

CS 61 discrete math HW 1

Textbook reading

Section 1: I haven't done algebra in a long time, so I had no idea how to simplify the problem, nor any indication how to approach getting closer to the solution...

$$(x - a)(x - b) = x^2 - ax - bx + ab$$

$$(x - a)(x - b)(x - c) = (x^2 - ax - bx + ab)(x - c) = x^3 - ax^2 - bx^2 + abx - cx^2 + acx + bcx + abc$$

So I assumed that this trend would continue in the following fashion

$$x^{26} - (a + b + c \dots + z)x^{25} \dots + (a * b * c \dots * z)$$

I am typing on Pages, which does not have a great equation editor, so I got a piece of paper to write out the 4th degree polynomial, which contains $-abx^2$, $-cx^3$, and $abcd$

I assume the answer can only be 0, since the question was presented to induce the wonder of math via reduction. This is not what makes math wonderful in my mind. Reductive simplifications are sometimes relieving when solving equations, but rarely spark any joy or awe. I like math because it finds and explores intricate patterns in simple things. I also like learning math because it will certainly help me identify patterns and understand information.

Section 2: I am not that amazing at spatial manipulation, so the challenge was hard. It seemed absurd to explain how to rearrange all the pieces after moving them around in my head. First they have no names, and second it's not obvious how to track the process verbally so as to retain the working memory.

Section 3: 0 does not any number except itself, but all numbers divide 0

Section 4: I am not fond of this book but there is no sense in complaining. I have the self-awareness to recognize that for people getting back to school, there is an adjustment period. I did not like my college and I really did not enjoy online classes. I am taking this class because it is required for my degree, and I want to make the most of it, so I will take this textbook with an open mind.

Part II:

To get the intuition for this type of problem, I first went day-by-day, before writing up a proper solution.

To solve the problem we review the facts.

1. The lion lies on Monday-Wednesday;
2. the unicorn lies on Thursday-Saturday
3. They both make the statement that their lying day was yesterday.
4. They both tell the truth on truth days and (can choose to) lie on lying days.

Since they are both claiming a lying day was yesterday, which is impossible since their lying days are mutually exclusive, one is lying while the other is telling the truth.

We see Thursday as a lying day for the unicorn, and knowing that Wednesday was a lying day for the lion, Thursday is a possibility.

Because both tell the truth on truth days, and because their lying days are mutually exclusive, then we can deduce that no other day of the week is a possibility.

According to the available information, today day is certainly Thursday.

For the second problem, we apply the concept of bounds to approach possible scenarios. The two main scenarios are either it is a truth day or a lying day (M-W). We see that it cannot be a truth day because the two statements are never both true because they claim a 4-day window of lying days. So it is certainly a lying day.

Because both claims together are false, either one of the claims is false. But because it is a lying day, we cannot derive any additional information about which lying day it is.

In the same vein as the second problem, the only time *either of* these two statements are true are within the lying day window, so all the statements grant is narrowing down the week to 3 days, (equivalent to about 1 bit of information).

The outcome is the same for the fourth question as the previous two, but with the change of logic that *both* statements are true only on lying days.

$3 \nmid 100$ is false because no integer c can be multiplied by 3 to produce 100, which is clear by the set of the prime factorization of 100: $\{2, 5\}$.

$-3 \mid 3$ is true because $-3 \cdot -1 = 3$

4.1a-d, 4.3, 4.4, 4.5

4.1a: if a is an odd integer and b is an even integer, then their product ab is an even integer.

4.1b: if c is an odd integer, then \sqrt{c} is odd

4.1c: if d is a prime number, then d^2 is not prime.

4.1d: if e, f are two negative integers, then their product ef is also negative.

4.3:

A: $x = 2$

B: x is even

4.4:

These statements can be proven to be identical using a truth table. The converse of any statement carries the same truth value.

They are true when 1) A and B are true, 2) A is false.

1) Because if A is true, then B must also be true for the statement to be true

2) If A is false, the statement is vacuously true.

4.5:

These statements can also be proven to be identical using a truth table because the contrapositive carries the same truth value.

They are true when 1) A and B are true, 2) A is false

The explanations are the same as above.