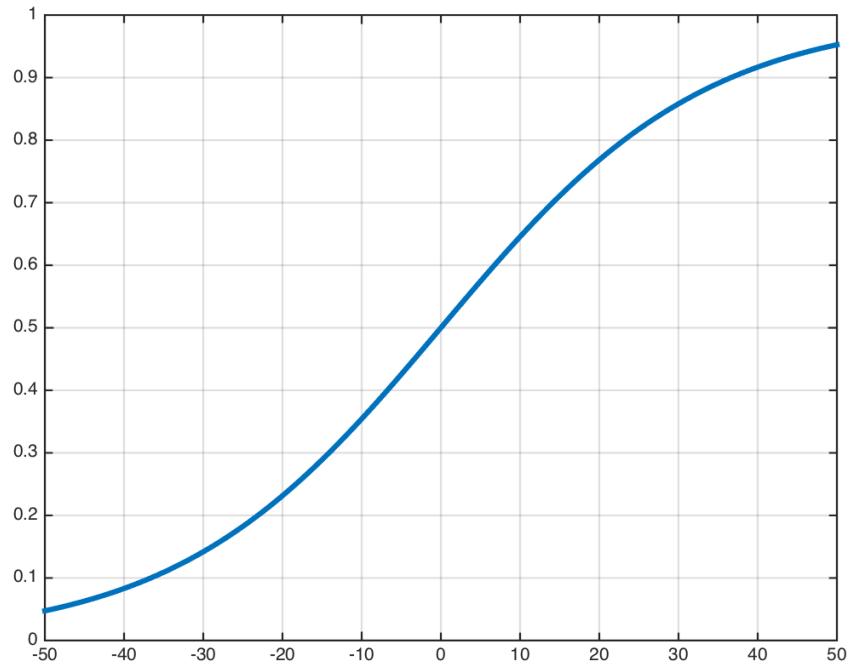


Figure 6: $\sigma(x, c_3) = \frac{1}{1 + \exp(-c_3 x)}$ with parameter, $c_3 = 0.06$. Also when $c_1 = 0$, $p(r = 1|w_1, w_2)$ collapses down to sigmoid

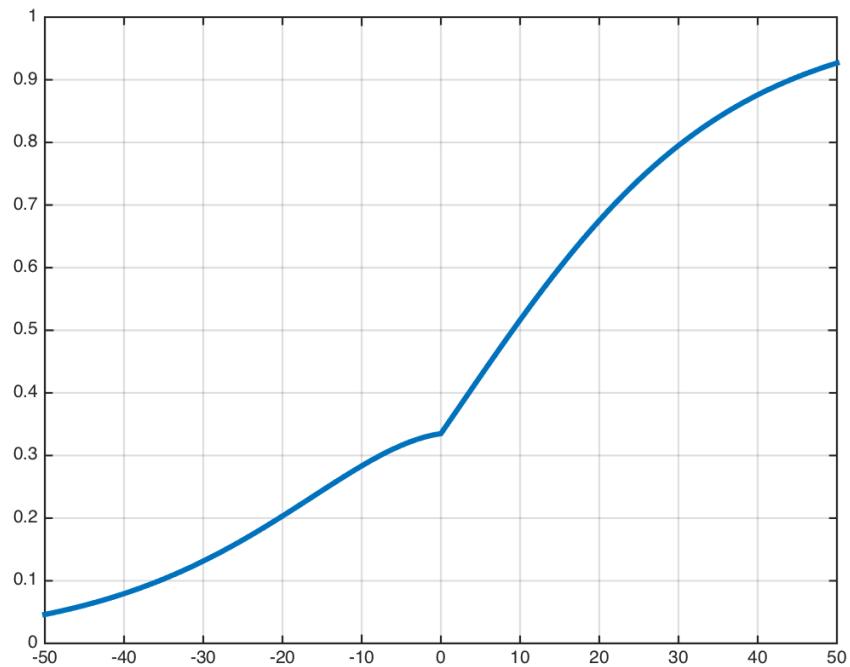


Figure 7: $p(r = 1|w_1, w_2)$ with parameters, $c_1 = 0.33$, $c_2 = 0.05$, $c_3 = 0.06$.

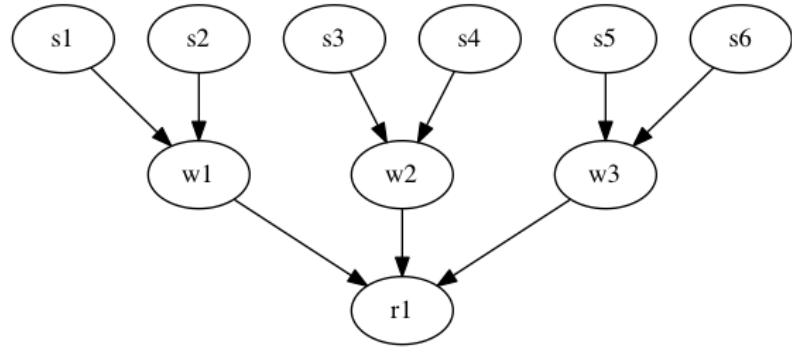


Figure 8: An example visual representation of a match with 3 teams each having 2 members. Notice that this match contains more than 2 competitors. See Figure 9 for decomposed version.

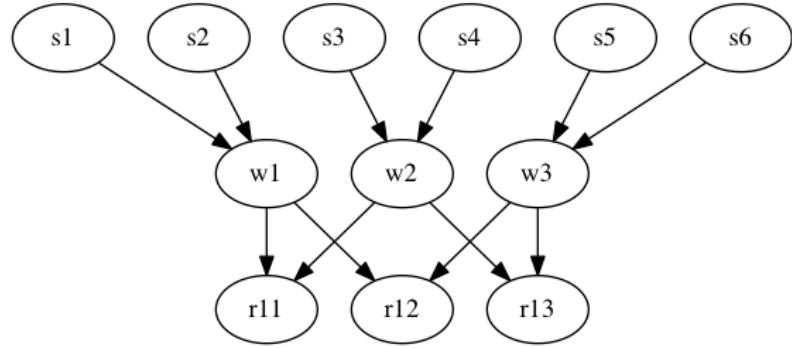


Figure 9: Decomposed version of the Bayesian network shown in Figure 8. Note that we perform decomposition in order to allow simple probability distribution functions for outcomes, i.e.

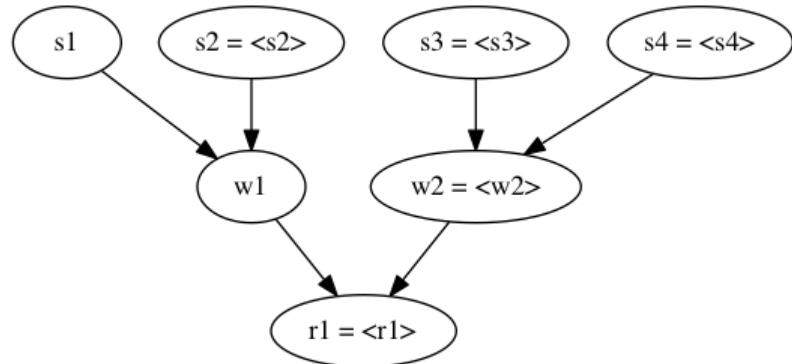


Figure 10: Example network with instantiated variables.

3.2 Inference

To estimate the skill distributions we are going to construct a Bayesian network whose terminal/result nodes are observed (i.e. fixed) and attempt to figure out the posterior distribution of skills of players. *Variational Bayesian methods* (approximate), *Monte Carlo methods* (approximate), *Belief Propagation algorithm* (exact), *Assumed Density Filtering* (approximate) and *Expectation Propagation* (approximate) are some significant methods for inference [6].

Exact methods in continuous domain are often impractical due to integrals intractable to calculate. For that reason approximate algorithms are preferred. Especially Metropolis-Hastings algorithm to sample from a given target distribution via forming an implicit Markov chain is very general. We implement this method in this project. Another MCMC method that we implement is Gibbs sampler because we can analytically come up with conditional distribution of individual skills and can sample from it comfortably which is a must for Gibbs sampler.

Furthermore we implemented Expectation Propagation algorithm as described in [3], therefore model assumed for this algorithm is different than we described in Section 3.1 and it is slightly more complex. You can find a brief description of *TrueSkill*'s model in Section 2.2.

3.2.1 Markov Chain Monte Carlo

Recall our joint probability distribution.

$$p(s|w, r = Data) = p(s_1, s_2, \dots, s_n|w_1, w_2, \dots, w_k, r_1 = \hat{r}_1, r_2 = \hat{r}_2, \dots)$$

$$p(s|w, r = Data) = \frac{p(w, r = Data|s)p(s)}{p(w, r = Data)}$$

$$p(s|w, r = Data) = \frac{p(w, r = Data|s)p(s)}{\int p(w, r = Data|s)p(s)ds}$$

Since $s \sim \mathcal{U}(0, 50)$, $p(s)$ does not vary. It is a constant. In addition, the relationship between w and s is deterministic. Given s , one can calculate w with confidence of 100%. Therefore this expression simplifies down to:

$$p(s|r = Data) = \frac{p(r = Data|s)}{\int p(r = Data|s)ds}$$

$$p(s|r = Data) \propto p(r = Data|s)$$

Metropolis-Hastings Algorithm The task in MH algorithm is fairly simple, see Figure 11 for description of the algorithm. It is insensitive if the target distribution is normalized or not which is a nice feature. We used a multivariate Gaussian distribution as our proposal. The mean of the Gaussian is the last point reached in the space and variance is given as input by the supervisor depending on the sensitivity of the target distribution. We observed in our tests that when the dimensionality of the data grows (i.e. more players and matches) small perturbations in the input/-variable result in enormous changes in the output/probability. This is a behaviour we would not favor because it increases the rejection rate. On the other side, one must be careful when choosing a less aggressive proposal because it can lead to inadequate exploration of the space.

When computing output/probability for a given configuration/point in the space, we may have to deal with really small numbers if the size of the dataset is large. To prevent errors originate from the representation of floating numbers in computers we transform some computations to logarithmic scale.

```

1: Initialise  $x^{(1)}$  arbitrarily
2: for  $n = 2, 3 \dots$  do
3:   Propose a candidate:  $x^{new} \sim q(x^{new}|x^{(n-1)})$ 
4:   Compute acceptance probability:

$$\alpha(x^{new}|x^{(n-1)}) = \min \left\{ 1, \frac{q(x^{(n-1)}|x^{new})\pi(x^{new})}{q(x^{new}|x^{(n-1)})\pi(x^{(n-1)})} \right\}$$

5:   Sample from uniform distribution:  $a \sim \mathcal{U}(0, 1)$ 
6:   if  $a < \alpha$  then
7:     Accept candidate:  $x^{(n)} \leftarrow x^{new}$ 
8:   else
9:     Reject candidate:  $x^{(n)} \leftarrow x^{(n-1)}$ 
10:  end if
11: end for
```

Figure 11: Metropolis-Hastings algorithm.

Gibbs Sampler We can go further and calculate conditional probabilities for individual skill variables. Consider the atomic match visualized in Bayes network in Figure 10

$$p(s_1|\hat{s}_2, \hat{s}_3, \hat{s}_4, \hat{r}_1) = \frac{p(\hat{r}_1, s_1, \hat{s}_2, \hat{s}_3, \hat{s}_4)}{\int_{s_1} p(\hat{r}_1, s_1, \hat{s}_2, \hat{s}_3, \hat{s}_4)}$$

Where \hat{v} denotes the instantiation of that variable. It is represented in the example network as $v = <\!v\!\!>$. The denominator is only a normalizing constant. In

fact $p(\hat{r}_1, s_1, \hat{s}_2, \hat{s}_3, \hat{s}_4) = p(\hat{r}_1|s_1, \hat{s}_2, \hat{s}_3, \hat{s}_4) * p(s_1) * p(\hat{s}_2) * p(\hat{s}_3) * p(\hat{s}_4)$ and since prior distributions of skills are a constant as well we can plug them in the normalizing constant Z . We get the following conditional distribution:

$$p(s_1|\hat{s}_2, \hat{s}_3, \hat{s}_4, \hat{r}_1) = \frac{p(\hat{r}_1|s_1, \hat{s}_2, \hat{s}_3, \hat{s}_4)}{Z}$$

We had defined this expression in the first place! (see Section 3.1, Figure 4). This indicates that we indeed can use Gibbs sampler once we can draw samples from the distribution given in Figure 4. Here we choose a numerical way for solving this problem. We compute the unnormalized cumulative distribution numerically by calculating the distribution function at various points and computing a running sum to obtain data points. These data points mimic cumulative distribution function. Then we apply multinomial resampling on obtained data points, see Figure 12. We transform these computations to logarithmic scale to prevent numerical truncation errors. When size of the dataset and number of the players involved grows we have to deal with really small probabilities.

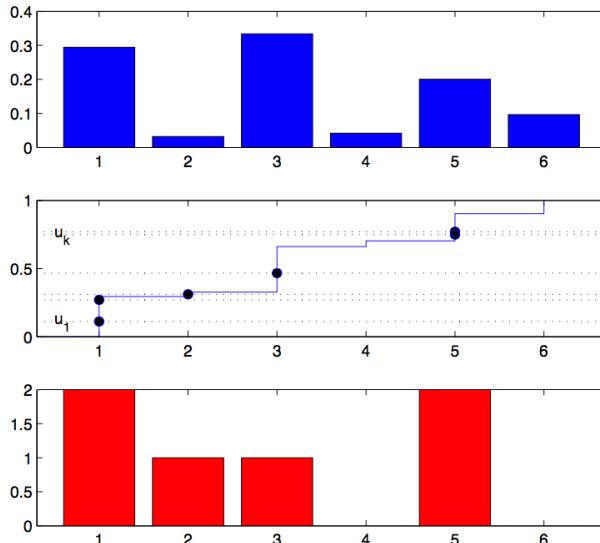


Figure 12: Multinomial resampling. Taken from [7]

```

1: Initialize  $x^{(1)} = (x_1, \dots, x_D)^{(1)}$  arbitrarily
2: for  $n = 2, 3 \dots$  do
3:    $x_1^{(n)} \sim p(x_1|x_2^{(n-1)}, x_3^{(n-1)}, \dots, x_D^{(n-1)})$ 
4:    $x_2^{(n)} \sim p(x_2|x_1^{(n)}, x_3^{(n-1)}, \dots, x_D^{(n-1)})$ 
5:    $\vdots$ 
6:    $x_D^{(n)} \sim p(x_D|x_1^{(n)}, x_2^{(n)}, \dots, x_{D-1}^{(n)})$ 
end for

```

Figure 13: Gibbs sampler

3.2.2 Expectation Propagation

Factor Graphs and Belief Propagation Factor graphs are bipartite graphs over variables and local factors. Local factor together form the global function over all variables when multiplied with each other. Therefore, factor graphs formalize the independence of local factors, Figure 14. Factor graphs are more general than Bayes networks therefore a Bayes network can be re-written as a factor graph when factors are defined as conditional probability distributions. The sum-product algorithm [4] (a.k.a. belief propagation) defines a procedure to marginalize product of factors (in Bayesian case, it corresponds to marginalization of joint probability distribution). Very roughly, it involves multiplication of factors and integrating out of variables which is intractable most of the time. Therefore in principle it works but in reality it is highly impractical.

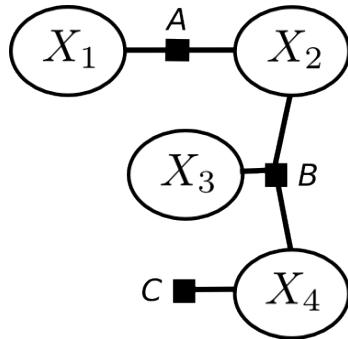


Figure 14: An example factor graph. $G(x_1, x_2, x_3, x_4) = A(x_1, x_2)B(x_2, x_3, x_4)C(x_4)$

EP follows this general approach: Given a factor graph whose marginal (we are interested in) is hard to compute, approximate it by a simpler graph whose marginal is easy to compute. We replace each factor f_α by an approximate factor \tilde{f}_α . Often we choose approximating distribution/factor to be from the exponential family.

A decent procedure to approximate while keeping the error on the marginal distribution minimum is as follows: Start with an empty factor graph. For each factor f_α , incorporate it into the current approximating distribution q by multiplication. This yields some new distribution q^i that is not in the approximating family (exponential family), so project it into the set of all approximating distributions using KL divergence. This is known as *Assumed density filtering*. A problem with this approach is that the order used to traverse factors affect how good the approximation of marginal will be.

Expectation propagation fixes this problem by looping through factors many times. Each of the time, old approximation of a factor f_α is removed from the global approximation q^{i-1} instead exact factor is plugged in back by multiplying. Then, the obtained global distribution (that is not in the family we would like it to

be in) is projected to the approximating family using KL divergence.

Algorithm 1: Assumed Density Filtering

Initialize. Set q^0 to uniform **foreach factor** f_α **do**

for $i=m\dots j+1$ **do**

- 1. *Refinement* Incorporate exact factor into the approximation

$$q^{+\alpha} \propto q^{i-1} f_\alpha$$
- 2. *Projection* Compute the new approximate distribution by projecting into approximating family

$$q = \arg \min_q \text{KL}(q^{+\alpha} \| q)$$

end

end

Algorithm 2: Expectation Propagation

Initialize. Set q^0 to uniform

repeat

- 1. Choose a factor f_α to refine
- 2. *Refinement.* Remove the previous approximation from global approximation. Incorporate the exact factor

$$q^{\backslash\alpha} \propto \frac{q^{i-1}}{f_\alpha}$$

$$q^{+\alpha} \propto f_\alpha q^{\backslash\alpha}$$
- 3. *Projection.* Compute the new global approximate distribution by projecting into approximating family

$$q = \arg \min_q \text{KL}(q^{+\alpha} \| q)$$

until *until convergence;*

4 Results

Synthetic Data We first, generated synthetic data in order to test the algorithms in various configurations. Generated synthetic players are assigned a "*reel skill*" value. Teams are formed randomly. The match results are generated according to the model we assumed in Figure 4.

Real-World Data Afterwards, we tested three algorithms on real-world data, namely, tennis data (match results in Grand Slam tournaments of 2012-2013), football data (match results in German, Spanish, Turkish, English primary national leagues), basketball data (match results of Nba 2015-2016 regular season). We fetched all datasets and preprocessed them so that they have the form shown in Table 3.

Team 1	Team 2	Result
Munich	Dortmund	1
Karlsruhe	Köln	0
Stuttgart	Hamburg	-1
...

Table 3: Form of the ready data (csv)

Since we have made extensive tests, we include only a small portion of them in this section. For all tests, see *Section 7: All Tests*.

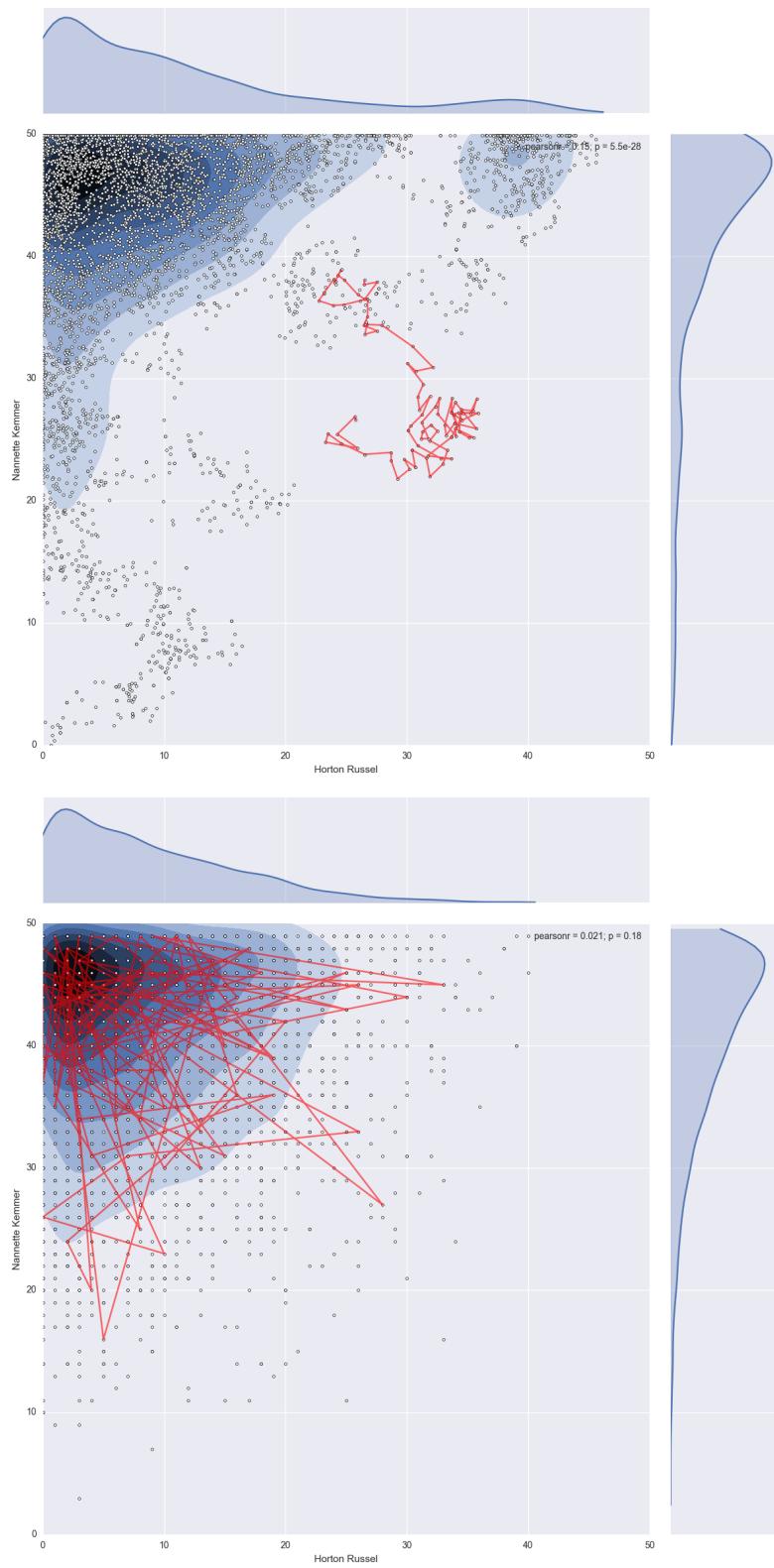


Figure 15: djsalid

Table 4: dsaşdsaa

Date	T8_23_Tue_May_31_22:15:43_2016
Algorithm	MH
Dataset	Synthetic
Number of matches	10
Number of players	2
Number of teams	2
Number of iterations	5000
Elapsed time	1
Variance of proposal	1.0
Proposals accepted	4747.0
Proposals rejected	253.0

#	Player	Reel Skill	#	P1	Skill	Uncertainty
1	Nannette Kemmer	40.00	1	Nannette Kemmer	40.65	10.64
2	Horton Russel	10.00	2	Horton Russel	10.84	11.01

Mean Skill Error	0.75	Correct	9
Std Skill Error	0.10	Incorrect	1
Mean Rank Error	0.00	Percentage	0.900
Std Rank Error	0.00	Mean of expected	0.857

Player	Reel Skill	Skill	Skill Error	RS Rank	Skill Rank	Rank Error
Nannette Kemmer	40.00	40.65	-0.65	1	1	0
Horton Russel	10.00	10.84	-0.84	2	2	0

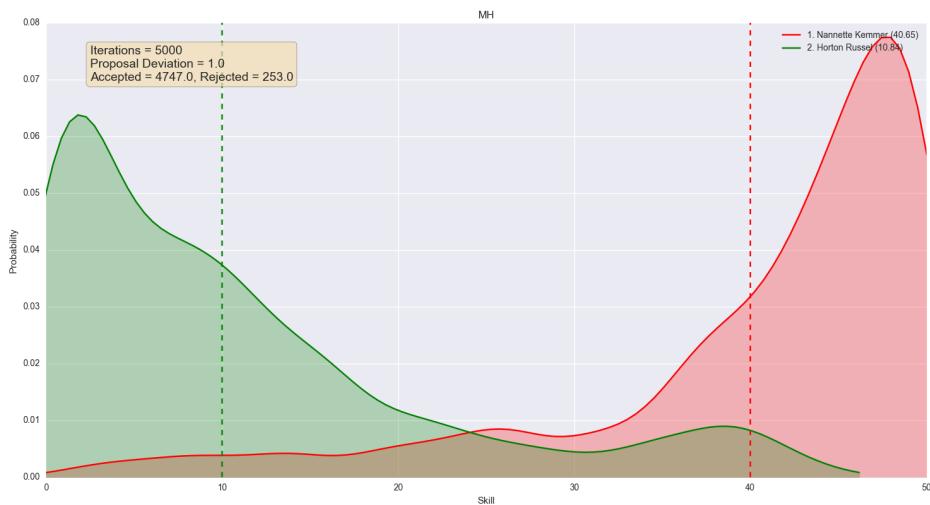


Table 5: dsadas

Date	T8_23_Tue_May_31_22:15:43_2016
Algorithm	Gibbs
Dataset	Synthetic
Number of matches	10
Number of players	2
Number of teams	2
Number of iterations	4000
Elapsed time	4
Variance of proposal	None
Proposals accepted	None
Proposals rejected	None

#	Player	Reel Skill	#	P1	Skill	Uncertainty
1	Nannette Kemmer	40.00	1	Nannette Kemmer	40.39	7.63
2	Horton Russel	10.00	2	Horton Russel	8.69	7.40

Mean Skill Error	0.85	Correct	9
Std Skill Error	0.46	Incorrect	1
Mean Rank Error	0.00	Percentage	0.900
Std Rank Error	0.00	Mean of expected	0.870

Player	Reel Skill	Skill	Skill Error	RS Rank	Skill Rank	Rank Error
Nannette Kemmer	40.00	40.39	-0.39	1	1	0
Horton Russel	10.00	8.69	1.31	2	2	0

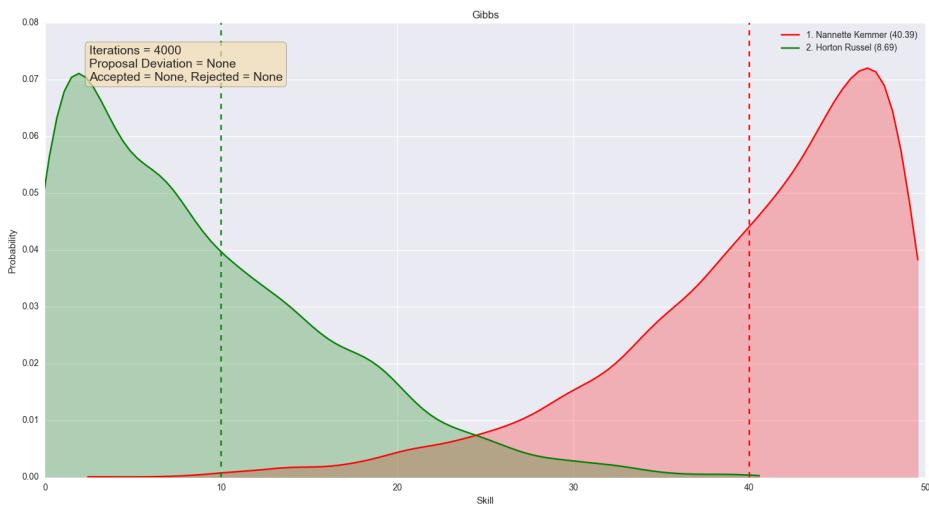


Table 6: dsadas

Date	T8_23_Tue_May_31_22:15:43_2016
Algorithm	EP
Dataset	Synthetic
Number of matches	10
Number of players	2
Number of teams	2
Number of iterations	None
Elapsed time	0
Variance of proposal	None
Proposals accepted	None
Proposals rejected	None

#	Player	Reel Skill	#	P1	Skill	Uncertainty
1	Nannette Kemmer	40.00	1	Nannette Kemmer	34.66	4.95
2	Horton Russel	10.00	2	Horton Russel	15.44	4.95

Mean Skill Error	5.39	Correct	9
Std Skill Error	0.05	Incorrect	1
Mean Rank Error	0.00	Percentage	0.900
Std Rank Error	0.00	Mean of expected	0.882

Player	Reel Skill	Skill	Skill Error	RS Rank	Skill Rank	Rank Error
Nannette Kemmer	40.00	34.66	5.34	1	1	0
Horton Russel	10.00	15.44	-5.44	2	2	0

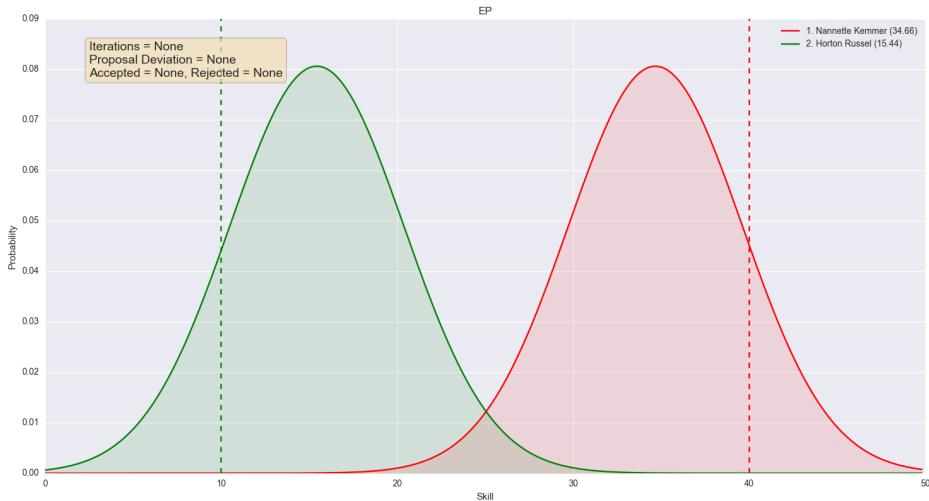


Table 7: dsaşdsaa

Date	T1_1_Wed_Jun__1_07:34:11_2016
Algorithm	MH
Dataset	Synthetic
Number of matches	300
Number of players	5
Number of teams	5
Number of iterations	5000
Elapsed time	31
Variance of proposal	1.0
Proposals accepted	3932.0
Proposals rejected	1068.0

#	Player	Reel Skill	#	P1	Skill	Uncertainty
1	Payton Doyle	45.00	1	Payton Doyle	44.96	3.92
2	Mr. Turner West II	35.00	2	Mr. Turner West II	33.77	4.93
3	Kinsey Block	25.00	3	Kinsey Block	22.09	4.52
4	Nannette Kemmer	15.00	4	Nannette Kemmer	8.20	4.54
5	Horton Russel	5.00	5	Horton Russel	5.45	4.41

Mean Skill Error	2.29	Correct	229
Std Skill Error	2.46	Incorrect	71
Mean Rank Error	0.00	Percentage	0.763
Std Rank Error	0.00	Mean of expected	0.758

Player	Reel Skill	Skill	Skill Error	RS Rank	Skill Rank	Rank Error
Payton Doyle	45.00	44.96	0.04	1	1	0
Mr. Turner West II	35.00	33.77	1.23	2	2	0
Kinsey Block	25.00	22.09	2.91	3	3	0
Nannette Kemmer	15.00	8.20	6.80	4	4	0
Horton Russel	5.00	5.45	-0.45	5	5	0

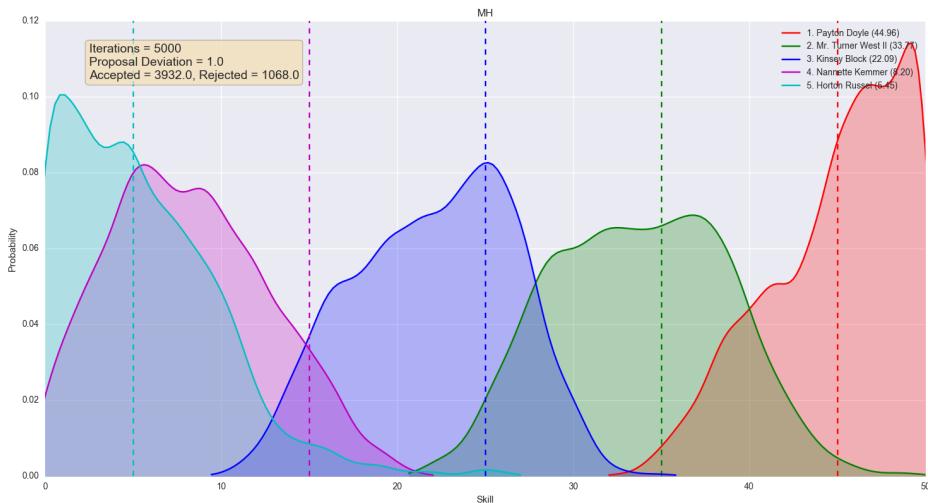


Table 8: dsadas

Date	T1_1_Wed_Jun__1_07:34:11_2016
Algorithm	Gibbs
Dataset	Synthetic
Number of matches	300
Number of players	5
Number of teams	5
Number of iterations	4000
Elapsed time	109
Variance of proposal	None
Proposals accepted	None
Proposals rejected	None

#	Player	Reel Skill	#	P1	Skill	Uncertainty
1	Payton Doyle	45.00	1	Payton Doyle	44.58	3.41
2	Mr. Turner West II	35.00	2	Mr. Turner West II	33.29	4.46
3	Kinsey Block	25.00	3	Kinsey Block	21.70	4.26
4	Nannette Kemmer	15.00	4	Nannette Kemmer	7.97	4.16
5	Horton Russel	5.00	5	Horton Russel	4.53	3.42

Mean Skill Error	2.59	Correct	229
Std Skill Error	2.46	Incorrect	71
Mean Rank Error	0.00	Percentage	0.763
Std Rank Error	0.00	Mean of expected	0.760

Player	Reel Skill	Skill	Skill Error	RS Rank	Skill Rank	Rank Error
Payton Doyle	45.00	44.58	0.42	1	1	0
Mr. Turner West II	35.00	33.29	1.71	2	2	0
Kinsey Block	25.00	21.70	3.30	3	3	0
Nannette Kemmer	15.00	7.97	7.03	4	4	0
Horton Russel	5.00	4.53	0.47	5	5	0

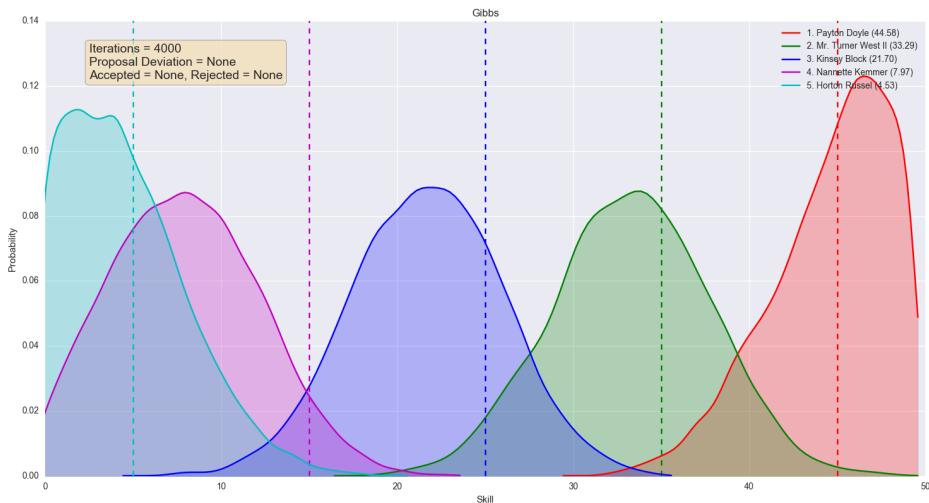


Table 9: dsadas

Date	T1_1_Wed_Jun__1_07:34:11_2016
Algorithm	EP
Dataset	Synthetic
Number of matches	300
Number of players	5
Number of teams	5
Number of iterations	None
Elapsed time	0
Variance of proposal	None
Proposals accepted	None
Proposals rejected	None

#	Player	Reel Skill	#	P1	Skill	Uncertainty
1	Payton Doyle	45.00	1	Payton Doyle	49.63	1.03
2	Mr. Turner West II	35.00	2	Mr. Turner West II	47.83	0.95
3	Kinsey Block	25.00	3	Kinsey Block	45.36	0.91
4	Nannette Kemmer	15.00	4	Nannette Kemmer	40.49	0.94
5	Horton Russel	5.00	5	Horton Russel	38.73	0.99

Mean Skill Error	19.41	Correct	229
Std Skill Error	10.05	Incorrect	71
Mean Rank Error	0.00	Percentage	0.763
Std Rank Error	0.00	Mean of expected	0.702

Player	Reel Skill	Skill	Skill Error	RS Rank	Skill Rank	Rank Error
Payton Doyle	45.00	49.63	-4.63	1	1	0
Mr. Turner West II	35.00	47.83	-12.83	2	2	0
Kinsey Block	25.00	45.36	-20.36	3	3	0
Nannette Kemmer	15.00	40.49	-25.49	4	4	0
Horton Russel	5.00	38.73	-33.73	5	5	0

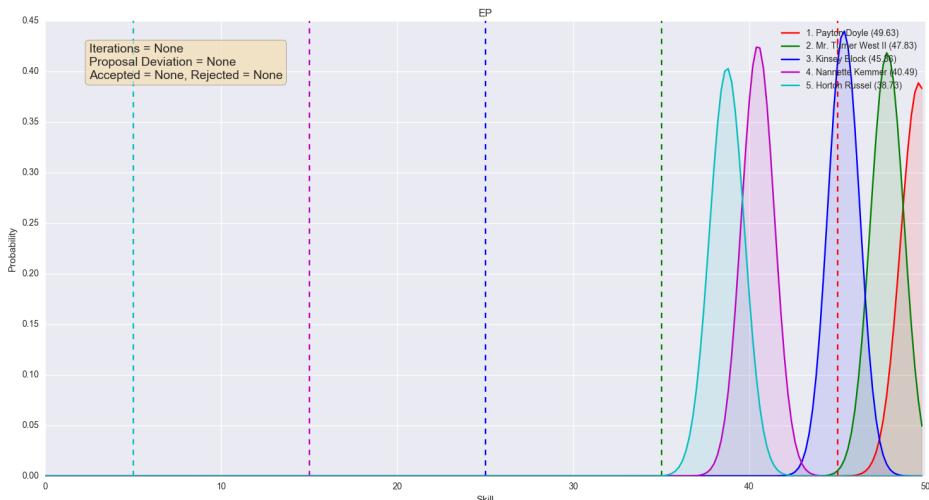


Table 10: dsaşdasa

Date	T1_5_Mon_May_30_18:33:50_2016
Algorithm	MH
Dataset	Synthetic
Number of matches	100
Number of players	5
Number of teams	5
Number of iterations	5000
Elapsed time	10
Variance of proposal	1.0
Proposals accepted	4289.0
Proposals rejected	711.0

#	Player	Reel Skill	#	P1	Skill	Uncertainty
1	Mr. Brandan Dickinson	45.00	1	Mr. Brandan Dickinson	41.43	5.82
2	Noemi Mante	35.00	2	Noemi Mante	40.08	5.93
3	Elbert Mraz	25.00	3	Antione Mertz	20.28	5.57
4	Antione Mertz	15.00	4	Elbert Mraz	18.21	5.85
5	Orvil Quigley	5.00	5	Orvil Quigley	4.82	4.38

Mean Skill Error	4.18	Correct	65
Std Skill Error	2.24	Incorrect	35
Mean Rank Error	0.40	Percentage	0.650
Std Rank Error	0.49	Mean of expected	0.708

Player	Reel Skill	Skill	Skill Error	RS Rank	Skill Rank	Rank Error
Mr. Brandan Dickinson	45.00	41.43	3.57	1	1	0
Noemi Mante	35.00	40.08	-5.08	2	2	0
Elbert Mraz	25.00	18.21	6.79	3	4	-1
Antione Mertz	15.00	20.28	-5.28	4	3	1
Orvil Quigley	5.00	4.82	0.18	5	5	0

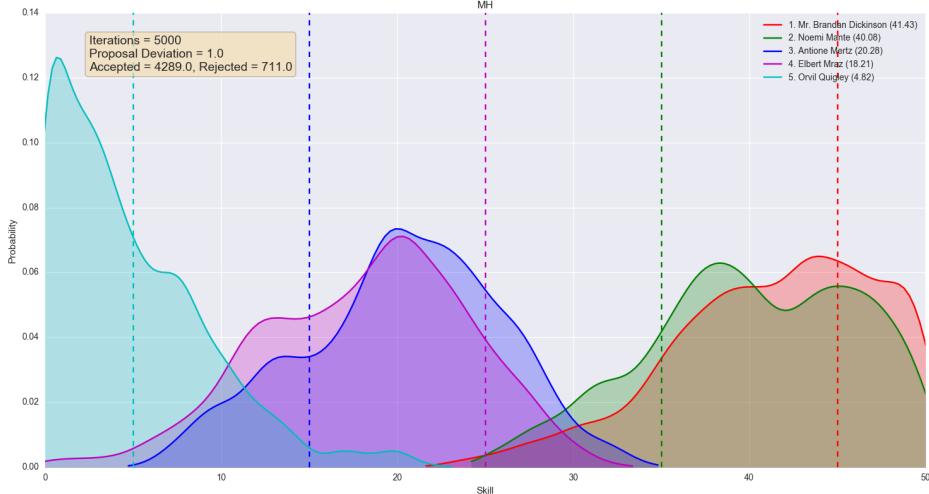


Table 11: dsadas

Date	T1_5_Mon_May_30_18:33:50_2016
Algorithm	Gibbs
Dataset	Synthetic
Number of matches	100
Number of players	5
Number of teams	5
Number of iterations	4000
Elapsed time	34
Variance of proposal	None
Proposals accepted	None
Proposals rejected	None

#	Player	Reel Skill	#	P1	Skill	Uncertainty
1	Mr. Brandan Dickinson	45.00	1	Mr. Brandan Dickinson	40.90	5.45
2	Noemi Mante	35.00	2	Noemi Mante	40.05	5.77
3	Elbert Mraz	25.00	3	Antione Mertz	22.53	6.76
4	Antione Mertz	15.00	4	Elbert Mraz	20.88	6.75
5	Orvil Quigley	5.00	5	Orvil Quigley	6.86	5.18

Mean Skill Error	4.53	Correct	65
Std Skill Error	1.83	Incorrect	35
Mean Rank Error	0.40	Percentage	0.650
Std Rank Error	0.49	Mean of expected	0.692

Player	Reel Skill	Skill	Skill Error	RS Rank	Skill Rank	Rank Error
Mr. Brandan Dickinson	45.00	40.90	4.10	1	1	0
Noemi Mante	35.00	40.05	-5.05	2	2	0
Elbert Mraz	25.00	20.88	4.12	3	4	-1
Antione Mertz	15.00	22.53	-7.53	4	3	1
Orvil Quigley	5.00	6.86	-1.86	5	5	0

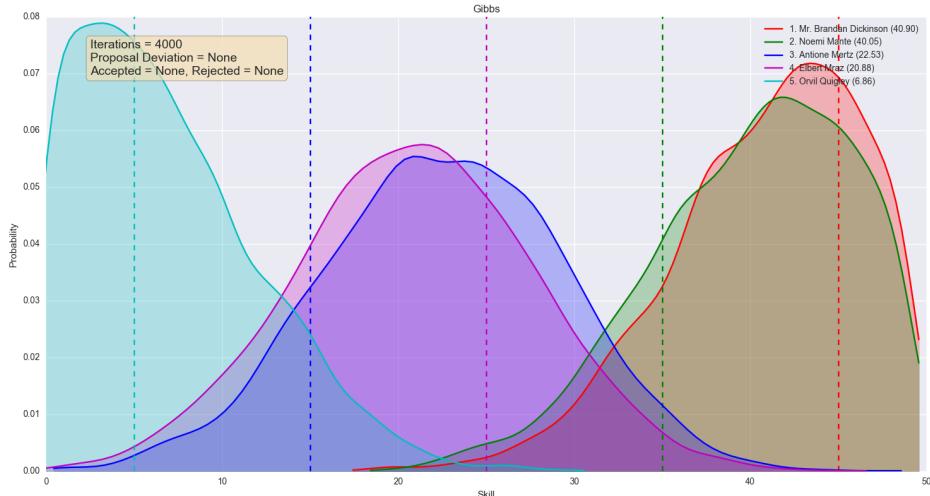


Table 12: dsadas

Date	T1_5_Mon_May_30_18:33:50_2016
Algorithm	EP
Dataset	Synthetic
Number of matches	100
Number of players	5
Number of teams	5
Number of iterations	None
Elapsed time	0
Variance of proposal	None
Proposals accepted	None
Proposals rejected	None

#	Player	Reel Skill	#	P1	Skill	Uncertainty
1	Mr. Brandan Dickinson	45.00	1	Noemi Mante	30.43	1.42
2	Noemi Mante	35.00	2	Mr. Brandan Dickinson	30.24	1.49
3	Elbert Mraz	25.00	3	Antione Mertz	25.96	1.41
4	Antione Mertz	15.00	4	Elbert Mraz	24.81	1.42
5	Orvil Quigley	5.00	5	Orvil Quigley	20.68	1.48

Mean Skill Error	9.23	Correct	57
Std Skill Error	5.98	Incorrect	43
Mean Rank Error	0.80	Percentage	0.570
Std Rank Error	0.40	Mean of expected	0.685

Player	Reel Skill	Skill	Skill Error	RS Rank	Skill Rank	Rank Error
Mr. Brandan Dickinson	45.00	30.24	14.76	1	2	-1
Noemi Mante	35.00	30.43	4.57	2	1	1
Elbert Mraz	25.00	24.81	0.19	3	4	-1
Antione Mertz	15.00	25.96	-10.96	4	3	1
Orvil Quigley	5.00	20.68	-15.68	5	5	0

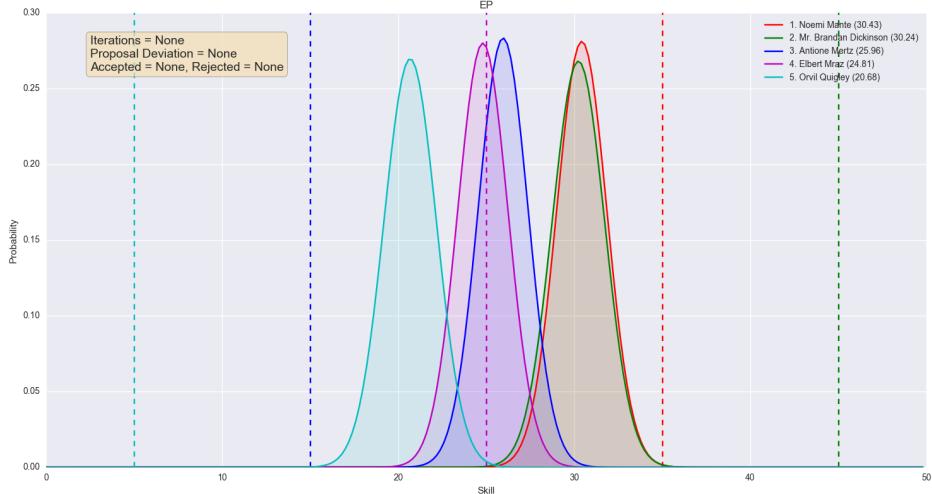


Table 13: dsaşdsa

Date	T2_2_Wed_Jun__1_07:48:16_2016
Algorithm	MH
Dataset	Synthetic
Number of matches	300
Number of players	5
Number of teams	5
Number of iterations	5000
Elapsed time	50
Variance of proposal	1.0
Proposals accepted	3701.0
Proposals rejected	1299.0

#	Player	Reel Skill	#	P1	Skill	Uncertainty
1	Payton Doyle	37.00	1	Payton Doyle	39.89	5.60
2	Horton Russel	25.00	2	Nannette Kemmer	24.74	5.92
3	Mr. Turner West II	21.00	3	Mr. Turner West II	21.22	5.74
4	Nannette Kemmer	20.00	4	Horton Russel	20.88	5.77
5	Kinsey Block	15.00	5	Kinsey Block	18.97	5.77

Mean Skill Error	3.19	Correct	185
Std Skill Error	1.60	Incorrect	115
Mean Rank Error	0.80	Percentage	0.617
Std Rank Error	0.98	Mean of expected	0.627

Player	Reel Skill	Skill	Skill Error	RS Rank	Skill Rank	Rank Error
Payton Doyle	37.00	39.89	-2.89	1	1	0
Horton Russel	25.00	20.88	4.12	2	4	-2
Mr. Turner West II	21.00	21.22	-0.22	3	3	0
Nannette Kemmer	20.00	24.74	-4.74	4	2	2
Kinsey Block	15.00	18.97	-3.97	5	5	0

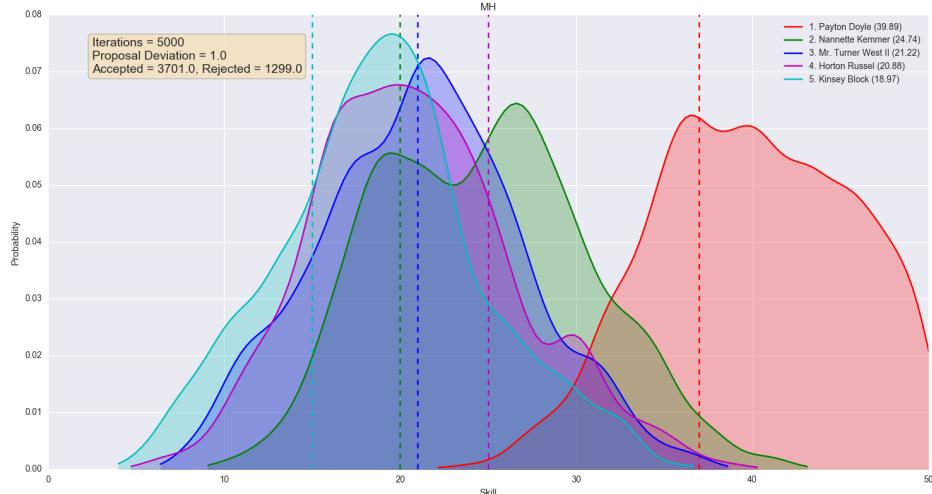


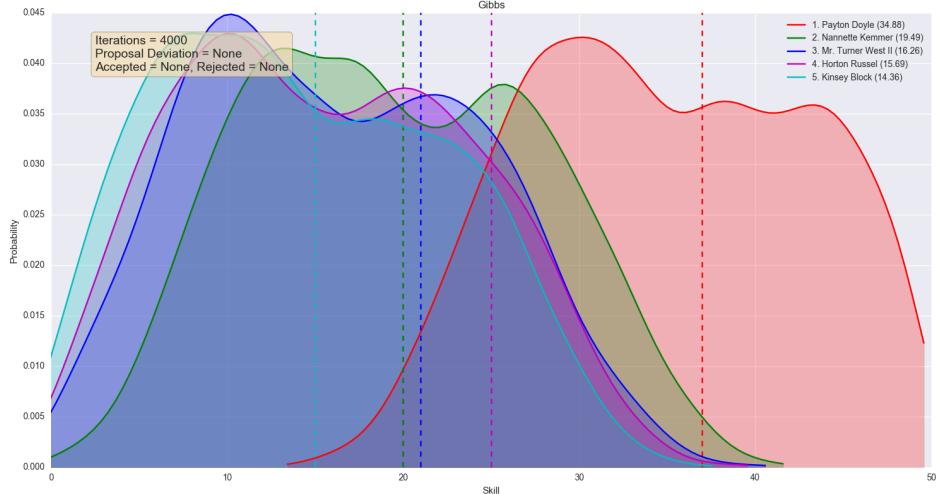
Table 14: Pivoting, Gibbs shifts because its narrow

Date	T2_2_Wed_Jun__1_07:48:16_2016
Algorithm	Gibbs
Dataset	Synthetic
Number of matches	300
Number of players	5
Number of teams	5
Number of iterations	4000
Elapsed time	115
Variance of proposal	None
Proposals accepted	None
Proposals rejected	None

#	Player	Reel Skill	#	P1	Skill	Uncertainty
1	Payton Doyle	37.00	1	Payton Doyle	34.88	7.93
2	Horton Russel	25.00	2	Nannette Kemmer	19.49	8.14
3	Mr. Turner West II	21.00	3	Mr. Turner West II	16.26	8.10
4	Nannette Kemmer	20.00	4	Horton Russel	15.69	8.13
5	Kinsey Block	15.00	5	Kinsey Block	14.36	8.03

Mean Skill Error	3.46	Correct	185
Std Skill Error	3.29	Incorrect	115
Mean Rank Error	0.80	Percentage	0.617
Std Rank Error	0.98	Mean of expected	0.624

Player	Reel Skill	Skill	Skill Error	RS Rank	Skill Rank	Rank Error
Payton Doyle	37.00	34.88	2.12	1	1	0
Horton Russel	25.00	15.69	9.31	2	4	-2
Mr. Turner West II	21.00	16.26	4.74	3	3	0
Nannette Kemmer	20.00	19.49	0.51	4	2	2
Kinsey Block	15.00	14.36	0.64	5	5	0



Date	T2_7_Mon_May_30_18:35:27_2016
Algorithm	EP
Dataset	Synthetic
Number of matches	30
Number of players	5
Number of teams	5
Number of iterations	None
Elapsed time	0
Variance of proposal	None
Proposals accepted	None
Proposals rejected	None

#	Player	Reel Skill	#	P1	Skill	Uncertainty
1	Mr. Khalid Friesen Sr.	34.00	1	Josiephine Heaney	26.74	2.76
2	Josiephine Heaney	31.00	2	Delwin Botsford	25.28	2.55
3	Delwin Botsford	30.00	3	Lorenzo Casper	25.20	2.61
4	Vergil Jacobs	19.00	4	Mr. Khalid Friesen Sr.	22.79	2.58
5	Lorenzo Casper	17.00	5	Vergil Jacobs	19.99	2.78

Mean Skill Error	5.88	Correct	23
Std Skill Error	3.51	Incorrect	7
Mean Rank Error	1.60	Percentage	0.767
Std Rank Error	0.80	Mean of expected	0.630

Player	Reel Skill	Skill	Skill Error	RS Rank	Skill Rank	Rank Error
Mr. Khalid Friesen Sr.	34.00	22.79	11.21	1	4	-3
Josiephine Heaney	31.00	26.74	4.26	2	1	1
Delwin Botsford	30.00	25.28	4.72	3	2	1
Vergil Jacobs	19.00	19.99	-0.99	4	5	-1
Lorenzo Casper	17.00	25.20	-8.20	5	3	2

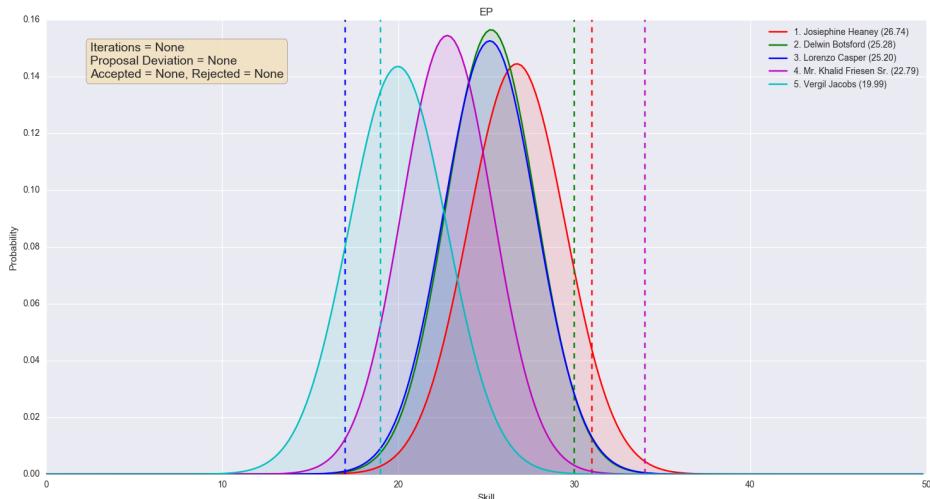


Table 15: dsadas

Table 16: dsaşdsa

Date	T3_9_Mon_May_30_18:36:48_2016
Algorithm	MH
Dataset	Synthetic
Number of matches	1050
Number of players	15
Number of teams	15
Number of iterations	5000
Elapsed time	152
Variance of proposal	1.0
Proposals accepted	2671.0
Proposals rejected	2329.0

Mean Skill Error	3.91	Correct	668
Std Skill Error	2.86	Incorrect	382
Mean Rank Error	1.20	Percentage	0.636
Std Rank Error	0.75	Mean of expected	0.649

Player	Reel Skill	Skill	Skill Error	RS Rank	Skill Rank	Rank Error
Mr. Carroll B.Jr.	38.00	38.29	-0.29	2	1	1
Levi Nader	38.00	33.81	4.19	1	2	-1
Luz Stehr	35.00	30.42	4.58	3	4	-1
Charla Hoeger	33.00	28.66	4.34	4	5	-1
Aydan Grimes	32.00	31.51	0.49	5	3	2
Lena Heller	28.00	27.52	0.48	6	6	0
Camilla Toy	27.00	19.51	7.49	7	9	-2
Mr. Ceasar S. V	25.00	18.85	6.15	8	10	-2
Dr. Shan B.Jr.	24.00	20.67	3.33	10	8	2
Ms. Marlo Kihn	24.00	21.05	2.95	9	7	2
Kingston McD.	20.00	10.73	9.27	12	13	-1
Norton Prosacco	20.00	18.22	1.78	11	11	0
Eber Stracke	19.00	10.11	8.89	13	14	-1
Daijah Huel	18.00	16.98	1.02	14	12	2
Chessie Wolf	11.00	7.64	3.36	15	15	0

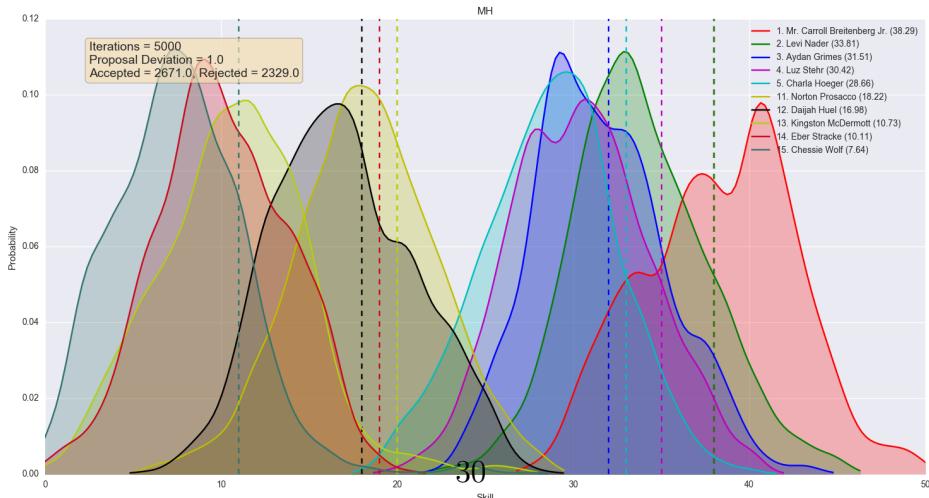


Table 17: Pivoting, Gibbs shifts because its narrow

Date	T3_9_Mon_May_30_18:36:48_2016
Algorithm	Gibbs
Dataset	Synthetic
Number of matches	1050
Number of players	15
Number of teams	15
Number of iterations	4000
Elapsed time	372
Variance of proposal	None
Proposals accepted	None
Proposals rejected	None

Mean Skill Error	3.03	Correct	664
Std Skill Error	2.29	Incorrect	386
Mean Rank Error	1.20	Percentage	0.632
Std Rank Error	0.91	Mean of expected	0.648

Player	Reel Skill	Skill	Skill Error	RS Rank	Skill Rank	Rank Error
Mr. Carroll B.Jr.	38.00	39.02	-1.02	2	1	1
Levi Nader	38.00	35.37	2.63	1	2	-1
Luz Stehr	35.00	31.14	3.86	3	4	-1
Charla Hoeger	33.00	29.73	3.27	4	5	-1
Aydan Grimes	32.00	32.68	-0.68	5	3	2
Lena Heller	28.00	28.64	-0.64	6	6	0
Camilla Toy	27.00	20.42	6.58	7	10	-3
Mr. Ceasar S.V	25.00	20.54	4.46	8	9	-1
Dr. Shan B.Jr.	24.00	21.95	2.05	10	7	3
Ms. Marlo Kihn	24.00	21.91	2.09	9	8	1
Kingston McD.	20.00	12.54	7.46	12	13	-1
Norton Prosacco	20.00	19.08	0.92	11	11	0
Eber Stracke	19.00	12.13	6.87	13	14	-1
Daijah Huel	18.00	17.61	0.39	14	12	2
Chessie Wolf	11.00	8.49	2.51	15	15	0

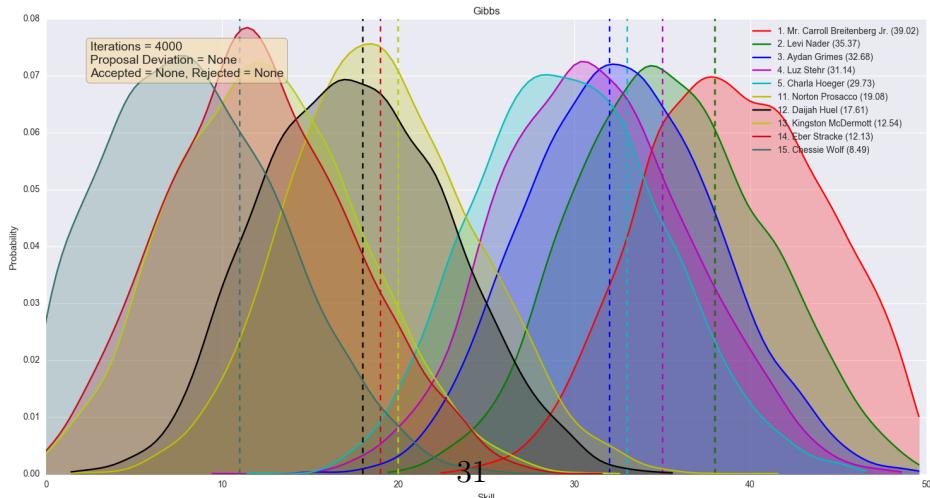


Table 18: dsaşdsa

Date	T4_11_Mon_May_30_18:46:01_2016
Algorithm	MH
Dataset	Synthetic
Number of matches	30
Number of players	9
Number of teams	3
Number of iterations	5000
Elapsed time	3
Variance of proposal	1.0
Proposals accepted	4539.0
Proposals rejected	461.0

Mean Skill Error	10.47	Correct	28
Std Skill Error	6.95	Incorrect	2
Mean Rank Error	2.89	Percentage	0.933
Std Rank Error	1.29	Mean of expected	0.843

Player	Reel Skill	Skill	Skill Error	RS Rank	Skill Rank	Rank Error
Tillie Ratke	45.00	30.04	14.96	1	4	-3
Estes Runolfsson	40.00	40.61	-0.61	2	1	1
Tiffanie Dicki	35.00	23.91	11.09	3	7	-4
Edward Tillman	30.00	25.38	4.62	4	5	-1
Squire Carter	25.00	17.59	7.41	5	8	-3
Xavier Anderson	20.00	34.06	-14.06	6	2	4
Dr. Mazie Batz PhD	15.00	13.46	1.54	7	9	-2
Miss Aline Ledner	10.00	30.59	-20.59	8	3	5
Andy Turner	5.00	24.33	-19.33	9	6	3

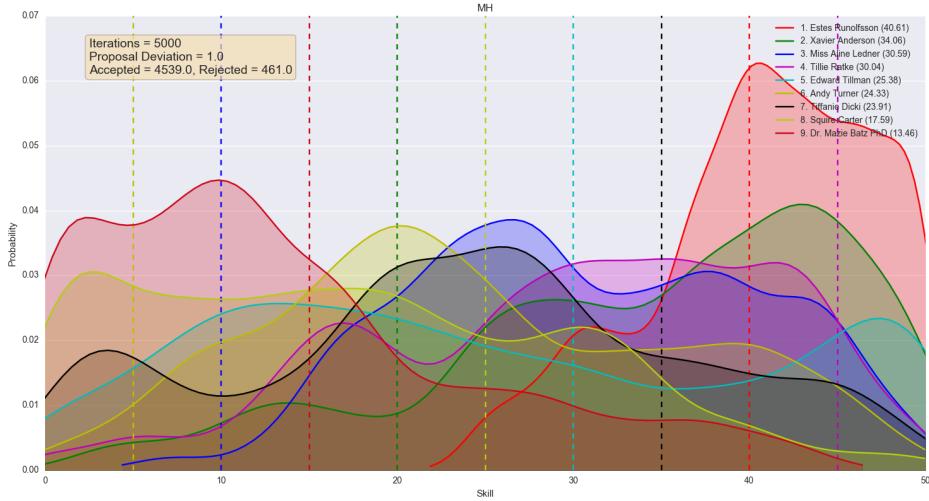


Table 19: Pivoting, Gibbs shifts because its narrow

Date	T4_11_Mon_May_30_18:46:01_2016
Algorithm	Gibbs
Dataset	Synthetic
Number of matches	30
Number of players	9
Number of teams	3
Number of iterations	4000
Elapsed time	35
Variance of proposal	None
Proposals accepted	None
Proposals rejected	None

Mean Skill Error	11.18	Correct	28
Std Skill Error	6.46	Incorrect	2
Mean Rank Error	2.00	Percentage	0.933
Std Rank Error	1.63	Mean of expected	0.872

Player	Reel Skill	Skill	Skill Error	RS Rank	Skill Rank	Rank Error
Tillie Ratke	45.00	26.66	18.34	1	3	-2
Estes Runolfsson	40.00	26.75	13.25	2	1	1
Tiffanie Dicki	35.00	22.78	12.22	3	4	-1
Edward Tillman	30.00	22.63	7.37	4	5	-1
Squire Carter	25.00	5.09	19.91	5	8	-3
Xavier Anderson	20.00	22.58	-2.58	6	6	0
Dr. Mazie Batz PhD	15.00	4.86	10.14	7	9	-2
Miss Aline Ledner	10.00	26.75	-16.75	8	2	6
Andy Turner	5.00	5.10	-0.10	9	7	2

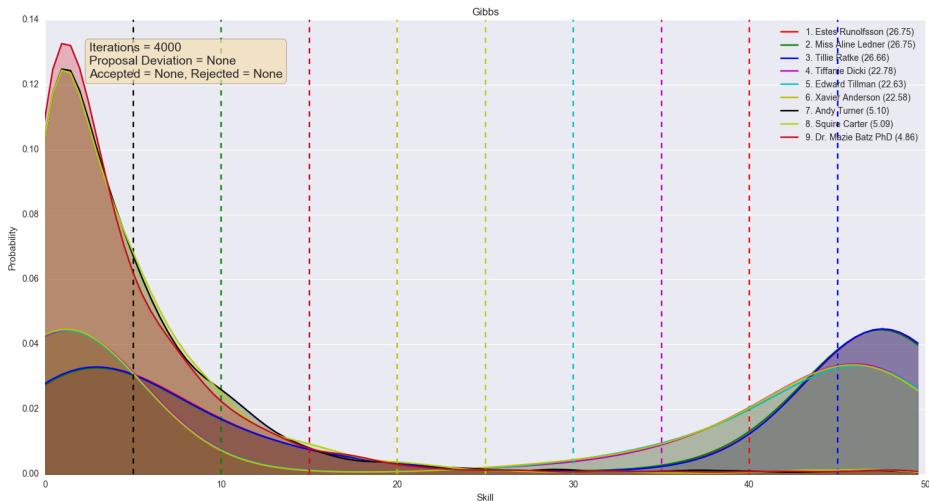


Table 20: dsaşdsa

Date	T5_13_Mon_May_30_18:48:57_2016
Algorithm	MH
Dataset	Tennis
Number of matches	251
Number of players	182
Number of teams	182
Number of iterations	5000
Elapsed time	27
Variance of proposal	1.0
Proposals accepted	3735.0
Proposals rejected	1265.0

#	P1	Skill	Uncertainty
1	Nadal	46.97	4.43
2	Wawrinka	44.78	5.39
3	Ferrer	43.96	5.05
4	Anderson	43.17	5.28
5	Berdych	42.97	5.81
178	Becker	8.63	7.39
179	Soeda	8.51	7.23
180	Monaco	7.61	6.24
181	Duckworth	7.60	6.37
182	Kuznetsov	6.77	6.23

Correct	150
Incorrect	90
Percentage	0.625
Mean of expected	0.663

#	P1	P2	Result	P(W)	P(L)	P(D)	Pred.	Correct
1	Becker	Murray	-1	0.13	0.87	0.00	-1	1
2	Ward	Lu	-1	0.45	0.55	0.00	-1	1
3	Mahut	Hajek	1	0.43	0.57	0.00	-1	0
4	Robredo	Bogomolov	1	0.73	0.27	0.00	1	1
5	Haase	Youzhny	-1	0.36	0.64	0.00	-1	1
6	Gicquel	Pospisil	-1	0.27	0.73	0.00	-1	1
7	Kuznetsov	Montanes	1	0.26	0.74	0.00	-1	0
8	Tipsarevic	Troicki	-1	0.17	0.83	0.00	-1	1
9	Baghdatis	Cilic	-1	0.23	0.77	0.00	-1	1
10	DeSchepper	Lorenzi	1	0.73	0.27	0.00	1	1

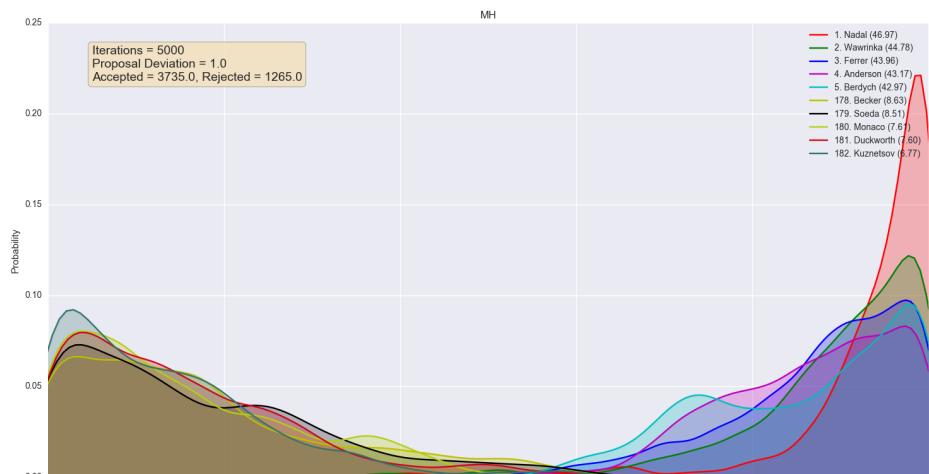


Table 21: dsaşds

Date	T5_13_Mon_May_30_18:48:57_2016
Algorithm	Gibbs
Dataset	Tennis
Number of matches	251
Number of players	182
Number of teams	182
Number of iterations	4000
Elapsed time	171
Variance of proposal	None
Proposals accepted	None
Proposals rejected	None

#	P1	Skill	Uncertainty
1	Nadal	45.92	3.18
2	Wawrinka	44.88	4.09
3	Ferrer	43.55	4.99
4	Djokovic	43.14	5.30
5	Federer	43.00	5.36
178	Kudla	14.49	11.72
179	Monaco	14.25	11.35
180	Reister	14.19	11.59
181	Kuznetsov	14.10	11.50
182	Johnson	14.07	11.25

Correct	153
Incorrect	87
Percentage	0.637
Mean of expected	0.638

#	P1	P2	Result	P(W)	P(L)	P(D)	Prediction	Correct
1	Becker	Murray	-1	0.21	0.79	0.00	-1	1
2	Ward	Lu	-1	0.43	0.57	0.00	-1	1
3	Mahut	Hajek	1	0.50	0.50	0.00	-1	0
4	Robredo	Bogomolov	1	0.74	0.26	0.00	1	1
5	Haase	Youzhny	-1	0.35	0.65	0.00	-1	1
6	Gicquel	Pospisil	-1	0.36	0.64	0.00	-1	1
7	Kuznetsov	Montanes	1	0.39	0.61	0.00	-1	0
8	Tipsarevic	Troicki	-1	0.36	0.64	0.00	-1	1
9	Baghdatis	Cilic	-1	0.30	0.70	0.00	-1	1
10	DeSchepper	Lorenzi	1	0.61	0.39	0.00	1	1

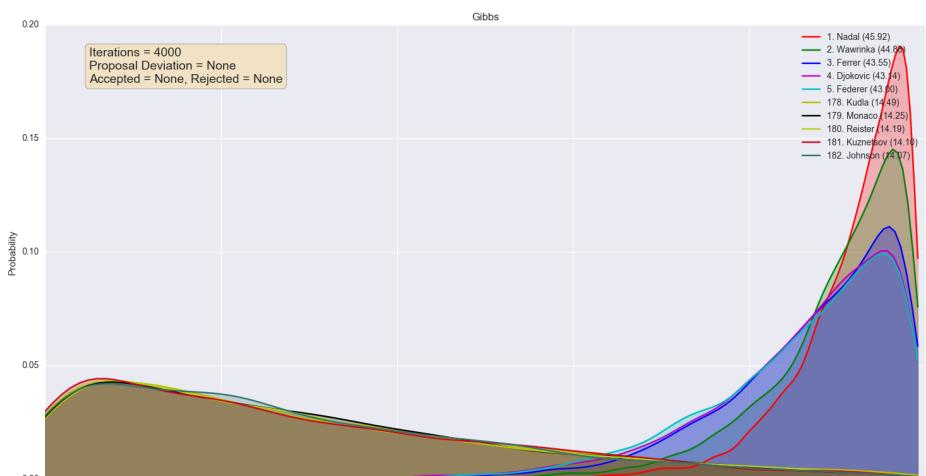


Table 22: dsaşdsa

Date	T5_13_Mon_May_30_18:48:57_2016
Algorithm	EP
Dataset	Tennis
Number of matches	251
Number of players	182
Number of teams	182
Number of iterations	None
Elapsed time	0
Variance of proposal	None
Proposals accepted	None
Proposals rejected	None

#	P1	Skill	Uncertainty
1	Nadal	42.41	3.04
2	Wawrinka	41.87	3.37
3	Ferrer	36.96	3.31
4	Robredo	36.52	3.56
5	Djokovic	36.42	3.52
6	Berdych	35.65	3.84
7	Tsonga	35.65	3.46
8	Nishikori	35.33	4.05
9	Federer	35.29	3.59
10	Dimitrov	33.47	3.81

Correct	153
Incorrect	87
Percentage	0.637
Mean of expected	0.687

#	P1	P2	Result	P(W)	P(L)	P(D)	Pred.	Correct
1	Becker	Murray	-1	0.06	0.84	0.10	-1	1
2	Ward	Lu	-1	0.42	0.48	0.10	-1	1
3	Mahut	Hajek	1	0.48	0.42	0.10	1	1
4	Robredo	Bogomolov	1	0.86	0.04	0.10	1	1
5	Haase	Youzhny	-1	0.21	0.69	0.10	-1	1
6	Gicquel	Pospisil	-1	0.34	0.56	0.10	-1	1
7	Kuznetsov	Montanes	1	0.26	0.64	0.10	-1	0
8	Tipsarevic	Troicki	-1	0.27	0.63	0.10	-1	1
9	Baghdatis	Cilic	-1	0.19	0.71	0.10	-1	1
10	DeSchepper	Lorenzi	1	0.71	0.19	0.10	1	1

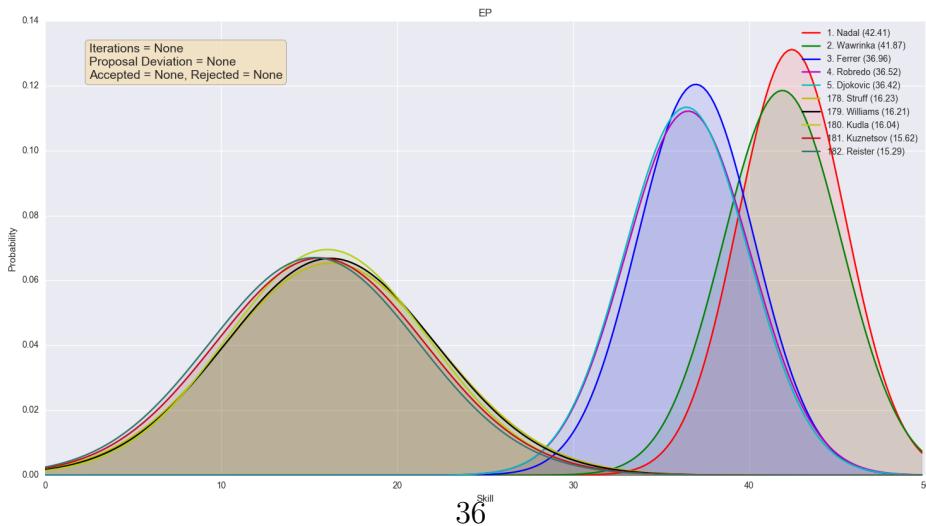


Table 23: dsaşdsa

Date	T5_15_Mon_May_30_18:56:44_2016
Algorithm	MH
Dataset	Tennis
Number of matches	491
Number of players	182
Number of teams	182
Number of iterations	5000
Elapsed time	54
Variance of proposal	1.0
Proposals accepted	3278.0
Proposals rejected	1722.0

#	P1	Skill	Uncertainty	RANKING	MOVE	COUNTRY	PLAYER	AGE	POINTS
1	Nadal	47.73	2.96	1	-	ESP	Novak Djokovic	29	11,120
2	Djokovic	46.01	3.99	2	-	CRO	Rafael Nadal	29	10,860
3	Ferrer	45.69	4.05	3	-	GBR	Andy Murray	29	7,075
4	Wawrinka	45.42	5.22	4	-	CRO	David Ferrer	34	6,710
5	Murray	45.19	4.65	5	▲ 1	CZE	Tomas Berdych	30	4,520
6	Robredo	44.75	4.63	6	▼ 1	SUI	Roger Federer	34	4,515
7	Tsonga	43.66	4.75	7	-	ARG	Juan Martin del Potro	27	4,425
8	Berdych	43.23	6.26	8	-	FRA	Jo-Wilfried Tsonga	31	3,325
9	Gasquet	43.17	5.83	9	▲ 1	SUI	Stan Wawrinka	31	3,150
10	Federer	42.63	5.63	10	▼ 1	FRA	Richard Gasquet	29	3,005

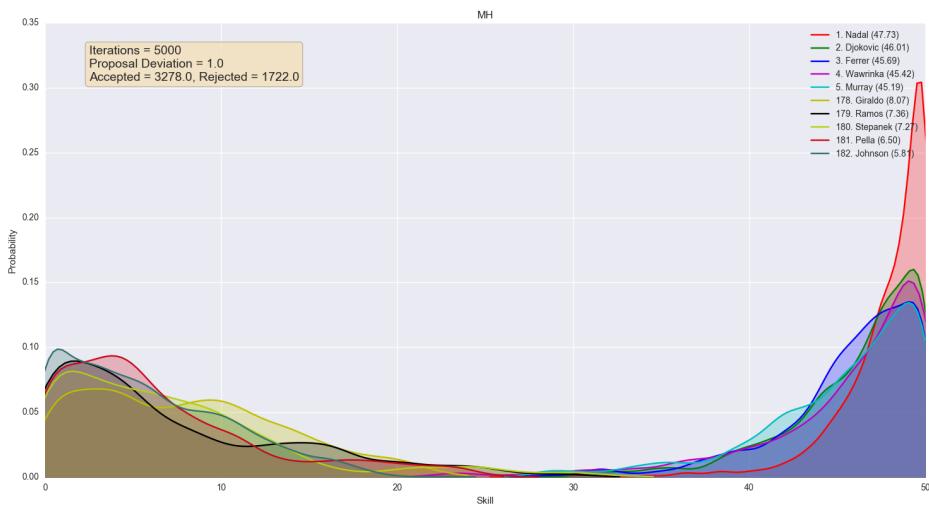


Table 24: dsaşdsa

Date	T5_15_Mon_May_30_18:56:44_2016
Algorithm	Gibbs
Dataset	Tennis
Number of matches	491
Number of players	182
Number of teams	182
Number of iterations	4000
Elapsed time	292
Variance of proposal	None
Proposals accepted	None
Proposals rejected	None

#	P1	Skill	Uncertainty
1	Nadal	46.85	2.43
2	Djokovic	46.11	2.94
3	Murray	45.76	3.33
4	Wawrinka	45.49	3.51
5	Ferrer	44.98	3.82
6	Robredo	43.47	4.95
7	Federer	43.16	4.92
8	Berdych	43.05	5.12
9	Gasquet	42.64	5.35
10	Tsonga	41.27	6.36

RANKING	MOVE	COUNTRY	PLAYER	AGE	POINTS
1	-	SRB	Novak Djokovic	29	11,120
2	-	SRB	Rafael Nadal	29	10,860
3	-	GBR	Andy Murray	29	7,075
4	-	CRO	David Ferrer	34	6,710
5	△ 1	CZE	Tomas Berdych	30	4,520
6	▼ 1	SUI	Roger Federer	34	4,515
7	-	ESP	Juan Martin del Potro	27	4,425
8	-	FRA	Jo-Wilfried Tsonga	31	3,325
9	△ 1	SUI	Stan Wawrinka	31	3,150
10	▼ 1	FRA	Richard Gasquet	29	3,005

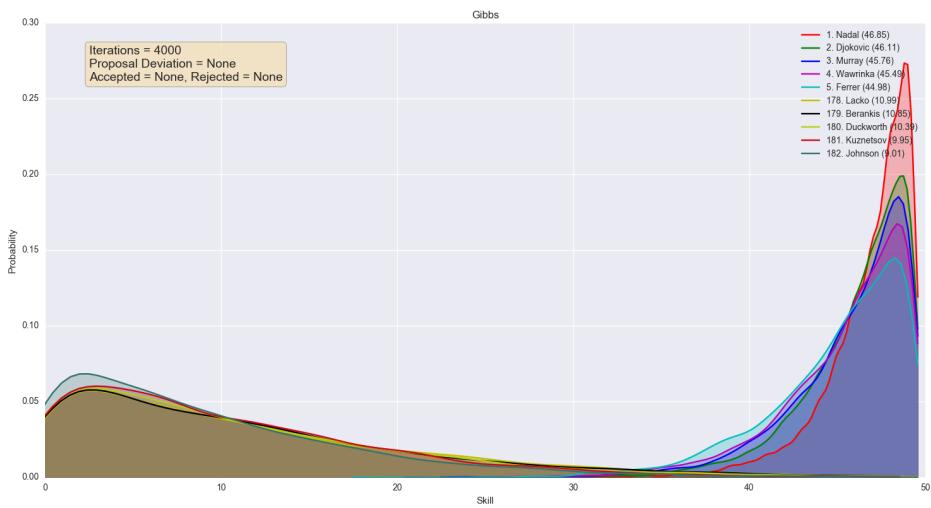


Table 25: dsaşds

Date	T5_15_Mon_May_30_18:56:44_2016
Algorithm	EP
Dataset	Tennis
Number of matches	491
Number of players	182
Number of teams	182
Number of iterations	None
Elapsed time	1
Variance of proposal	None
Proposals accepted	None
Proposals rejected	None

#	P1	Skill	Uncertainty
1	Nadal	41.09	2.38
2	Wawrinka	39.45	2.56
3	Djokovic	38.39	2.23
4	Murray	37.22	2.69
5	Robredo	36.10	2.65
6	Ferrer	35.97	2.55
7	Berdych	34.83	2.67
8	Federer	34.71	2.71
9	Gasquet	34.13	2.68
10	BautistaAgut	34.03	4.19

RANKING	MOVE	COUNTRY	PLAYER	AGE	POINTS
1	-	SRB	Novak Djokovic	29	11,120
2	-	ESP	Rafael Nadal	29	10,860
3	-	GBR	Andy Murray	29	7,075
4	-	CZE	David Ferrer	34	6,710
5	≈ 1	CZE	Tomas Berdych	30	4,520
6	∨ 1	SUI	Roger Federer	34	4,515
7	-	ARG	Juan Martin del Potro	27	4,425
8	-	FRA	Jo-Wilfried Tsonga	31	3,325
9	≈ 1	SUI	Stan Wawrinka	31	3,150
10	∨ 1	ESP	Richard Gasquet	29	3,005

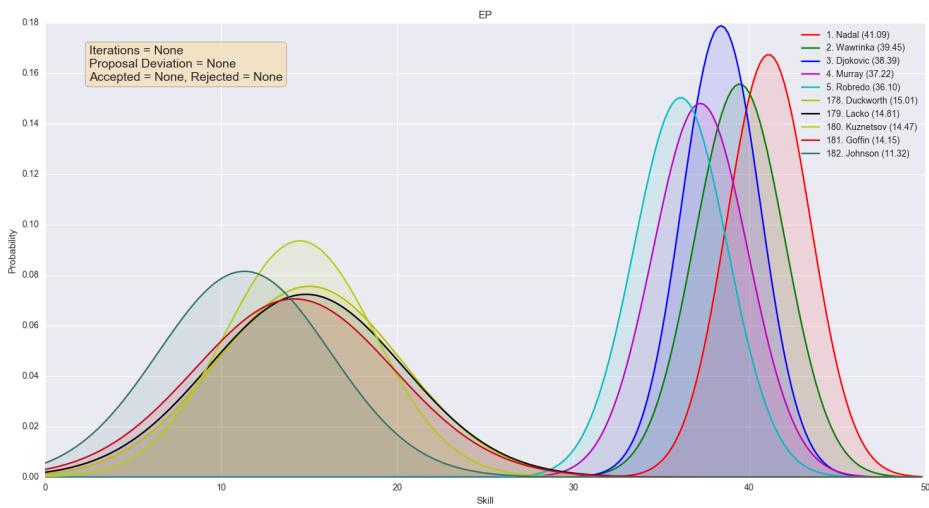


Table 26: dsaşdsa

Date	T6_1_Mon_May_30_19:05:10_2016
Algorithm	MH
Dataset	Football
Number of matches	190
Number of players	20
Number of teams	20
Number of iterations	5000
Elapsed time	21
Variance of proposal	1.0
Proposals accepted	3640.0
Proposals rejected	1360.0

#	P1	Skill	Uncertainty
1	Arsenal	41.52	6.30
2	Leicester	38.95	5.95
3	Man City	34.98	6.61
4	Tottenham	34.36	6.70
5	Liverpool	30.31	6.67
16	Swansea	21.51	7.14
17	Southampton	20.69	7.78
18	Newcastle	14.19	6.56
19	Aston Villa	7.71	5.52
20	Sunderland	7.64	5.58

Correct	82
Incorrect	108
Percentage	0.432
Mean of expected	0.509

#	P1	P2	Result	P(Win P1)	P(Loss P1)	P(Draw)	Prediction	Correct
1	Arsenal	Newcastle	1	0.77	0.15	0.08	1	1
2	Leicester	Bournemouth	0	0.62	0.23	0.15	1	0
3	Man United	Swansea	1	0.45	0.31	0.24	1	1
4	Norwich	Southampton	1	0.41	0.32	0.27	1	1
5	Sunderland	Aston Villa	1	0.33	0.34	0.33	-1	0
6	Watford	Man City	-1	0.30	0.49	0.22	-1	1
7	West Brom	Stoke	1	0.31	0.45	0.24	-1	0
8	West Ham	Liverpool	1	0.32	0.43	0.26	-1	0
9	Crystal Palace	Chelsea	-1	0.45	0.31	0.24	1	0
10	Everton	Tottenham	0	0.31	0.45	0.24	-1	0

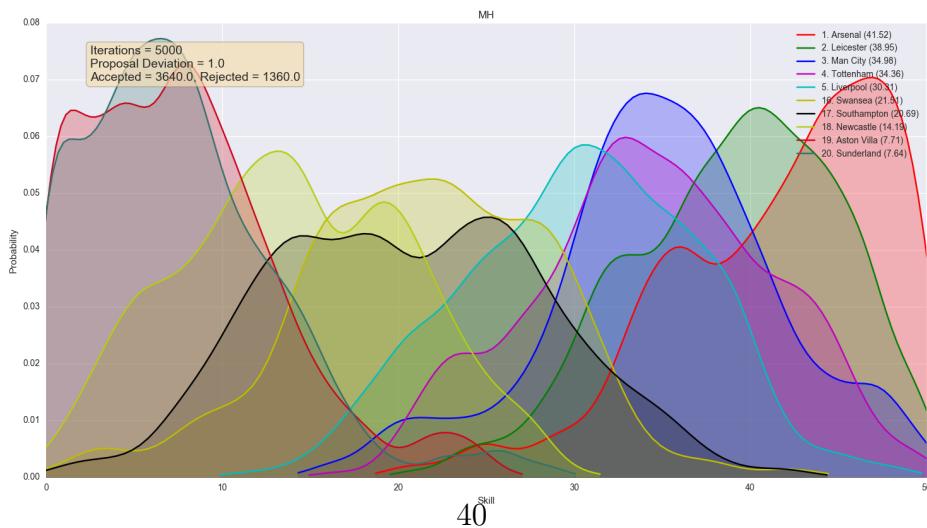


Table 27: dsaşdasa

Date	T6_1_Mon_May_30_19:05:10_2016
Algorithm	Gibbs
Dataset	Football
Number of matches	190
Number of players	20
Number of teams	20
Number of iterations	4000
Elapsed time	75
Variance of proposal	None
Proposals accepted	None
Proposals rejected	None

#	P1	Skill	Uncertainty
1	Arsenal	35.67	7.63
2	Leicester	34.74	7.22
3	Man City	34.11	7.79
4	Tottenham	32.35	7.23
5	Liverpool	29.90	7.40
16	Swansea	23.48	7.41
17	Chelsea	23.45	7.69
18	Newcastle	20.13	7.86
19	Sunderland	13.27	7.64
20	Aston Villa	13.04	7.56

Correct	83
Incorrect	107
Percentage	0.437
Mean of expected	0.457

#	P1	P2	Result	P(Win P1)	P(Loss P1)	P(Draw)	Prediction	Correct
1	Arsenal	Newcastle	1	0.61	0.24	0.15	1	1
2	Leicester	Bournemouth	0	0.52	0.28	0.20	1	0
3	Man United	Swansea	1	0.45	0.31	0.24	1	1
4	Norwich	Southampton	1	0.33	0.34	0.32	-1	0
5	Sunderland	Aston Villa	1	0.34	0.33	0.33	1	1
6	Watford	Man City	-1	0.31	0.46	0.24	-1	1
7	West Brom	Stoke	1	0.33	0.39	0.28	-1	0
8	West Ham	Liverpool	1	0.33	0.38	0.29	-1	0
9	Crystal Palace	Chelsea	-1	0.44	0.31	0.25	1	0
10	Everton	Tottenham	0	0.32	0.42	0.26	-1	0

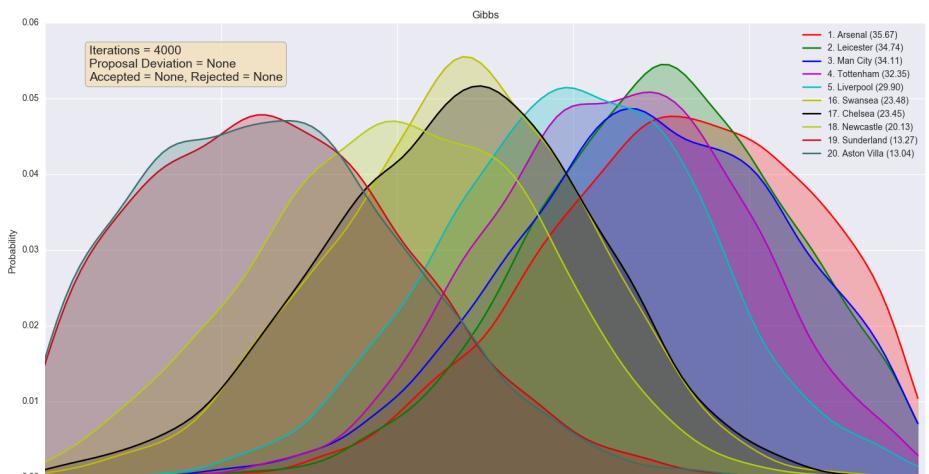


Table 28: dsaşdasa

Date	T6_1_Mon_May_30_19:05:10_2016
Algorithm	EP
Dataset	Football
Number of matches	190
Number of players	20
Number of teams	20
Number of iterations	None
Elapsed time	0
Variance of proposal	None
Proposals accepted	None
Proposals rejected	None

#	P1	Skill	Uncertainty
1	Leicester	29.35	1.88
2	Arsenal	28.66	1.94
3	Tottenham	28.42	1.79
4	Man City	28.22	2.01
5	Liverpool	28.08	1.80
16	Swansea	23.12	1.83
17	Norwich	23.10	1.91
18	Newcastle	22.52	1.93
19	Aston Villa	20.13	1.90
20	Sunderland	19.37	1.98

Correct	90
Incorrect	100
Percentage	0.474
Mean of expected	0.628

#	P1	P2	Result	P(Win P1)	P(Loss P1)	P(Draw)	Prediction	Correct
1	Arsenal	Newcastle	1	0.83	0.07	0.10	1	1
2	Leicester	Bournemouth	0	0.80	0.10	0.10	1	0
3	Man United	Swansea	1	0.70	0.20	0.10	1	1
4	Norwich	Southampton	1	0.41	0.49	0.10	-1	0
5	Sunderland	Aston Villa	1	0.45	0.45	0.10	1	1
6	Watford	Man City	-1	0.32	0.58	0.10	-1	1
7	West Brom	Stoke	1	0.47	0.43	0.10	1	1
8	West Ham	Liverpool	1	0.37	0.53	0.10	-1	0
9	Crystal Palace	Chelsea	-1	0.69	0.21	0.10	1	0
10	Everton	Tottenham	0	0.34	0.56	0.10	-1	0

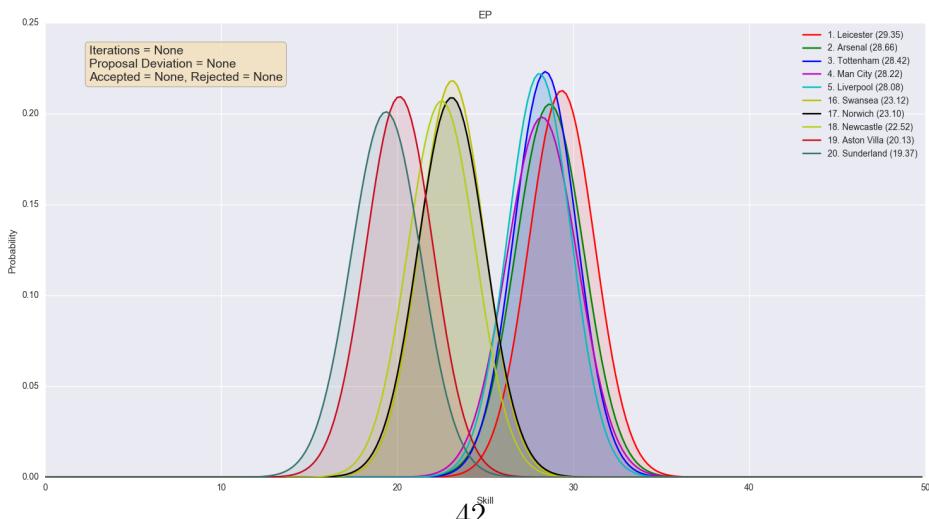


Table 29: dsaşdsa

Date	T6_2_Mon_May_30_19:06:51_2016
Algorithm	MH
Dataset	Football
Number of matches	153
Number of players	18
Number of teams	18
Number of iterations	5000
Elapsed time	17
Variance of proposal	1.0
Proposals accepted	3808.0
Proposals rejected	1192.0

#	P1	Skill	Uncertainty
1	Bayern Munich	46.13	4.55
2	Dortmund	43.54	5.22
3	Hertha	36.97	7.47
4	M'gladbach	33.00	8.77
5	Schalke 04	28.11	8.90
14	Hoffenheim	20.59	7.45
15	Darmstadt	19.03	7.18
16	Werder Bremen	16.67	9.36
17	Stuttgart	15.19	9.59
18	Hannover	11.50	6.65

Correct	73
Incorrect	80
Percentage	0.477
Mean of expected	0.511

#	P1	P2	Result	P(Win P1)	P(Loss P1)	P(Draw)	Prediction	Correct
1	Hamburg	Bayern Munich	-1	0.19	0.71	0.11	-1	1
2	FC Koln	Stuttgart	-1	0.48	0.30	0.22	1	0
3	Hannover	Darmstadt	-1	0.30	0.47	0.23	-1	1
4	Hertha	Augsburg	0	0.58	0.25	0.17	1	0
5	Hoffenheim	Leverkusen	0	0.30	0.47	0.23	-1	0
6	Ingolstadt	Mainz	1	0.34	0.33	0.33	1	1
7	M'gladbach	Dortmund	-1	0.28	0.53	0.19	-1	1
8	Ein Frankfurt	Wolfsburg	1	0.32	0.41	0.27	-1	0
9	Schalke 04	Werder Bremen	-1	0.54	0.27	0.19	1	0
10	Mainz	M'gladbach	1	0.28	0.53	0.19	-1	0

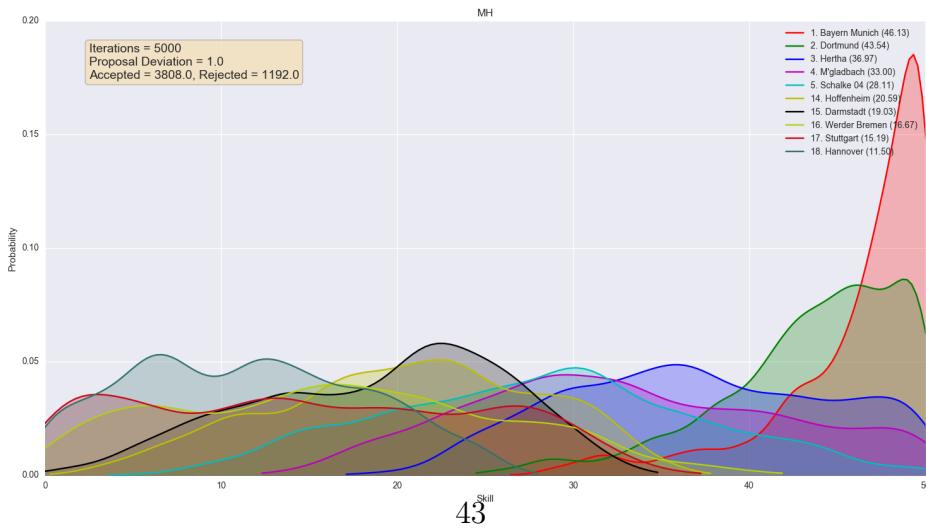


Table 30: dsaşdsa

Date	T6_2_Mon_May_30_19:06:51_2016
Algorithm	Gibbs
Dataset	Football
Number of matches	153
Number of players	18
Number of teams	18
Number of iterations	4000
Elapsed time	61
Variance of proposal	None
Proposals accepted	None
Proposals rejected	None

#	P1	Skill	Uncertainty
1	Bayern Munich	41.96	6.05
2	Dortmund	33.30	8.64
3	Hertha	25.97	8.89
4	M'gladbach	22.67	8.47
5	Leverkusen	21.67	7.92
14	Ein Frankfurt	16.01	7.27
15	Hoffenheim	14.27	6.76
16	Werder Bremen	12.18	7.16
17	Stuttgart	11.51	6.82
18	Hannover	11.02	7.03

Correct	75
Incorrect	78
Percentage	0.490
Mean of expected	0.475

#	P1	P2	Result	P(Win P1)	P(Loss P1)	P(Draw)	Prediction	Correct
1	Hamburg	Bayern Munich	-1	0.17	0.73	0.10	-1	1
2	FC Koln	Stuttgart	-1	0.46	0.30	0.23	1	0
3	Hannover	Darmstadt	-1	0.31	0.43	0.25	-1	1
4	Hertha	Augsburg	0	0.51	0.29	0.21	1	0
5	Hoffenheim	Leverkusen	0	0.30	0.47	0.23	-1	0
6	Ingolstadt	Mainz	1	0.33	0.34	0.32	-1	0
7	M'gladbach	Dortmund	-1	0.28	0.53	0.19	-1	1
8	Ein Frankfurt	Wolfsburg	1	0.32	0.40	0.28	-1	0
9	Schalke 04	Werder Bremen	-1	0.50	0.29	0.21	1	0
10	Mainz	M'gladbach	1	0.32	0.42	0.26	-1	0

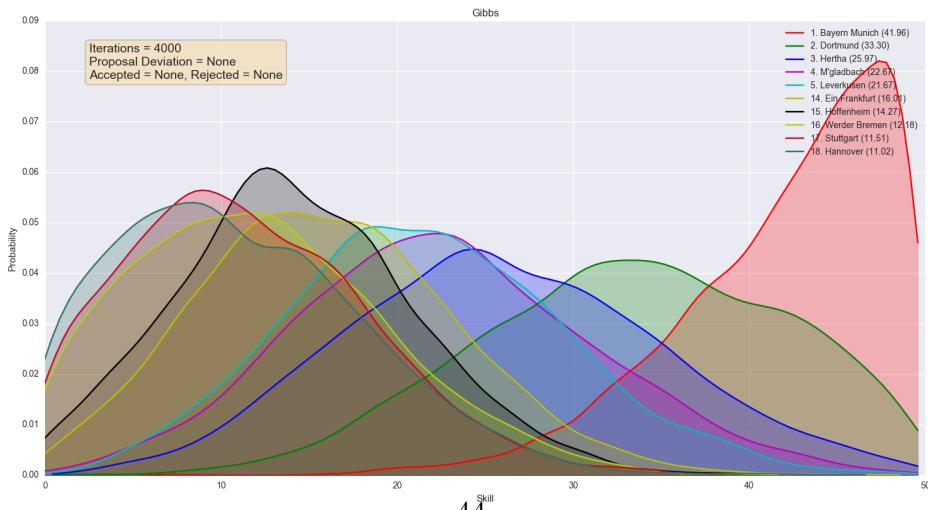


Table 31: dsaşdasa

Date	T6_2_Mon_May_30_19:06:51_2016
Algorithm	EP
Dataset	Football
Number of matches	153
Number of players	18
Number of teams	18
Number of iterations	None
Elapsed time	0
Variance of proposal	None
Proposals accepted	None
Proposals rejected	None

#	P1	Skill	Uncertainty
1	Bayern Munich	35.39	2.57
2	Dortmund	29.18	2.20
3	Hertha	28.16	2.17
4	M'gladbach	26.50	2.13
5	Leverkusen	26.33	2.13
14	Darmstadt	23.42	1.95
15	Hoffenheim	23.37	2.01
16	Stuttgart	22.77	2.06
17	Hannover	22.34	2.17
18	Werder Bremen	21.84	2.16

Correct	76
Incorrect	77
Percentage	0.497
Mean of expected	0.615

#	P1	P2	Result	P(Win P1)	P(Loss P1)	P(Draw)	Prediction	Correct
1	Hamburg	Bayern Munich	-1	0.07	0.83	0.10	-1	1
2	FC Koln	Stuttgart	-1	0.65	0.25	0.10	1	0
3	Hannover	Darmstadt	-1	0.44	0.46	0.10	-1	1
4	Hertha	Augsburg	0	0.75	0.15	0.10	1	0
5	Hoffenheim	Leverkusen	0	0.33	0.57	0.10	-1	0
6	Ingolstadt	Mainz	1	0.51	0.39	0.10	1	1
7	M'gladbach	Dortmund	-1	0.34	0.56	0.10	-1	1
8	Ein Frankfurt	Wolfsburg	1	0.39	0.51	0.10	-1	0
9	Schalke 04	Werder Bremen	-1	0.73	0.17	0.10	1	0
10	Mainz	M'gladbach	1	0.37	0.53	0.10	-1	0

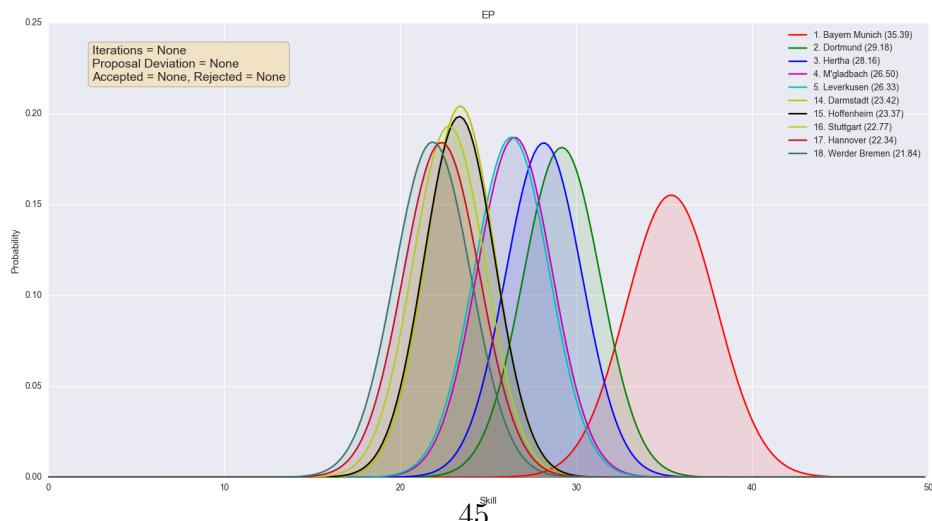


Table 32: dsaşdsa

Date	T6_3_Mon_May_30_19:08:14_2016
Algorithm	MH
Dataset	Football
Number of matches	153
Number of players	18
Number of teams	18
Number of iterations	5000
Elapsed time	16
Variance of proposal	1.0
Proposals accepted	3763.0
Proposals rejected	1237.0

#	P1	Skill	Uncertainty	Correct	60
1	Besiktas	43.85	5.91	Correct	60
2	Fenerbahce	41.10	8.42	Incorrect	93
3	Akhisar Belediyespor	35.86	9.16	Percentage	0.392
4	Kasimpasa	34.19	7.97	Mean of expected	0.537
5	Galatasaray	32.11	7.29		
14	Sivasspor	22.07	6.19		
15	Genclerbirligi	12.67	7.82		
16	Bursaspor	11.41	7.29		
17	Mersin Idman Yurdu	11.38	6.54		
18	Eskisehirspor	5.97	5.18		

#	P1	P2	Result	P(Win P1)	P(Loss P1)	P(Draw)	Prediction	Correct
1	Akhisar Belediyespor	Konyaspor	-1	0.49	0.29	0.21	1	0
2	Galatasaray	Sivasspor	1	0.52	0.28	0.20	1	1
3	Kasimpasa	Gaziantepspor	-1	0.39	0.33	0.29	1	0
4	Kayserispor	Osmanspor	1	0.36	0.33	0.31	1	1
5	Antalyaspor	Buyuksehry	-1	0.33	0.38	0.30	-1	1
6	Bursaspor	Trabzonspor	1	0.21	0.67	0.13	-1	0
7	Rizespor	Genclerbirligi	-1	0.62	0.24	0.15	1	0
8	Eskisehirspor	Fenerbahce	-1	0.10	0.84	0.06	-1	1
9	Konyaspor	Kayserispor	1	0.36	0.33	0.31	1	1

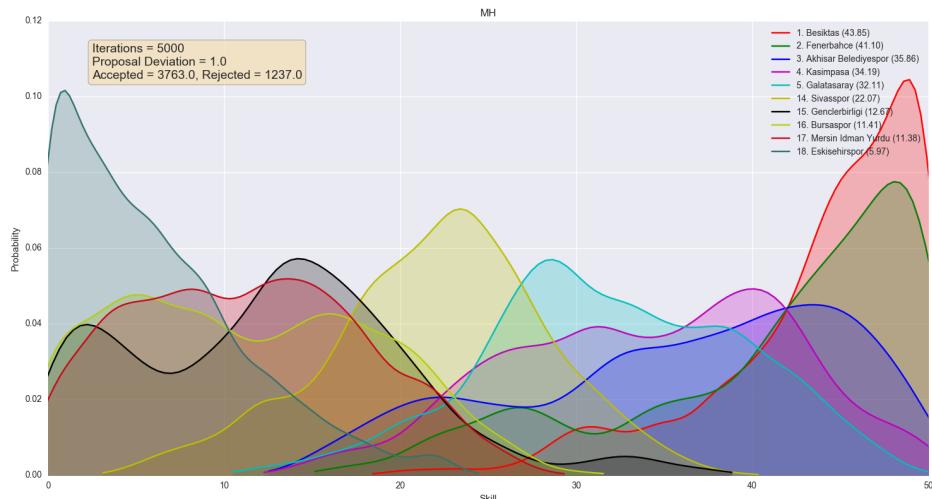


Table 33: dsaşdsa

Date	T6_3_Mon_May_30_19:08:14_2016
Algorithm	Gibbs
Dataset	Football
Number of matches	153
Number of players	18
Number of teams	18
Number of iterations	4000
Elapsed time	59
Variance of proposal	None
Proposals accepted	None
Proposals rejected	None

#	P1	Skill	Uncertainty
1	Besiktas	40.07	6.50
2	Fenerbahce	37.52	6.98
3	Kasimpasa	30.69	7.15
4	Akhisar Belediyespor	29.79	7.20
5	Galatasaray	29.66	7.33
14	Sivasspor	21.18	6.76
15	Genclerbirligi	16.32	7.80
16	Bursaspor	15.22	8.41
17	Mersin Idman Yurdu	13.89	7.55
18	Eskisehirspor	9.32	6.71

Correct	65
Incorrect	88
Percentage	0.425
Mean of expected	0.492

#	P1	P2	Result	P(Win P1)	P(Loss P1)	P(Draw)	Prediction	Correct
1	Akhisar Belediyespor	Konyaspor	-1	0.40	0.32	0.27	1	0
2	Galatasaray	Sivasspor	1	0.49	0.29	0.22	1	1
3	Kasimpasa	Gaziantepspor	-1	0.42	0.32	0.26	1	0
4	Kayserispor	Osmanspor	1	0.38	0.33	0.30	1	1
5	Antalyaspor	Buyuksehry	-1	0.33	0.39	0.28	-1	1
6	Bursaspor	Trabzonspor	1	0.27	0.54	0.19	-1	0
7	Rizespor	Genclerbirligi	-1	0.51	0.29	0.20	1	0
8	Eskisehirspor	Fenerbahce	-1	0.14	0.78	0.08	-1	1
9	Konyaspor	Kayserispor	1	0.37	0.33	0.30	1	1

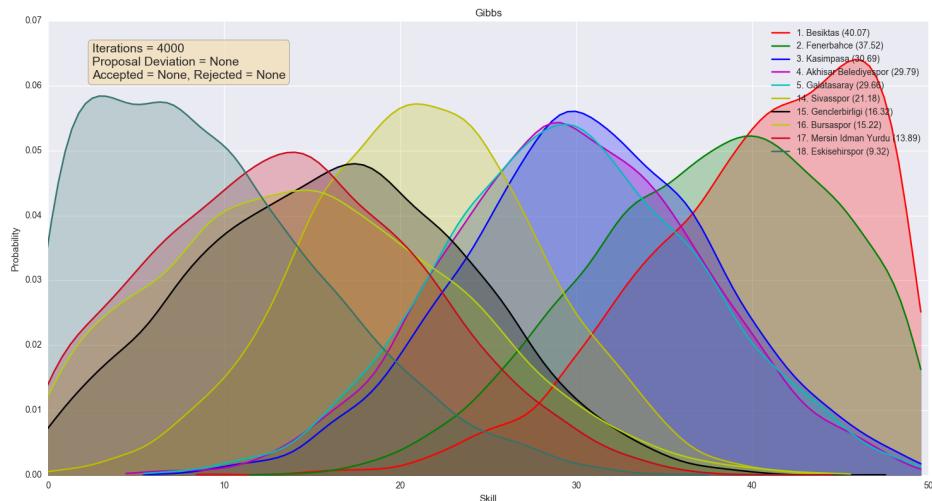


Table 34: dsaşdasa

Date	T6_3_Mon_May_30_19:08:14_2016
Algorithm	EP
Dataset	Football
Number of matches	153
Number of players	18
Number of teams	18
Number of iterations	None
Elapsed time	0
Variance of proposal	None
Proposals accepted	None
Proposals rejected	None

#	P1	Skill	Uncertainty	Correct	70
1	Fenerbahce	30.92	2.22	Correct	70
2	Besiktas	30.28	2.38	Incorrect	83
3	Kasimpasa	28.03	2.06	Percentage	0.458
4	Buyuksehyr	27.68	2.19	Mean of expected	0.647
5	Galatasaray	27.50	1.88		
14	Sivasspor	23.42	1.93		
15	Bursaspor	22.20	2.14		
16	Genclerbirligi	21.40	1.96		
17	Mersin Idman Yurdu	20.39	2.04		
18	Eskisehirspor	19.73	2.28		

#	P1	P2	Result	P(Win P1)	P(Loss P1)	P(Draw)	Prediction	Correct
1	Akhisar Belediyespor	Konyaspor	-1	0.54	0.36	0.10	1	0
2	Galatasaray	Sivasspor	1	0.74	0.16	0.10	1	1
3	Kasimpasa	Gaziantepspor	-1	0.67	0.23	0.10	1	0
4	Kayserispor	Osmanspor	1	0.56	0.34	0.10	1	1
5	Antalyaspor	Buyuksehyr	-1	0.33	0.57	0.10	-1	1
6	Bursaspor	Trabzonspor	1	0.31	0.59	0.10	-1	0
7	Rizespor	Genclerbirligi	-1	0.74	0.16	0.10	1	0
8	Eskisehirspor	Fenerbahce	-1	0.05	0.85	0.10	-1	1
9	Konyaspor	Kayserispor	1	0.58	0.32	0.10	1	1

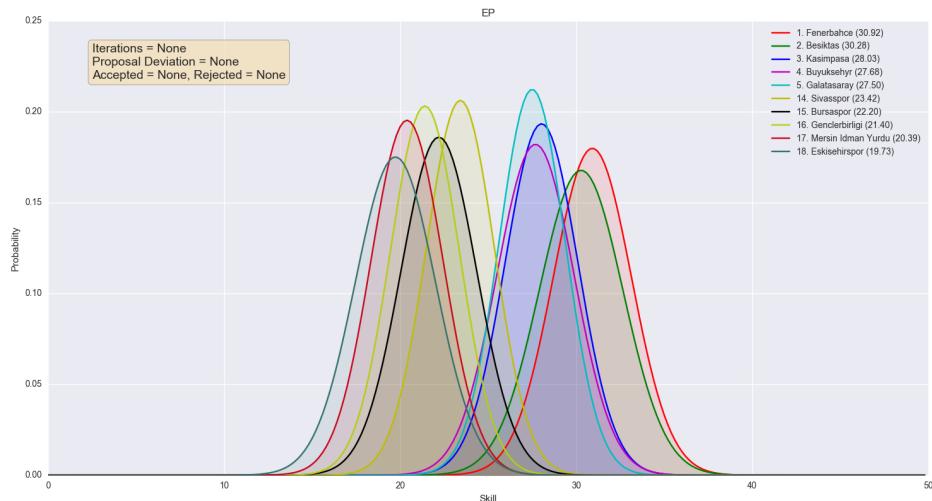


Table 35: dsaşdsa

Date	T6_4_Mon_May_30_19:09:34_2016
Algorithm	MH
Dataset	Football
Number of matches	190
Number of players	20
Number of teams	20
Number of iterations	5000
Elapsed time	21
Variance of proposal	1.0
Proposals accepted	3668.0
Proposals rejected	1332.0

#	P1	Skill	Uncertainty
1	Ath Madrid	43.91	4.56
2	Barcelona	43.05	5.81
3	Real Madrid	41.90	5.62
4	Villarreal	38.30	6.91
5	Celta	33.14	6.69
16	Granada	16.56	7.65
17	Espanol	14.61	5.66
18	Sp Gijon	12.17	6.17
19	Vallecano	12.08	6.79
20	Levante	12.06	6.75

Correct	94
Incorrect	96
Percentage	0.495
Mean of expected	0.539

#	P1	P2	Result	P(Win P1)	P(Loss P1)	P(Draw)	Prediction	Correct
1	Sevilla	Malaga	1	0.38	0.33	0.29	1	1
2	Sociedad	La Coruna	0	0.27	0.54	0.19	-1	0
3	Villarreal	Betis	0	0.55	0.27	0.18	1	0
4	Barcelona	Ath Bilbao	1	0.61	0.24	0.15	1	1
5	Getafe	Espanol	1	0.56	0.26	0.17	1	1
6	Las Palmas	Ath Madrid	-1	0.22	0.65	0.13	-1	1
7	Real Madrid	Sp Gijon	1	0.79	0.13	0.07	1	1
8	Valencia	Vallecano	0	0.65	0.22	0.13	1	0
9	Eibar	Granada	1	0.61	0.24	0.15	1	1
10	Sp Gijon	Sociedad	1	0.31	0.46	0.24	-1	0

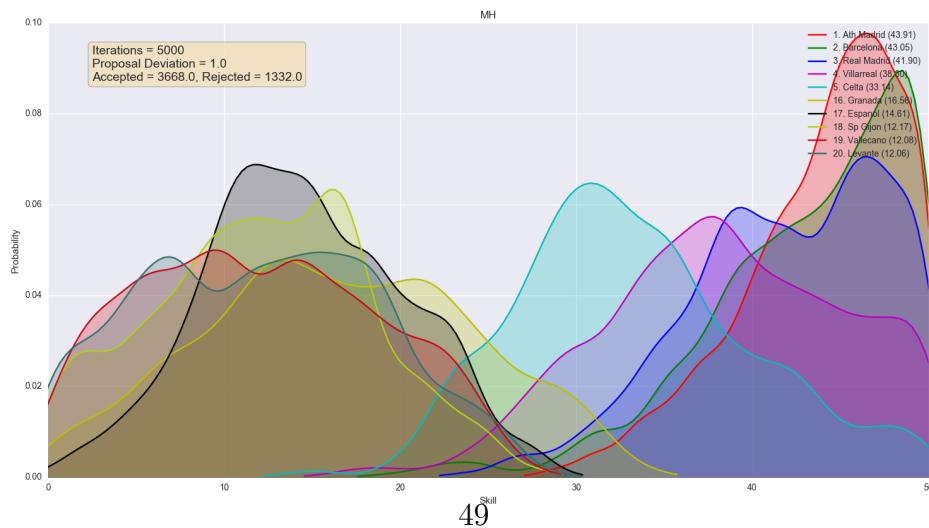


Table 36: dsaşdsa

Date	T6_4_Mon_May_30_19:09:34_2016
Algorithm	Gibbs
Dataset	Football
Number of matches	190
Number of players	20
Number of teams	20
Number of iterations	4000
Elapsed time	73
Variance of proposal	None
Proposals accepted	None
Proposals rejected	None

#	P1	Skill	Uncertainty	Correct	93
1	Ath Madrid	37.77	7.16	Correct	93
2	Barcelona	36.23	7.80	Incorrect	97
3	Real Madrid	32.85	7.94	Percentage	0.489
4	Villarreal	30.98	8.03	Mean of expected	0.486
5	Eibar	24.22	7.00		
16	Espanol	15.92	7.65		
17	Granada	14.67	6.81		
18	Levante	13.69	7.02		
19	Sp Gijon	12.31	7.06		
20	Vallecano	12.29	6.84		

#	P1	P2	Result	P(Win P1)	P(Loss P1)	P(Draw)	Prediction	Correct
1	Sevilla	Malaga	1	0.36	0.33	0.31	1	1
2	Sociedad	La Coruna	0	0.31	0.45	0.24	-1	0
3	Villarreal	Betis	0	0.56	0.26	0.18	1	0
4	Barcelona	Ath Bilbao	1	0.60	0.25	0.16	1	1
5	Getafe	Espanol	1	0.39	0.33	0.29	1	1
6	Las Palmas	Ath Madrid	-1	0.20	0.68	0.12	-1	1
7	Real Madrid	Sp Gijon	1	0.68	0.20	0.12	1	1
8	Valencia	Vallecano	0	0.49	0.29	0.21	1	0
9	Eibar	Granada	1	0.51	0.29	0.20	1	1
10	Sp Gijon	Sociedad	1	0.32	0.41	0.27	-1	0

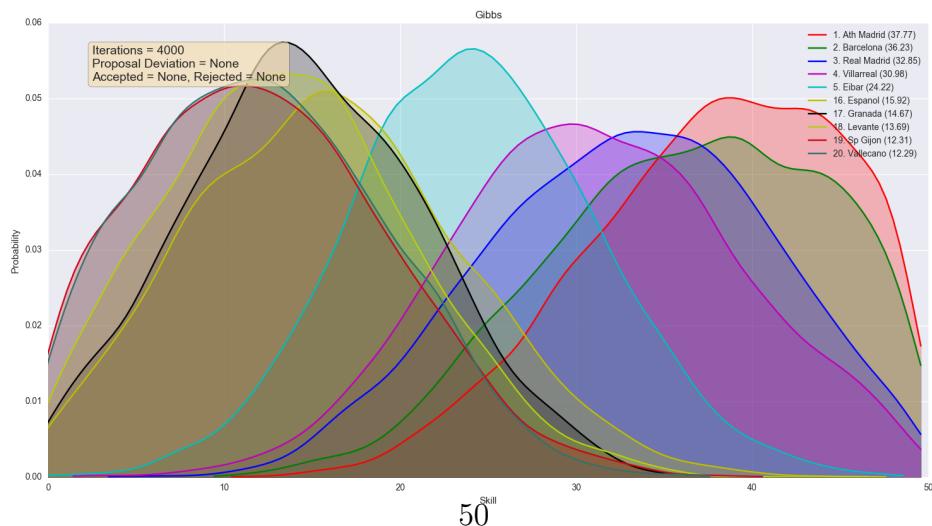


Table 37: dsaşdsa

Date	T6_4_Mon_May_30_19:09:34_2016
Algorithm	EP
Dataset	Football
Number of matches	190
Number of players	20
Number of teams	20
Number of iterations	None
Elapsed time	0
Variance of proposal	None
Proposals accepted	None
Proposals rejected	None

#	P1	Skill	Uncertainty
1	Barcelona	32.41	2.00
2	Ath Madrid	31.75	2.04
3	Real Madrid	30.53	1.99
4	Villarreal	29.93	2.00
5	Eibar	27.98	1.93
16	Granada	24.42	1.86
17	Las Palmas	24.35	1.82
18	Levante	22.63	1.84
19	Vallecano	21.59	2.00
20	Sp Gijon	21.51	2.04

Correct	92
Incorrect	98
Percentage	0.484
Mean of expected	0.646

#	P1	P2	Result	P(Win P1)	P(Loss P1)	P(Draw)	Prediction	Correct
1	Sevilla	Malaga	1	0.48	0.42	0.10	1	1
2	Sociedad	La Coruna	0	0.37	0.53	0.10	-1	0
3	Villarreal	Betis	0	0.80	0.10	0.10	1	0
4	Barcelona	Ath Bilbao	1	0.83	0.07	0.10	1	1
5	Getafe	Espanol	1	0.52	0.38	0.10	1	1
6	Las Palmas	Ath Madrid	-1	0.13	0.77	0.10	-1	1
7	Real Madrid	Sp Gijon	1	0.92	-0.02	0.10	1	1
8	Valencia	Vallecano	0	0.76	0.14	0.10	1	0
9	Eibar	Granada	1	0.71	0.19	0.10	1	1
10	Sp Gijon	Sociedad	1	0.33	0.57	0.10	-1	0

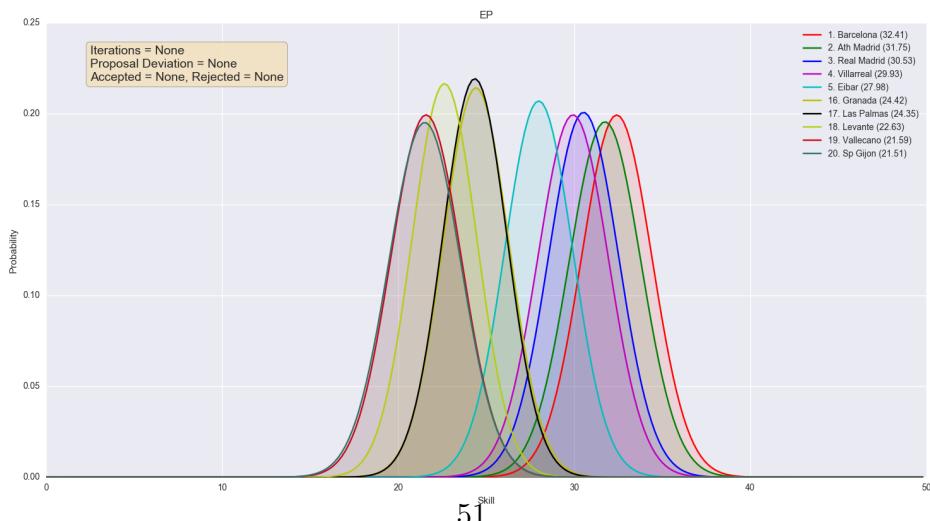


Table 38: dsaşds

Date	T6_5_Mon_May_30_19:16:57_2016
Algorithm	MH
Dataset	Football
Number of matches	306
Number of players	18
Number of teams	18
Number of iterations	5000
Elapsed time	37
Variance of proposal	1.0
Proposals accepted	3357.0
Proposals rejected	1643.0

#	P1	Skill	Uncertainty
1	Besiktas	44.36	4.98
2	Fenerbahce	37.85	5.87
3	Konyaspor	31.89	5.17
4	Buyuksehry	30.90	5.03
5	Galatasaray	27.60	5.45
14	Kayserispor	18.19	5.12
15	Gaziantepspor	14.99	5.97
16	Trabzonspor	14.52	7.31
17	Eskisehirspor	10.86	5.80
18	Mersin Idman Yurdu	8.20	5.79

#	Takim	OM	G	B	M	AG	YG	A	P
1	Besiktas	34	25	4	5	75	35	40	79
2	Fenerbahce	34	22	8	4	60	27	33	74
3	Konyaspor	34	19	9	6	44	33	11	66
4	Bagaktehir	34	16	11	7	54	36	18	59
5	Osmanspor	34	14	10	10	52	36	16	52
6	Galatasaray	34	13	12	9	69	49	20	51
7	Kasimpasa	34	14	8	12	50	40	10	50
8	Aksaray Bid	34	11	13	10	42	41	1	46
9	Antalyaspor	34	12	9	13	53	52	1	45
10	Genclerbirligi	34	13	6	15	42	42	0	45
11	Bursaspor	34	13	5	16	47	55	-8	44
12	Trabzonspor	34	12	4	18	40	59	-19	40
13	Rizespor	34	9	10	15	39	48	-9	37
14	Gaziantepspor	34	9	9	16	31	50	-19	36
15	Kayserispor	34	7	13	14	25	41	-16	34
16	Sivasspor	34	6	13	15	34	48	-14	31
17	Eskisehirspor	34	8	6	20	39	64	-25	30
18	Mersin Idman Yurdu	34	5	6	23	31	71	-40	21

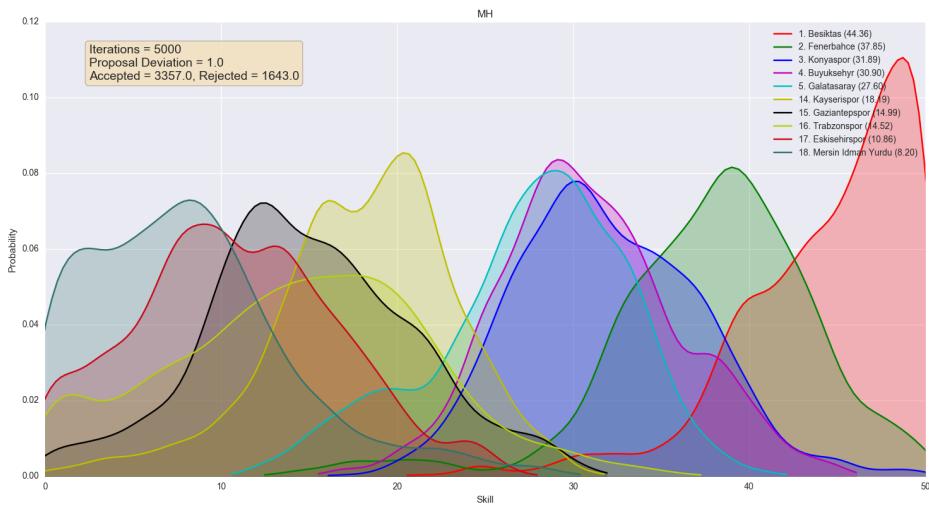


Table 39: dsaşdsa

Date	T6_5_Mon_May_30_19:16:57_2016
Algorithm	Gibbs
Dataset	Football
Number of matches	306
Number of players	18
Number of teams	18
Number of iterations	4000
Elapsed time	116
Variance of proposal	None
Proposals accepted	None
Proposals rejected	None

#	P1	Skill	Uncertainty
1	Besiktas	36.24	7.43
2	Fenerbahce	29.74	7.56
3	Konyaspor	25.89	6.95
4	Buyuksehir	24.24	6.76
5	Galatasaray	22.62	6.68
14	Rizespor	18.49	6.67
15	Trabzonspor	17.66	7.15
16	Gaziantepspor	17.35	6.61
17	Eskisehirspor	14.65	6.89
18	Mersin Idman Yurdu	11.05	6.72

#	Takım	OM	G	B	M	AG	YG	A	P
1	Besiktas	34	25	4	5	75	35	40	79
2	Fenerbahce	34	22	8	4	60	27	33	74
3	Konyaspor	34	19	9	6	44	33	11	66
4	Bagaiktehir	34	16	11	7	54	36	18	59
5	Osmanspor	34	14	10	10	52	36	16	52
6	Galatasaray	34	13	12	9	69	49	20	51
7	Kasimpasa	34	14	8	12	50	40	10	50
8	Akhisar Bid	34	11	13	10	42	41	1	46
9	Antalyaspor	34	12	9	13	53	52	1	45
10	Genclerbirligi	34	13	6	15	42	42	0	45
11	Bursaspor	34	13	5	16	47	55	-8	44
12	Trabzonspor	34	12	4	18	40	59	-19	40
13	Rizespor	34	9	10	15	39	48	-9	37
14	Gaziantepspor	34	9	9	16	31	50	-19	36
15	Kayserispor	34	7	13	14	25	41	-16	34
16	Sivasspor	34	6	13	15	34	48	-14	31
17	Eskisehirspor	34	8	6	20	39	64	-25	30
18	Mersin Idman Yurdu	34	5	6	23	31	71	-40	21

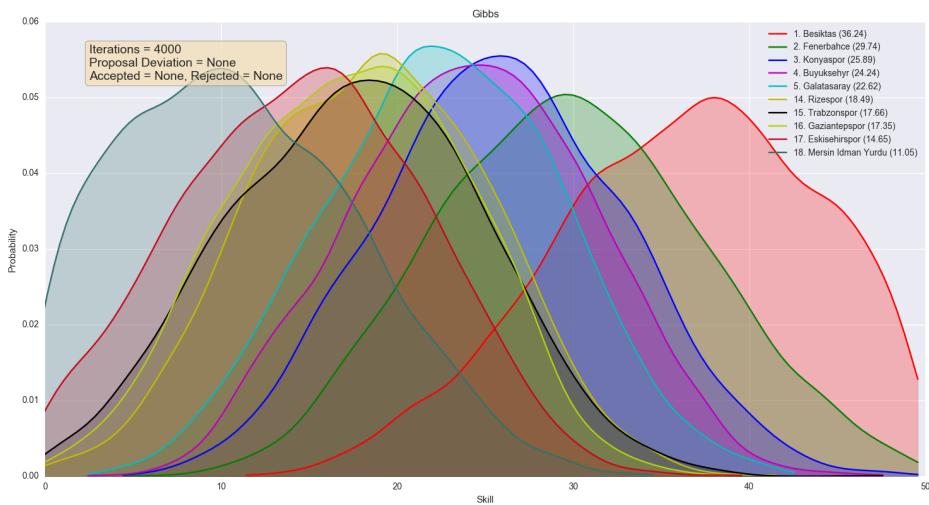


Table 40: dsaşdsa

Date	T6_5_Mon_May_30_19:16:57_2016
Algorithm	EP
Dataset	Football
Number of matches	306
Number of players	18
Number of teams	18
Number of iterations	None
Elapsed time	1
Variance of proposal	None
Proposals accepted	None
Proposals rejected	None

#	P1	Skill	Uncertainty
1	Besiktas	32.53	1.53
2	Fenerbahce	31.71	1.44
3	Konyaspor	29.97	1.42
4	Buyuksehry	29.37	1.32
5	Osmanlispor	28.22	1.38
14	Sivasspor	25.76	1.28
15	Gaziantepspor	25.57	1.32
16	Trabzonspor	25.54	1.38
17	Eskisehirspor	24.25	1.42
18	Mersin Idman Yurdu	23.44	1.53

#	Takim	OM	G	B	M	AG	YG	A	P
1	Besiktas	34	25	4	5	75	35	40	79
2	Fenerbahce	34	22	8	4	60	27	33	74
3	Konyaspor	34	19	9	6	44	33	11	66
4	Bakirkoy	34	16	11	7	54	36	18	59
5	Osmanlispor	34	14	10	10	52	36	16	52
6	Galatasaray	34	13	12	9	69	49	20	51
7	Kasimpasa	34	14	8	12	50	40	10	50
8	Akhisar Bld	34	11	13	10	42	41	1	46
9	Antalyaspor	34	12	9	13	53	52	1	45
10	Genclerbirligi	34	13	6	15	42	42	0	45
11	Bursaspor	34	13	5	16	47	55	-8	44
12	Trabzonspor	34	12	4	18	40	59	-19	40
13	Rizespor	34	9	10	15	39	48	-9	37
14	Gaziantepspor	34	9	9	16	31	50	-19	36
15	Kayserispor	34	7	13	14	25	41	-16	34
16	Sivasspor	34	6	13	15	34	48	-14	31
17	Eskisehirspor	34	8	6	20	39	64	-25	30
18	Mersin Idman Yurdu	34	5	6	23	31	71	-40	21

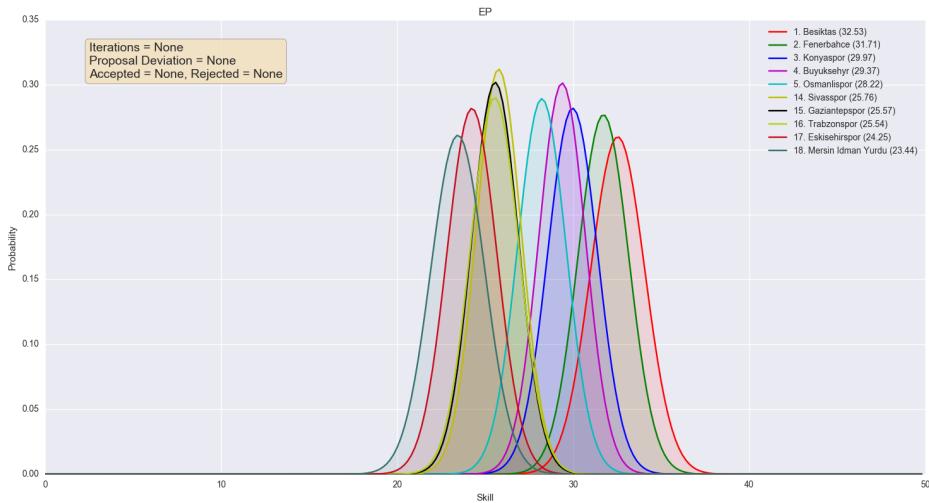


Table 41: dsaşdsa

Date	T7_6_Mon_May_30_19:19:36_2016
Algorithm	MH
Dataset	Basketball
Number of matches	615
Number of players	30
Number of teams	30
Number of iterations	5000
Elapsed time	67
Variance of proposal	1.0
Proposals accepted	3040.0
Proposals rejected	1960.0

#	P1	Skill	Uncertainty	Correct	392
1	Warriors	46.98	3.68		
2	Hawks	46.97	3.86		
3	Trail Blazers	45.93	3.75		
4	Grizzlies	43.32	5.92		
5	Wizards	42.77	5.58		
26	Celtics	8.82	5.73		
27	Lakers	7.60	7.07		
28	76ers	3.18	4.10		
29	Timberwolves	2.74	4.44		
30	Knicks	2.58	4.56		

#	P1	P2	Result	P(Win P1)	P(Loss P1)	P(Draw)	Prediction	Correct
1	Celtics	Clippers	-1	0.13	0.79	0.08	-1	1
2	Mavericks	Grizzlies	1	0.33	0.39	0.29	-1	0
3	Raptors	Bucks	1	0.49	0.29	0.21	1	1
4	Pelicans	Knicks	-1	0.71	0.18	0.11	1	0
5	Lakers	Suns	-1	0.18	0.71	0.11	-1	1
6	Kings	Trail Blazers	-1	0.12	0.82	0.07	-1	1
7	76ers	Wizards	-1	0.08	0.87	0.05	-1	1
8	Spurs	Nuggets	1	0.63	0.23	0.14	1	1
9	Thunder	Heat	1	0.43	0.32	0.26	1	1
10	Pacers	Hawks	-1	0.10	0.85	0.05	-1	1

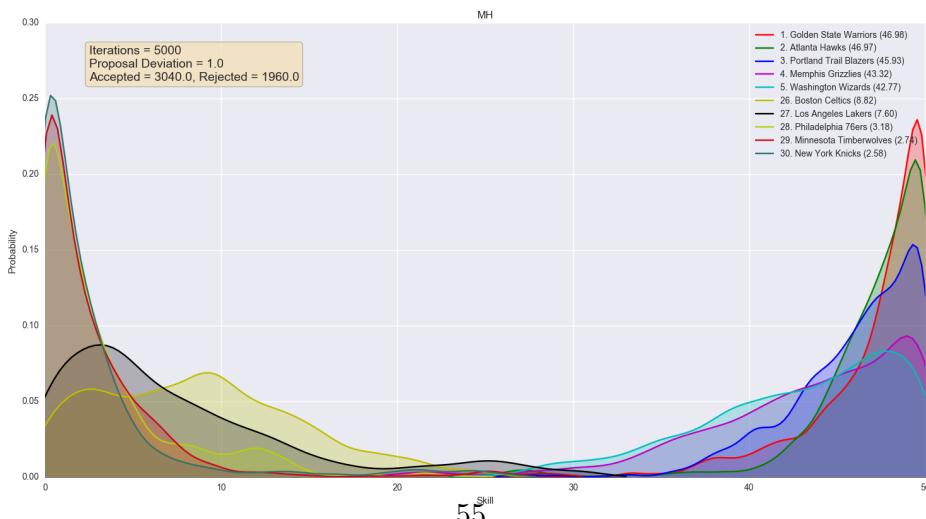


Table 42: dsaşdsa

Date	T7_6_Mon_May_30_19:19:36_2016
Algorithm	Gibbs
Dataset	Basketball
Number of matches	615
Number of players	30
Number of teams	30
Number of iterations	4000
Elapsed time	224
Variance of proposal	None
Proposals accepted	None
Proposals rejected	None

#	P1	Skill	Uncertainty
1	Warriors	45.54	3.23
2	Hawks	44.76	3.73
3	Grizzlies	41.87	4.82
4	Trail Blazers	41.46	4.97
5	Rockets	39.52	5.38
26	Magic	13.16	5.78
27	Lakers	12.84	5.95
28	76ers	5.17	4.13
29	Timberwolves	4.05	3.59
30	Knicks	3.19	3.08

Correct	403
Incorrect	212
Percentage	0.655
Mean of expected	0.569

#	P1	P2	Result	P(Win P1)	P(Loss P1)	P(Draw)	Prediction	Correct
1	Celtics	Clippers	-1	0.17	0.73	0.10	-1	1
2	Mavericks	Grizzlies	1	0.32	0.41	0.27	-1	0
3	Raptors	Bucks	1	0.51	0.29	0.20	1	1
4	Pelicans	Knicks	-1	0.74	0.17	0.10	1	0
5	Lakers	Suns	-1	0.24	0.61	0.15	-1	1
6	Kings	Blazers	-1	0.18	0.71	0.11	-1	1
7	76ers	Wizards	-1	0.12	0.82	0.07	-1	1
8	Spurs	Nuggets	1	0.57	0.26	0.17	1	1
9	Thunder	Heat	1	0.41	0.32	0.27	1	1
10	Pacers	Hawks	-1	0.12	0.81	0.07	-1	1

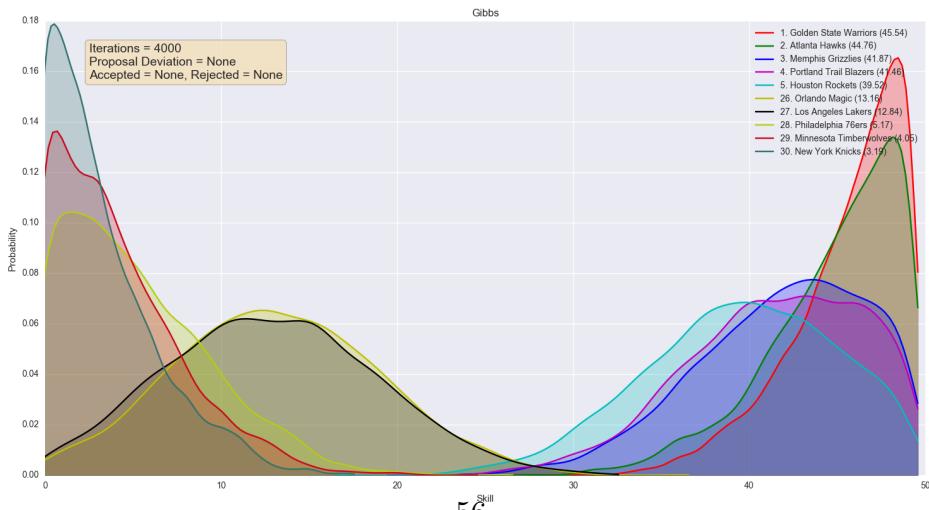


Table 43: dsaşda

Date	T7_6_Mon_May_30_19:19:36_2016
Algorithm	EP
Dataset	Basketball
Number of matches	615
Number of players	30
Number of teams	30
Number of iterations	None
Elapsed time	2
Variance of proposal	None
Proposals accepted	None
Proposals rejected	None

#	P1	Skill	Uncertainty	Correct	393
1	Warriors	33.80	1.79		
2	Hawks	31.54	1.50		
3	Blazers	31.26	1.49		
4	Grizzlies	31.08	1.42		
5	Rockets	30.45	1.49		
26	Pacers	22.68	1.37	Incorrect	222
27	Celtics	22.55	1.48	Percentage	0.639
28	76ers	19.09	1.66	Mean of expected	0.686
29	Timberwolves	18.52	1.60		
30	Knicks	17.28	1.63		

#	P1	P2	Result	P(Win P1)	P(Loss P1)	P(Draw)	Prediction	Correct
1	Celtics	Clippers	-1	0.13	0.77	0.10	-1	1
2	Mavericks	Grizzlies	1	0.41	0.49	0.10	-1	0
3	Raptors	Bucks	1	0.69	0.21	0.10	1	1
4	Pelicans	Knicks	-1	0.93	-0.03	0.10	1	0
5	Lakers	Suns	-1	0.32	0.58	0.10	-1	1
6	Kings	Trail Blazers	-1	0.14	0.76	0.10	-1	1
7	76ers	Wizards	-1	0.05	0.85	0.10	-1	1
8	Spurs	Nuggets	1	0.73	0.17	0.10	1	1
9	Thunder	Heat	1	0.60	0.30	0.10	1	1
10	Pacers	Hawks	-1	0.08	0.82	0.10	-1	1

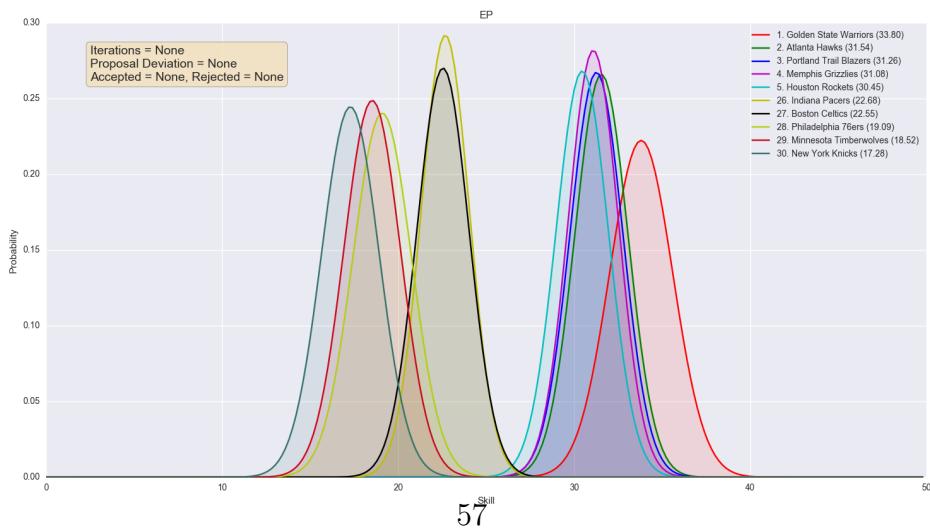


Table 44: dsaşds

Date	T7_7_Mon_May_30_19:24:35_2016
Algorithm	MH
Dataset	Basketball
Number of matches	1230
Number of players	30
Number of teams	30
Number of iterations	5000
Elapsed time	138
Variance of proposal	1.0
Proposals accepted	2385.0
Proposals rejected	2615.0

#	P1	Skill	Uncertainty	EASTERN CONFERENCE		WESTERN CONFERENCE	
				Eastern	Western		
				W	L	PCT	
1	Warriors	48.70	2.34	Cleveland 1e	57	25	0.695
2	Rockets	46.23	3.56	Toronto 2a	56	26	0.683
3	Hawks	45.52	4.20	Miami 3se	48	34	0.585
4	Clippers	45.33	4.07	Atlanta 4x	48	34	0.585
5	Grizzlies	44.14	6.62	Boston 5x	48	34	0.585
26	Magic	5.45	4.26	Charlotte 6x	48	34	0.585
27	Lakers	3.41	3.05	Indiana 7x	45	37	0.549
28	Knicks	2.51	5.15	Detroit 8x	44	38	0.537
29	76ers	1.98	2.81	Chicago 9	42	40	0.512
30	Timberwolves	1.62	2.88	Washington 0	41	41	0.500
				Orlando 0	35	47	0.427
				Milwaukee 0	33	49	0.402
				New York 0	32	50	0.390
				Brooklyn 0	21	61	0.256
				Philadelphia 0	10	72	0.122
				EASTERN CONFERENCE		WESTERN CONFERENCE	
				Eastern	W	L	PCT
				Cleveland 1e	57	25	0.695
				Toronto 2a	56	26	0.683
				Miami 3se	48	34	0.585
				Atlanta 4x	48	34	0.585
				Boston 5x	48	34	0.585
				Charlotte 6x	48	34	0.585
				Indiana 7x	45	37	0.549
				Detroit 8x	44	38	0.537
				Chicago 9	42	40	0.512
				Washington 0	41	41	0.500
				Orlando 0	35	47	0.427
				Milwaukee 0	33	49	0.402
				New York 0	32	50	0.390
				Brooklyn 0	21	61	0.256
				Philadelphia 0	10	72	0.122
				EASTERN CONFERENCE		WESTERN CONFERENCE	
				Eastern	W	L	PCT
				Cleveland 1e	57	25	0.695
				Toronto 2a	56	26	0.683
				Miami 3se	48	34	0.585
				Atlanta 4x	48	34	0.585
				Boston 5x	48	34	0.585
				Charlotte 6x	48	34	0.585
				Indiana 7x	45	37	0.549
				Detroit 8x	44	38	0.537
				Chicago 9	42	40	0.512
				Washington 0	41	41	0.500
				Orlando 0	35	47	0.427
				Milwaukee 0	33	49	0.402
				New York 0	32	50	0.390
				Brooklyn 0	21	61	0.256
				Philadelphia 0	10	72	0.122

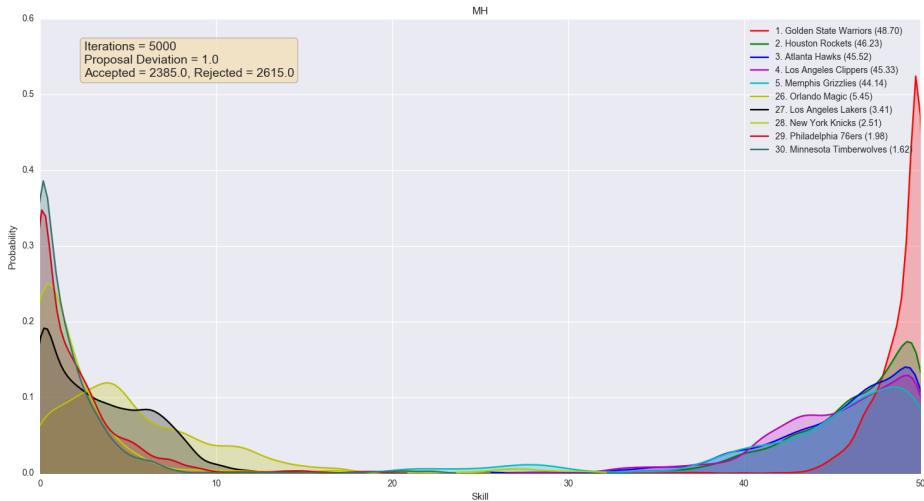


Table 45: dsaşds

Date	T7_7_Mon_May_30_19:24:35_2016
Algorithm	Gibbs
Dataset	Basketball
Number of matches	1230
Number of players	30
Number of teams	30
Number of iterations	4000
Elapsed time	419
Variance of proposal	None
Proposals accepted	None
Proposals rejected	None

#	P1	Skill	Uncertainty	EASTERN CONFERENCE	WESTERN CONFERENCE
				Eastern	Western
1	Warriors	46.84	2.19	Cleveland 1e	Golden State 1w
2	Hawks	42.17	4.05	Toronto 2a	San Antonio 2sw
3	Rockets	40.77	4.18	Miami 3se	Oklahoma City 3nw
4	Clippers	40.61	4.27	Atlanta 4x	L.A. Clippers 4x
5	Grizzlies	39.79	4.31	Boston 5x	Portland 5x
26	Magic	10.31	4.36	Charlotte 6x	Dallas 6x
27	Lakers	7.73	4.29	Indiana 7x	Memphis 7x
28	76ers	4.41	3.41	Detroit 8x	Houston 8x
29	Timberwolves	4.06	3.20	Chicago 9	Utah 9
30	Knicks	3.68	3.07	Washington 10	Sacramento 10
				Orlando 11	Denver 11
				Milwaukee 12	New Orleans 12
				New York 13	Minnesota 13
				Brooklyn 14	Phoenix 14
				Philadelphia 15	L.A. Lakers 15

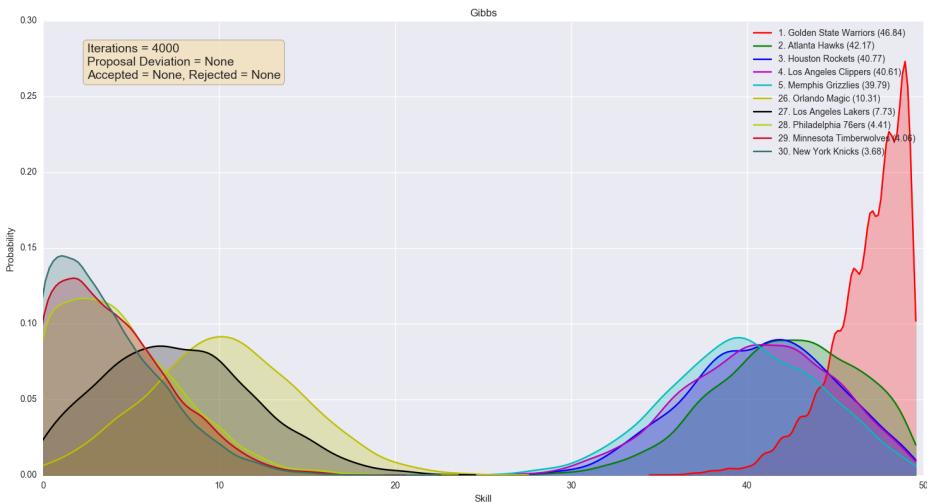
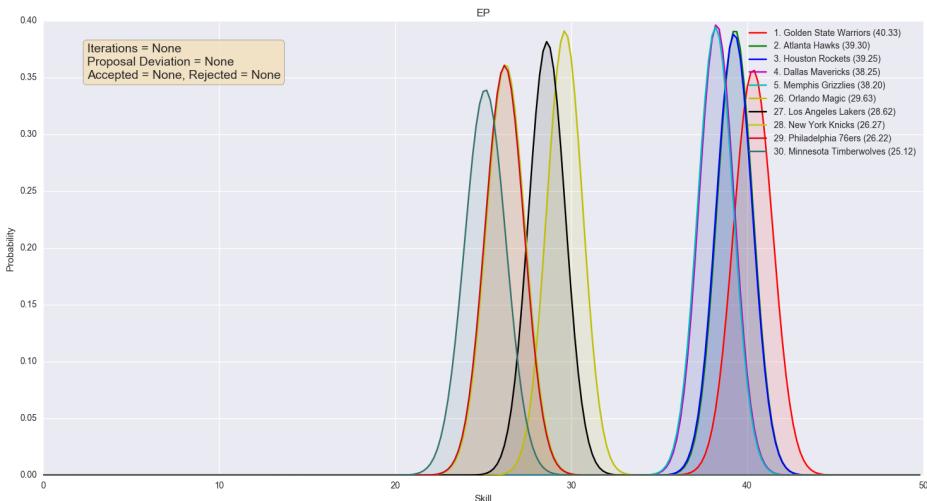


Table 46: dsaşds

Date	T7_7_Mon_May_30_19:24:35_2016
Algorithm	EP
Dataset	Basketball
Number of matches	1230
Number of players	30
Number of teams	30
Number of iterations	None
Elapsed time	3
Variance of proposal	None
Proposals accepted	None
Proposals rejected	None

#	P1	Skill	Uncertainty
1	Warriors	40.33	1.12
2	Hawks	39.30	1.02
3	Rockets	39.25	1.03
4	Mavericks	38.25	1.01
5	Grizzlies	38.20	1.01
26	Magic	29.63	1.02
27	Lakers	28.62	1.05
28	Knicks	26.27	1.10
29	76ers	26.22	1.11
30	Timberwolves	25.12	1.17

EASTERN CONFERENCE				WESTERN CONFERENCE			
Eastern	W	L	PCT	Western	W	L	PCT
Cleveland 1e	57	25	0.695	Golden State 1w	73	9	0.890
Toronto 2a	56	26	0.683	San Antonio 2sw	67	15	0.817
Miami 3se	48	34	0.585	Oklahoma City 3nw	55	27	0.671
Atlanta 4x	48	34	0.585	L.A. Clippers 4x	53	29	0.646
Boston 5x	48	34	0.585	Portland 5x	44	38	0.537
Charlotte 6x	48	34	0.585	Dallas 6x	42	40	0.512
Indiana 7x	45	37	0.549	Memphis 7x	42	40	0.512
Detroit 8x	44	38	0.537	Houston 8x	41	41	0.500
Chicago 9	42	40	0.512	Utah 9	40	42	0.488
Washington 9	41	41	0.500	Sacramento 9	33	49	0.402
Orlando 9	35	47	0.427	Denver 9	33	49	0.402
Milwaukee 9	33	49	0.402	New Orleans 9	30	52	0.366
New York 9	32	50	0.390	Minnesota 9	29	53	0.354
Brooklyn 9	21	61	0.256	Phoenix 9	23	59	0.280
Philadelphia 9	10	72	0.122	L.A. Lakers 9	17	65	0.207



5 Conclusion & Discussion

6 Future Work

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7 All Tests

8 Appendices