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Some Critical Aspects in Sliding Mode Control Design for the Boost Inverter.

N. Vázquez* J. Alvarez.** C. Aguilar*** J. Arau***

*Instituto Tecnológico de Celaya.
Depto. de sistemas e informática.
Celaya, México.
Phone: 52 (461) 175-75
Fax: 52 (461) 179-79
E-mail: nvazquez@itc.mx

**CINVESTAV-IPN.
Depto. de ingenieria electrica.
México, D.F.
Phone: 52 (5) 747-7000
Fax: 52 (5) 747-7089
E-Mail: jalvarez@ctrl.cinvestav.mx

*** CENIDET
Depto. de ingenieria electrónica.
Cuernavaca, México
Phone: 52 (731) 877-41
Fax: 52 (731) 224-34
E-Mail: j.arau@ieee.org

Abstract. — In this paper some critical aspects for the design of a sliding mode control applied to the boost inverter are presented, including a theoretical analysis and the implementation of the control strategy. The SMC has been proposed in different papers as an option to improve the dynamic response of the electronics circuits. In this paper several design issues related to the theory as well as to the practical implementation of the SMC are discussed. A converter, which has excellent features like boost and inverting functions at the same time (boost inverter), is used as an example design.

I. INTRODUCTION

The sliding mode control (SMC) has been used to improve the robustness and the dynamic response of different converters such as inverters and cd/cd converters [1-5]. This is because the SMC does not include a low pass band filter in the control circuit; that is, the state variables are not introduced in a low pass band filter that reduces the dynamic response of the converter. In those papers, it is shown that the SMC introduces those good characteristics, and a converter is analyzed and designed. Some of them mention different aspects to consider in the physical implementation of the SMC. In this paper the mathematical analysis and physical implementation of the SMC are discussed. In particular some critical aspect in the design and implementation of the SMC for the boost inverter are analyzed.

In this work the sliding mode control is applied to the boost inverter; this converter has some excellent features like the boosting and inverting functions at the same time. A brief operation description of this converter is also given.

In the next two sections the operation of the SMC and the boost inverter is presented, in the fourth section the design of the boost inverter controlled by the SMC is explained: first we deal with the theoretical part and then the implementation is commented.

II. THE SLIDING MODE CONTROL

Sliding mode control offers some advantages such as: stability, robustness, good dynamic response and simple implementation. However, the control theory involved is rather complex compared with the traditional control theory. Many papers have presented a variety of sliding mode control design steps [6-8], but they could be summarized as follows:

i) Propose the sliding surface.
ii) Verify the existence of a sliding mode.
iii) Analyze the stability in the sliding surface.

Typically, the sliding surface proposed is a linear combination of the state variables (step i); this is because of the easy in implementation and theoretical analysis. The SMC forces the system to be held in this surface (step ii) and then the system is driven to the equilibrium point (step iii), of course the sliding surface must include the equilibrium point.

III. THE BOOST INVERTER

The converter used to implement the SMC is an integrated inverter topology with both boosting and inverting functions. This topology

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was introduced in [9]. The boost DC - AC converter, referred to as *the boost inverter*, features an excellent property: it naturally generates an output AC voltage lower or larger than the DC input voltage, depending on its duty cycle [9]. This property is not found in the classical voltage source inverter which produces an instantaneous AC output voltage always lower than the input DC voltage.

The boost inverter achieves DC - AC conversion as follows: the power stage consists of two current bi-directional boost converter and the load is connected differentially across them (see Fig. 1). These converters produce a DC - biased sinusoidal waveform, so that each converter produces an unipolar voltage. The modulation of each converter is 180 degrees out of phase with respect to the other, which maximizes the voltage excursion over the load (Fig. 2) [9-10].

IV. THE BOOST INVERTER CONTROLLED BY THE SLIDING MODE CONTROL

This section is divided in two parts: The first presents the analysis of the converter with the SMC (only theoretically) and in the second one the physical implementation is presented.

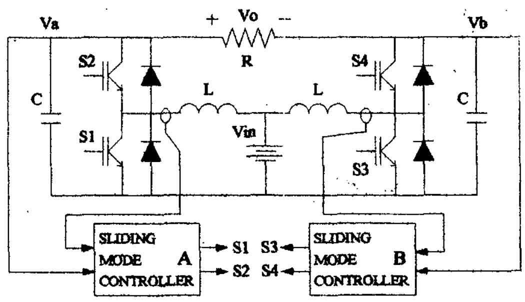

Fig. 1. Boost inverter with sliding mode control.

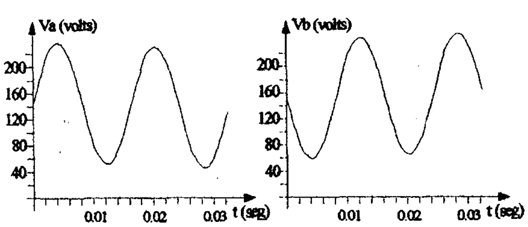

Fig 2. Output voltage for each DC-DC converter.

Analysis of the converter.

a) *System modeling.*

The boost inverter is modeled as two cd/cd boost converters, but one is modeled as an ideal sinusoidal voltage source plus a dc component (Fig 3). As it can be seen in figure 3 there are two possible positions for the switch (-1 and 1).

The system model is given by [10]:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & -w_0/2 \\ w_0/2 & -w_1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & w_0/2 \\ -w_0/2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} u + \begin{bmatrix} b \\ c \end{bmatrix} \quad (1)$$

where: $x_1 = I_L\sqrt{L}$, $x_2 = Vc\sqrt{C}$,

$$w_0 = \frac{1}{\sqrt{LC}},\ w_1 = \frac{1}{RC},$$

$$b = \frac{Vin}{\sqrt{L}},\ c = \frac{Vb}{R\sqrt{C}}.$$

b) *Design of the Sliding mode controller.*

As it was mentioned in section two the sliding mode control design can be summarized as follows:

i) The sliding surface. The sliding surface proposed is a linear combination of the state variables and the reference variables, that is:

$$\sigma = \mathbf{SX} - \mathbf{SX}_r = \mathbf{SeX} \quad (2)$$

where: $\mathbf{S} = [s_1 \quad s_2]$,

$\mathbf{X}$ = State variables,

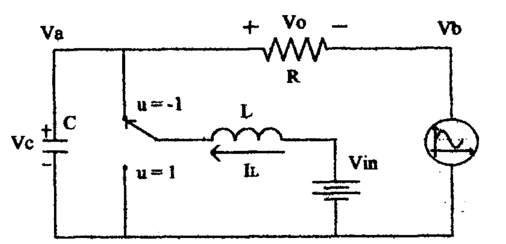

Fig. 3. Simplified circuit of the boost inverter.

$\mathbf{X}_r$ = Reference variables,

$\mathbf{eX} = \begin{bmatrix} ex_1 & ex_2 \end{bmatrix}^T$.

The control law proposed is:

$$u = u_{eq} + u_N \tag{3}$$

where: u_{eq} = *Equivalent control*,

$u_N = -\operatorname{sgn}\sigma$

This control law is composed by two terms, the first is only valid on the sliding surface (u_{eq}) and the other assures the existence of a sliding mode.

ii) Existence of a sliding mode. Existence of a sliding mode implies that the following condition is fulfilled [6-7]:

$$\sigma\dot{\sigma} \langle 0, \tag{4}$$

It can be easily shown that in order to guarantee the existence conditions of a sliding mode the following inequality must be fulfilled:

$$s_1x_2 - s_2x_1 \rangle 0 \tag{5}$$

In order to assure the existence conditions, s_1 must be positive since x_2 is always positive. The term s_1x_2 must be higher than s_2x_1. This inequality shows the compromise between the existence condition and the state variables

iii) Stability analysis in the sliding surface. A tool developed to describe the movement in the sliding surface is the equivalent control [8]. The equivalent control is valid when an analysis of the dynamics of the system is made assuming the system is on the sliding surface ($\sigma = 0$), hence $\dot{\sigma} = 0$. Therefore, the equivalent control (u_{eq}) is obtained from $\dot{\sigma} = 0$, resulting:

$$u_{eq} = -[\mathbf{SBX}]^{-1}\left[\mathbf{SAX} + \mathbf{SC} - \mathbf{S}\dot{\mathbf{X}}_r\right] \tag{6}$$

The condition $\mathbf{SBX} \neq 0$ must be satisfied to avoid singularities in the equivalent control. The equivalent control is substituted in the model of the system, and the stability analysis must be made under this condition.

A formal stability analysis is not presented in the present paper, but to assure the stability and the right operation form of the converter, s_2 must be positive.

c) *Simulation results.*

The system has been simulated with the following parameters: L=360 μH, C=28μF, R=48Ω, V_{in}=50V, V_o=170Vac, f_o = 60 Hz.

Simulation results with s_1 = 4 and s_2 = 1 show the existence of a sliding mode, but the system has a slow response (see Fig. 4). In the phase plane graph it is shown that the system reaches the sliding surface and it is maintained there.

Also, the system has been simulated with the parameters s_1 = 1 and s_2 = 1 that demonstrate that if s_1 becomes lower by maintaining s_2 constant, the system is faster, as we can see in Fig. 5. In the phase plane graph it is shown that the existence is fulfilled in steady state, but it does not in transient operation. However even the existence condition is not fulfilled in the transient operation the system is stable.

A simulation with s_1 = 0.26 and s_2 = 1 demonstrates that if s_1 becomes too low by maintaining s_2 constant there is no a sliding mode even in the steady state (Fig. 6). A phase plane graph show that the existence is not fulfilled. It is important to note that when the system stays in the sliding surface the system dynamics is the fastest.

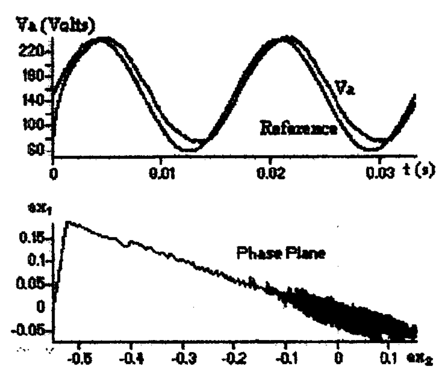

Fig. 4. Simulated results with the parameters s_1 =3 and s_2 =1.

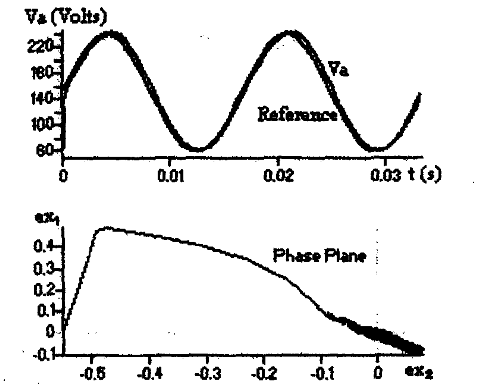

Fig. 5 .Simulations with $s_1 = 1$ and $s_2 = 1$.

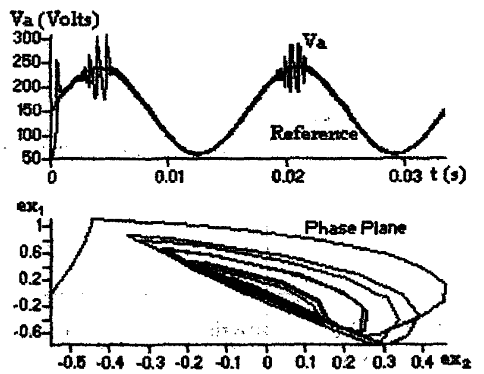

Fig. 6. Simulations with $s_1 = 0.4$ and $s_2 = 1$.

As it can be seen in the simulations made, there is a strong compromise between the existence condition and the fastness of the system.

A simulation with a load variation was done in which the load has changed from R = 48Ω to R=1000Ω, and viceversa (Figure 7). This simulation demonstrates the fast dynamics and robustness of the system by using the sliding mode control.

Physical implementation.

For the physical implementation is important to consider the reference generation and the frequency limitation in addition to the sliding surface generation.

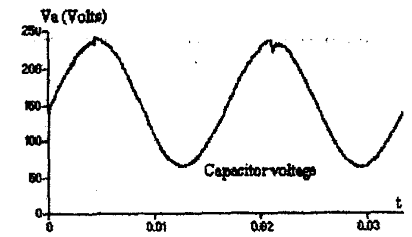

Fig. 7. Simulation with load variation. $s_1 = 1$ y $s_2 = 1$.

a) Reference generation.

The sliding mode control uses two state variables to control the boost inverter; therefore, it is necessary to include the part that generates the references.

One state variable is the capacitor voltage, its reference is a sinusoidal voltage plus a dc component due to the behavior of the boost inverter. This reference voltage is independent of the load and easy to implement; it is just necessary a sinusoidal signal generator and an operational amplifier to add the sinusoidal signal and a dc voltage.

The other state variable is the inductor current. The level of this current depends on the load, that is the current is a signal that changes as a function of the load. This is difficult to implement. Moreover the inductor current has an strange form as can be observed in the figure 9.

To avoid the dependence of the current with the load, an idea to produce the error (current inductor minus reference) was proposed in [2]; there, the current is measured and a *high* pass band filter is used to obtain the ripple at switching frequency that simulate the error.

In [2] is used the before mentioned technique in a cd/cd converter. In the boost inverter case is more complicated to use the filter, because the current is not a flat line as in the case of a cd/cd converter (see Fig 9). The cutoff frequency of the high pass band filter must be chosen carefully. It must to be high to eliminate the 60Hz frequency of the current, but not too high that distort the current ripple that simulate the error (at least 25 higher than 60Hz).

The filter order must be one, or do not introduce a lag over 180°. This is necessary to avoid little oscillations at the output voltage.

b) Boundedness of switching frequency

There are different techniques to bound the frequency: histeresys, delay and holding constant the time the switch is in the ON position [11]. In this work, the last technique is used. This technique allows to easily know the switching frequency and it is easy to bound it. It consists in to hold a constant time (t_{on}) the switch at u=1 position according to the figure 3, and to regulate the output voltage the switch at u=-1 position must be controlled.

c) Surface generation.

The surface is easy to generate because it has been chosen as a linear combination, of the error signals, so it is only necessary to use an operational amplifier to implement this part.

d) Example of implementation.

The design example consider a switching frequency (f_s) much more higher than 60Hz, a resistive load, Po = 300W (R=48Ω), f_{smax} =30KHz, Vin=48V, and Vo=120Vrms. The implementation is based on the maximum ripple desired in the inductor current and the capacitor voltage.

Firstly the dc component of the capacitor voltage (Vdc) must be calculated [10]:

$$Vdc \geq \frac{Vop}{2} + Vin \qquad (7)$$

where: Vop = peak output voltage ≅170V.

Using (7) Vdc results 133V, to avoid a saturation in the output voltage is chosen 150V. The maximum capacitor voltage and inductor current is determined by [10]:

$$Vc_{max} = Vdc + \frac{Vop}{2} \qquad (8)$$

$$I_{Lmax} = \frac{2D_{max} - Gm'(1-D_{max})}{(1-D_{max})^2} \cdot \frac{Vin}{R} \qquad (9)$$

where: $D_{max} = 1 - \dfrac{Vin}{Vdc + \dfrac{Vop}{2}}$, duty cycle $_{max}$

$$Gm' = \frac{2(Vdc - Vin)}{Vin}.$$

Resolving is obtained a $D_{max} \cong 0.8$, Vc_{max}=235V, I_{lmax}=18.75A. The inductance and the capacitance are calculated with a 20% and 1.5% of ripple respectively.

$$L = \frac{t_{on}}{0.2\, I_{Lmax}} Vin \qquad (10)$$

$$C = \frac{t_{on}}{0.015 V_{cmax}} I_{op} \qquad (11)$$

where: I_{op} = peak output current ≅ 3.53 A.

But firstly the t_{on} must be calculated with:

$$t_{on} = \frac{D_{max}}{f_{smax}} \qquad (12)$$

Then $t_{on} \cong 28\mu s$. Substituting (12) in (10) and (11) is obtained L ≅ 360μH and C ≅ 28 μF.

To calculate the controller parameters is necessary to found the boundary of the existence of the sliding mode; determined by:

$$s_1 = \frac{s_2 x_{1P}}{x_{2P}} \qquad (13)$$

where: $x_{1P} = I_{Lmax}\sqrt{L}$,

$x_{2P} = Vc_{max}\sqrt{C}$.

Using (13) and chosen s_2 equal to one is obtained that $s_1 \cong 0.28$; the parameter s_1 must be higher than 0.28 to avoid that there is no a sliding mode due to parasitic elements. Therefore s_1 is chosen equal one.

V. EXPERIMENTAL RESULTS

The boost inverter controlled by sliding mode control was implemented. The converter parameters have the followings values: L= 360uH, C=28uF, Vin= 48V, Vo=120Vrms, fo=60Hz.

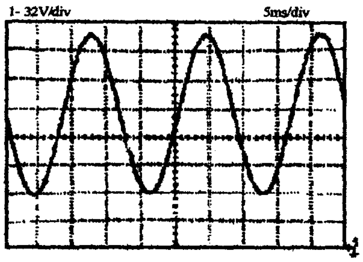

Fig 8 Capacitor voltage, Po=200W

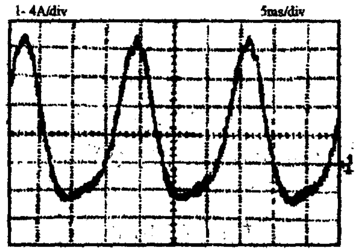

Fig. 9 Inductor current, Po=200W.

In figure 8 is shown the capacitor voltage. In figure 9 is shown the inductor current, its important to note its strange form.

In order to show the good characteristics of the closed loop system a load variation was made (see Fig. 10).

VI. Conclusions

The sliding mode control has been extensively used to increase the response of the system. In this paper, the control has been implemented in a converter which can boost and invert a signal at the same time. The aspects for designing the sliding mode control applied to the boost inverter has been presented, including a theoretical analysis and the physical implementation of the control strategy.

The physical implementation must consider the problems related to the reference generation and frequency bounds. The converter controlled by sliding mode control was verified with simulation and experimental results.

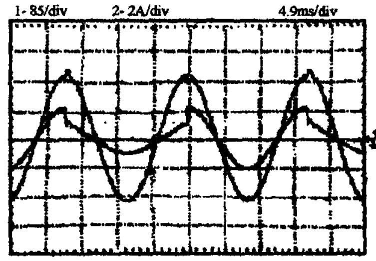

Fig 10 Output voltage and current under load variation Po=200W to 100W and viceversa.

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