Dependence everywhere: Copulas and its applications

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EdinbR talk
22 January 2026

- Lecturer in Statistics (FHEA) at the School of Mathematics and has been teaching mathematics students across different subjects.
- Outside of university teaching, I was a co-organiser of Technology Enhanced Mathematical Sciences Education (TEMSE) seminars in School of Math for 2023-2025, nowadays having the roles of Academic Cohort Lead (ACL), Generative AI TEMSE co-lead, EdinbR community and RSS Edinburgh local group member.
- Recently, Artificial Intelligence in Statistics Education (AI) SIG member within Researchers of Statistics Education Network (RoSE) Network, one of the Teaching and Learning scholar of SICSA, working group (WG) membership for a COST action project.
- Previously, postdoc positions at Padova University (2021) and KU Leuven (2020), after completing PhD in Statistics (2018) at Middle East Technical University.

More about my teaching and research from here: `https://oevkaya.netlify.app/`

A Historical Timeline

Progress of Copulas

1951; 1972
M. Fréchet,
G. Dall'Aglio

(Sklar, 1959)
introduce the
notion of a copula

(Schweizer & Sklar, 1974).
In the framework of the theory of Probabilistic Metric spaces

(Schweizer & Wollf, 1981)
Comptes Rendus de l'Académie des Sciences (1976) and Wolff's Ph.D. Dissertation

1997; 1999;2001;2009
-Harry Joe (Mult. Models & Mult. Dep. Concepts)
-Roger B. Nelsen (Intro. to Copulas)
-Paul Embrechts (1998 RiskLab Report)
-Li (Li Credit Portfoli Model)

At end of the nineties, the notion of copulas became increasingly popular.

Copulas (Sklar, 1959)

Copulas are mathematical tools that provide a flexible and powerful way to answer

- how to construct multivariate distributions with different margins ?
- how to separate the dependency structure from the margins ?

2-D case: Let F be a bivariate distribution with marginals F_1 and F_2
There exist two dimensional copula $C(.,.)$ s.t. $\forall(x_1, x_2) \in \mathbb{R}^2$

$$F(x_1, x_2) = C(F_1(x_1), F_2(x_2)) \tag{1}$$

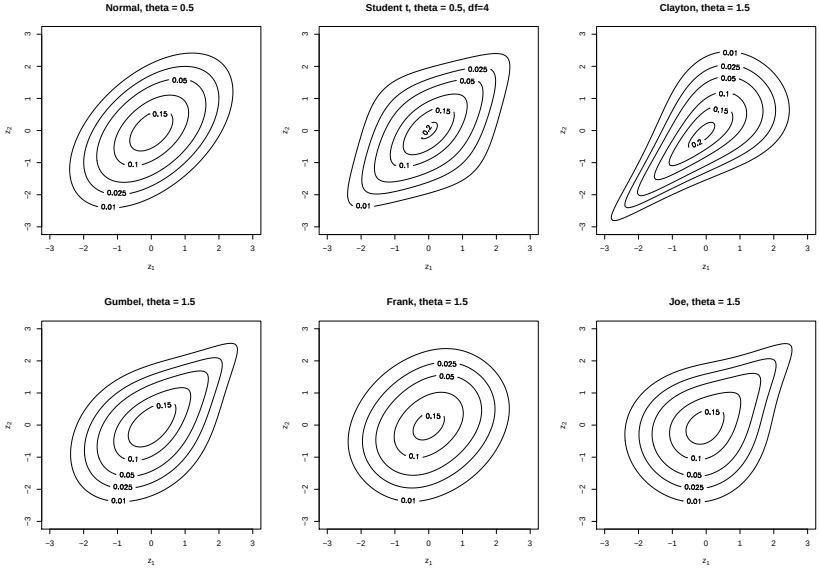
If F_1 and F_2 are continuous, the copula C is unique.

1. Elliptical Copula Families, derived from elliptical distributions
2. Archimedean Copula Families, based on archimedean generators

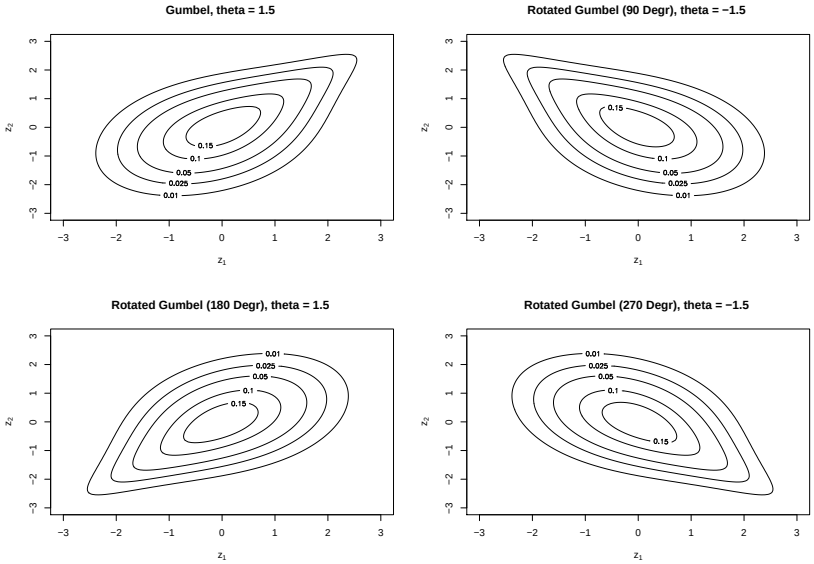
"This couples (hence the Latinate term copula) the individual probabilities associated with A and B to come up with a single number."

Various applications in; Financial Risk Management, Hydrology, Data Mining, Medical and Enviromental studies.

Different Tails



Rotated Families



Vine Copula:R-vine Copula

Definition 1: Regular Vine Distribution

The joint distribution F for the d dimensional random vector $X = (X_1, X_2, ...X_d)$ has a regular vine distribution, if we can specify the triplet (F, υ, B) such that:

1. Marginal distributions: $F = (F_1, F_2, ...F_d)$ is a vector of continuous invertible marginal distribution functions, representing the marginal distribution functions of the random variable $X_i, i = 1, 2, .., d$
2. Regular vine tree sequence: υ is an R-vine tree sequence on d elements.
3. Bivariate Copulas:The set $B = \{C_e | e \in E_i, i = 1, 2, ..., d-1\}$ where C_e is a symmetric bivariate copula with density. Here E_i is the edge set of tree T_i in the R-vine tree sequence υ.
4. Relationship between R-vine tree sequence υ and the set B of bivariate copulas: For each $e \in E_i, i = 1, 2, ..., d-1, e = \{a, b\}$, C_e is the copula associated with the conditional distribution of $X_{C_{e,a}}$ and $X_{C_{e,b}}$ given X_{D_e}=x_{D_e}. Further, $C_e(.,.)$ does not depend on the specific value of x_{D_e} .

Definition 2: Pair copula and copula density associated with edge e

We will denote the copula C_e corresponding to edge e by $C_{C_{e,a},C_{e,b};D_e}$ and the corresponding density by $c_{C_{e,a},C_{e,b};D_e}$, respectively. This copula is also called a pair copula.

Vine Copula:R-vine Copula

R-Vine Copula

- Every vine copula has a Regular Vine structure (R-vine).
- 2 specific subgenres of R-vine:Canonical Vine and Drawable Vine.

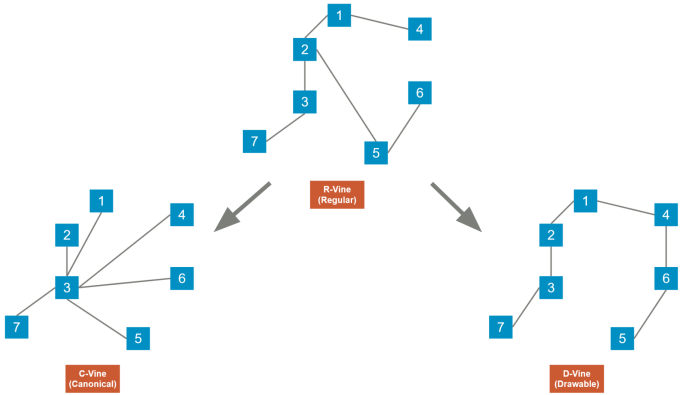
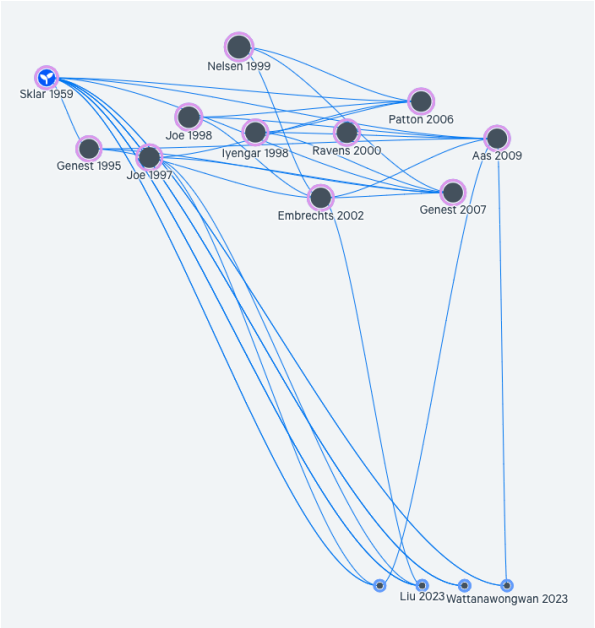


Figure: R-vine,C-vine,D-vine

Source:https://hudsonthames.org/a-practical-intro-to-vine-copula/

Pioneering Works



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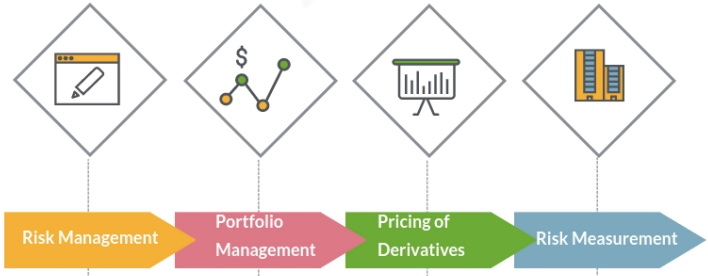
Major Areas of Appl. in Finance

-the lack of normality & dependence btw extreme values of assets



Book by McNeil, Frey & Embrechts

2005

ie.credit,market,operational risk, risk aggregation.

-stimulated by Basel II Accors

Paper by A. Patton

2004

dependence btw financial markets, different classes of assets & currencies

-Bivariate equity portfolio management

Book by Cheburini, Luciano & Vecchiato

2004

ie. pricing exotic options, CDOs, credit default swaps

Paper by Embrechts, Höing & Juri

2003

discussing VAR, expected shortfall and financial contagion

Misuse can be disaster

A formula in statistics, misunderstood and misused, has devastated the global economy

- In the years before 2008, it was hardly unthinkable that a math wizard like David X. Li might someday earn a Nobel Prize. Today, though, as dazed bankers, politicians, regulators, and investors survey the wreckage of the biggest financial meltdown since the Great Depression, Li is probably thankful he still has a job in finance at all
- For five years, Li's formula, known as a Gaussian copula function, looked like an unambiguously positive breakthrough, a piece of financial technology that allowed hugely complex risks to be modeled with more ease and accuracy than ever before
- And Li's Gaussian copula formula will go down in history as instrumental in causing the unfathomable losses that brought the world financial system to its knees.

The formula that killed Wall Street, by Felix Solmon, Significance, 2012

My Research Papers

PhD Thesis

Worked on finite mixture model with vine copulas.

COMMUNICATIONS IN STATISTICS: CASE STUDIES, DATA ANALYSIS AND APPLICATIONS
2019, VOL. 5, NO. 3, 245–260
https://doi.org/10.1080/23737484.2019.1635923

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Drought analysis using copula approach: a case study for Turkey

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ABSTRACT
Drought is one of the most drastic natural phenomenon, having devastating economic and environmental damages. Most studies related to drought monitoring consider quantitative indices, primarily based on univariate drought indicator. Besides, drought characteristics are important to monitor the impacts of drought on any specific area. The objective of this study is to introduce a new drought index using copula approach. Additionally, drought duration, severity and peak intensity are analyzed based on various drought indices. The dependence among characteristics are exploited and return period calculations are compared. For that purpose, monthly meteorological data for Aksehir Station in Konya are investigated.

ARTICLE HISTORY
Received 14 November 2018
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KEYWORDS
drought analysis; copulas; drought index; return period

1. Introduction

Droughts, common climatic extremes, which results from a lack of precipitation as compared with the normal amount and often spread across large time and spatial scales. Drought is the costliest natural hazard in the world according to its corresponding damage (Wilhite 2000). For each year, some part of the world experiences dry periods and suffers from the huge economical results from drought. Historical droughts have large impacts over the population, often leading to significant damages (50% of the mortality due to natural hazards), whereas almost 7% of economic losses have been linked to their occurrence worldwide (Nunez et al. 2011). Monitoring and understanding the effect of drought and its properties have vital importance in well-designed drought management plans.

As a 7% of the total area of Turkey, Konya Closed Basin is located the in Central Anatolia region and covers an area of 53, 850 km^2. Aksehir station, located in the south-west of the basin, is one of the stations faced with various dry periods (Selim 2013). Besides, Konya Closed Basin is an important area

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Color versions of one or more of the figures in the article can be found online at www.tandfonline.com/ucas.
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Sectoral electricity consumption modeling with D-vine quantile regression: The US electricity market case

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School of Mathematics, University of Edinburgh, Edinburgh, UK; Mathematics Department, TU Kaiserslautern, Kaiserslautern, Germany; Industrial Engineering Department, Gazi University, Ankara, Turkey

ABSTRACT
Efficient electricity demand planning is crucial for energy market actors. However, it is difficult as a consequence of climate change. We aim at investigating how climate variables (heating and cooling degree days) may affect electricity demand. By examining electricity consumption in various US sectors, we explore this relationship using parametric and non-parametric D-vine quantile regression models that exploits the dependence between covariates and allows sequential covariate selection. The results are compared against the classical linear quantile regression. We find a positive effect of the climatic variables on electricity consumption that is as heating and cooling degree days increase electricity demand rises in all sectors, and cooling need has a greater impact than heating need. Evidence suggests that residential and commercial electricity consumptions are affected the most, while industrial and transport sector consumptions are less sensitive. The D-vine quantile regression performs better than the linear quantile regression for almost all sectors.

KEYWORDS
Climate change; cooling degree days (CDD); D-vine quantile regression; electricity demand forecasting; heating degree days (HDD); linear quantile regression

1. Introduction

Efficient operation of an electricity market requires a forecast of future electricity demand to prepare short-, medium-, and long-term production plans. A critical component to these plans is an understanding of the customer demand to predict future usage in a way as to not over or under supply power since electricity is not storable. Factors that affect electricity consumption have been examined in various studies (see, e.g., Elkamel et al. 2020; Zhang et al. 2020). These factors comprise population, income, urbanization, demographic structure, price of fuel, and technological advances as well as those associated with climate change now identified as having the most significant influence (Tan et al. 2020).

Climate change poses many challenges to electricity markets. On the supply side, the growing sector of renewable means of generating electricity depends on a variety of weather conditions (precipitation, wind, air temperature, etc.) that are prone to extreme events as the global temperature rises. Furthermore, changes in these variables also cause considerable variations in electricity demand. For example, it is known that energy demand versus air temperature form a "U" shaped relationship due to heating buildings when the air is cold in the winter and the use of air conditioning units in the summer. However, as the climate warms, higher temperatures mean electricity usage for cooling increases in summer, whilst also reducing heating needs in winter (De Cian and Wing 2019). Hou et al. (2014) predicted this significant effect on electricity demand due to global warming when forecasting electricity consumption for Shanghai between 2011 and 2050. Therefore, it is essential to distinguish how demand in electricity markets is likely to shift due to changes in the temperature.

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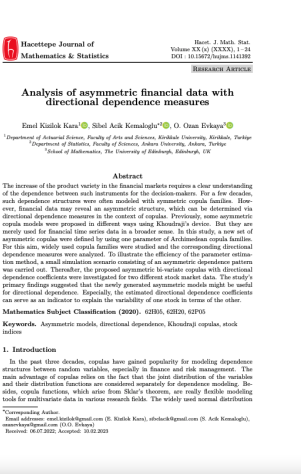

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Volume XX (x) (XXXX), 1 – 24
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Research Article

Analysis of asymmetric financial data with directional dependence measures

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Department of Statistics, Faculty of Sciences, Ankara University, Ankara, Turkiye
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Abstract
The increase of the product variety in the financial markets requires a clear understanding of the dependence between such instruments for the decision-makers. For a few decades, such dependence structures were often modeled with symmetric copula families. However, financial data may reveal an asymmetric structure, which can be determined via directional dependence measures in the context of copulas. Previously, some asymmetric copula models were proposed in different ways using Khoudraji's device. But they are merely used for financial time series data in a broader sense. In this study, a new set of asymmetric copulas were defined by using one parameter of Archimedean copula families. For this aim, widely used copula families were studied and the corresponding directional dependence measures were analyzed. To illustrate the efficiency of the parameter estimation method, a small simulation scenario consisting of an asymmetric dependence pattern was carried out. Thereafter, the proposed asymmetric bi-variate copulas with directional dependence coefficients were investigated for two different stock market data. The study's primary findings suggested that the newly generated asymmetric models might be useful for directional dependence. Especially, the estimated directional dependence coefficients can serve as an indicator to explain the variability of one stock in terms of the other.

Mathematics Subject Classification (2020). 62H05, 62H20, 62P05

Keywords. Asymmetric models, directional dependence, Khoudraji copulas, stock indices

1. Introduction

In the past three decades, copulas have gained popularity for modeling dependence structures between random variables, especially in finance and risk management. The main advantage of copulas relies on the fact that the joint distribution of the variables and their distribution functions are considered separately for dependence modeling. Besides, copula functions, which arise from Sklar's theorem, are really flexible modeling tools for multivariate data in various research fields. The widely used normal distribution

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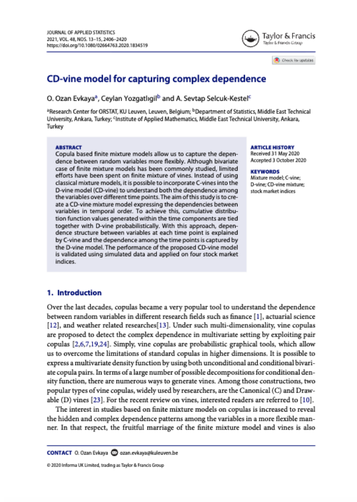
My Research Papers



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CD-vine model for capturing complex dependence

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ABSTRACT
Copula based finite mixture models allow us to capture the dependence between random variables more flexibly. Although bivariate case of finite mixture models has been commonly studied, limited efforts have been spent on finite mixture of vines. Instead of using classical mixture models, it is possible to incorporate C-vines into the D-vine model (CD-vine) to understand both the dependence among the variables over different time points. The aim of this study is to create a CD-vine mixture model expressing the dependencies between variables in temporal order. To achieve this, cumulative distribution function values generated within the time components are tied together with D-vine probabilistically. With this approach, dependence structure between variables at each time point is explained by C-vine and the dependence among the time points is captured by the D-vine model. The performance of the proposed CD-vine model is validated using simulated data and applied on four stock market indices.

ARTICLE HISTORY
Received 31 May 2020
Accepted 3 October 2020

KEYWORDS
Mixture model; C-vine; D-vine; CD-vine mixture; stock market indices

1. Introduction

Over the last decades, copulas became a very popular tool to understand the dependence between random variables in different research fields such as finance [1], actuarial science [12], and weather related researches[13]. Under such multi-dimensionality, vine copulas are proposed to detect the complex dependence in multivariate setting by exploiting pair copulas [2,6,7,19,24]. Simply, vine copulas are probabilistic graphical tools, which allow us to overcome the limitations of standard copulas in higher dimensions. It is possible to express a multivariate density function by using both unconditional and conditional bivariate copula pairs. In terms of a large number of possible decompositions for conditional density function, there are numerous ways to generate vines. Among those constructions, two popular types of vine copulas, widely used by researchers, are the Canonical (C) and Drawable (D) vines [23]. For the recent review on vines, interested readers are referred to [10].

The interest in studies based on finite mixture models on copulas is increased to reveal the hidden and complex dependence patterns among the variables in a more flexible manner. In that respect, the fruitful marriage of the finite mixture model and vines is also

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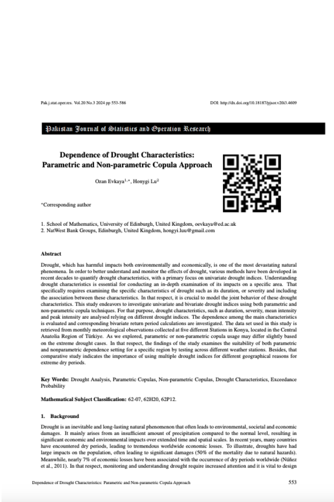

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Pakistan Journal of Statistics and Operation Research

Dependence of Drought Characteristics: Parametric and Non-parametric Copula Approach

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Abstract

Drought, which has harmful impacts both environmentally and economically, is one of the most devastating natural phenomena. In order to better understand and monitor the effects of drought, various methods have been developed in recent decades to quantify drought characteristics, with a primary focus on univariate drought indices. Understanding drought characteristics is essential for conducting an in-depth examination of its impacts on a specific area. That specifically requires examining the specific characteristics of drought such as its duration, or severity and including the association between these characteristics. In that respect, it is crucial to model the joint behavior of these drought characteristics. This study endeavors to investigate univariate and bivariate drought indices using both parametric and non-parametric copula techniques. For that purpose, drought characteristics, such as duration, severity, mean intensity and peak intensity are analysed relying on different drought indices. The dependence among the main characteristics is evaluated and corresponding bivariate return period calculations are investigated. The data set used in this study is retrieved from monthly meteorological observations collected at five different Stations in Konya, located in the Central Anatolia Region of Türkiye. As we explored, parametric or non-parametric copula usage may differ slightly based on the extreme drought cases. In that respect, the findings of the study examines the suitability of both parametric and nonparametric dependence setting for a specific region by testing across different weather stations. Besides, that comparative study indicates the importance of using multiple drought indices for different geographical reasons for extreme dry periods.

Key Words: Drought Analysis, Parametric Copulas, Non-parametric Copulas, Drought Characteristics, Exceedance Probability

Mathematical Subject Classification: 62-07, 62H20, 62P12.

1. Background

Drought is an inevitable and long-lasting natural phenomenon that often leads to environmental, societal and economic damages. It mainly arises from an insufficient amount of precipitation compared to the normal level, resulting in significant economic and environmental impacts over extended time and spatial scales. In recent years, many countries have encountered dry periods, leading to tremendous worldwide economic losses. To illustrate, droughts have had large impacts on the population, often leading to significant damages (50% of the mortality due to natural hazards). Meanwhile, nearly 7% of economic losses have been associated with the occurrence of dry periods worldwide (Núñez et al., 2011). In that respect, monitoring and understanding drought require increased attention and it is vital to design

Dependence of Drought Characteristics: Parametric and Non-parametric Copula Approach 553

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Vine Copula Approach to Understand the Financial Dependence of the Istanbul Stock Exchange Index

Ozan Evkaya[1] · İsmail Gür[2] · Bükre Yıldırım Külekci[3,4] · Gülden Poyraz[5]

Accepted: 21 December 2023 / Published online: 6 February 2024

Abstract
Recently, the complex dependence patterns among various stocks gained more importance. Measuring the dependency structure is critical for investors to manage their portfolio risks. Since the global financial crisis, researchers have been more interested in studying the dynamics of dependency within stock markets by using novel methodologies. This study aims to investigate a Regular-Vine copula approach to estimate the interdependence structure of the Istanbul Stock Exchange index (ISE100). For this purpose, we consider 32 stocks related to 6 sectors belonging to ISE100. To reflect the time-varying impacts of the 2008–2009 global financial crisis, the dependence analysis is conducted over pre-, during-, and post-global financial crisis periods. Portfolio analysis is considered via a rolling window approach to capture the changes in the dependence. We compare the Regular-Vine-based generalized autoregressive conditional heteroskedasticity (GARCH) against the conventional GARCH model with different innovations. Value at risk and expected shortfall risk measures are used to validate the models. Additionally, for the constructed portfolios, return performance is summarized using both Sharpe and Sortino ratios. To test the ability of the considered Regular-Vine approach on ISE100, another evaluation has been done during the COVID-19 pandemic crisis with various parameter settings. The main findings across different risky periods illustrate the suitability of using the Regular-vine GARCH approach to model the complex dependence among stocks in emerging market conditions.

Keywords R-Vine copula · Global financial crisis · Istanbul stock exchange · Value-at-risk · Expected shortfall

1 Introduction

In the past decade, financial markets experienced many crises due to underestimation of risk (MacKenzie & Spears, 2014; Jickling, 2009). Since the global financial crisis (GFC), researchers and practitioners have increasingly sought to develop new

Extended author information available on the last page of the article

Springer

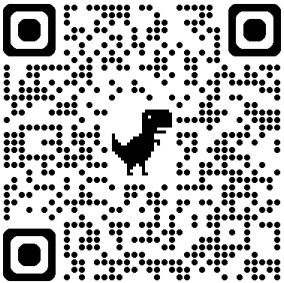
Ongoing Research

- Cluster-specific ranking and variable importance for Scottish regional deprivation via vine mixtures (joint work-under publication)
- Semi-parametric approach for copula directed acyclic graphs (joint work with Gerda C. and Irene G. - under preparation)
- Dependency Modeling of Global Liquidity Conditions on Stock Markets During the Covid-19 Pandemic (joint work-under preparation)

Student Projects

- Exploiting clustering with vine copulas to examine the Scottish Multiple Deprivation Index 2020, 2025-2026
- Drought Analysis of UK by exploiting the benefits of copulas, 2025-2026
- Clustering Dementia Characteristics Using Vine Copula Mixture Models, 2024-2025
- Turkey Stock market Dependencies: Study based on wavelet-vine copula approach, 2023-2024
- Wavelet based copula approach for modeling market risk: Turkey ISE-100 Case Study, SoR MSc Thesis, Summer 2024

How about the computational aspect?



https://github.com/oevkaya/EdinbR-talks-20Jan26