

Dependence everywhere: Copulas and its applications

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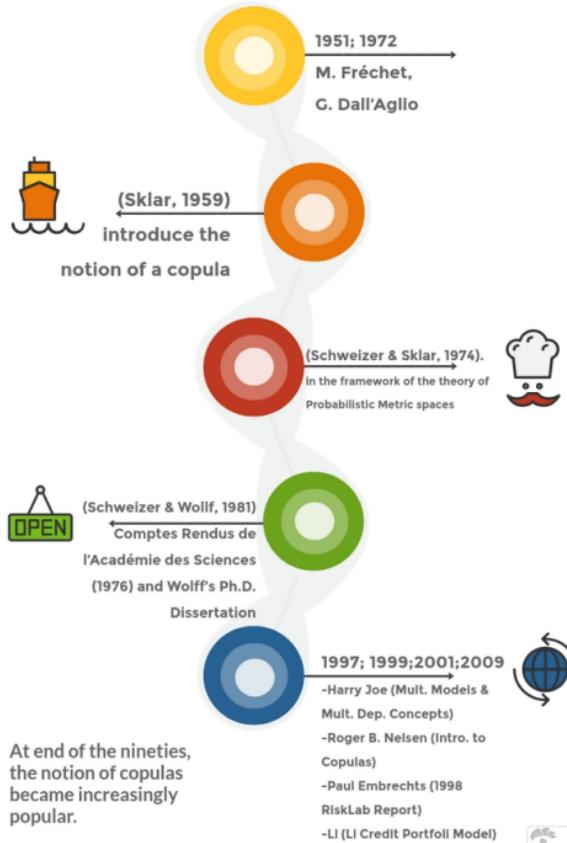
EdinbR talk
22 January 2026

- Lecturer in Statistics (FHEA) at the School of Mathematics and has been teaching mathematics students across different subjects.
- Outside of university teaching, I was a co-organiser of Technology Enhanced Mathematical Sciences Education (TEMSE) seminars in School of Math for 2023-2025, nowadays having the roles of Academic Cohort Lead (ACL), Generative AI TEMSE co-lead, EdinbR community and RSS Edinburgh local group member.
- Recently, Artificial Intelligence in Statistics Education (AI) SIG member within Researchers of Statistics Education Network (RoSE) Network, one of the Teaching and Learning scholar of SICSA, working group (WG) membership for a COST action project.
- Previously, postdoc positions at Padova University (2021) and KU Leuven (2020), after completing PhD in Statistics (2018) at Middle East Technical University.

More about my teaching and research from here: <https://oevkaya.netlify.app/>

A Historical Timeline

Progress of Copulas



Copulas (Sklar, 1959)

Copulas are mathematical tools that provide a flexible and powerful way to answer

- how to construct multivariate distributions with different margins ?
- how to separate the dependency structure from the margins ?

2-D case: Let F be a bivariate distribution with marginals F_1 and F_2

There exist two dimensional copula $C(.,.)$ s.t. $\forall(x_1, x_2) \in \mathbb{R}^2$

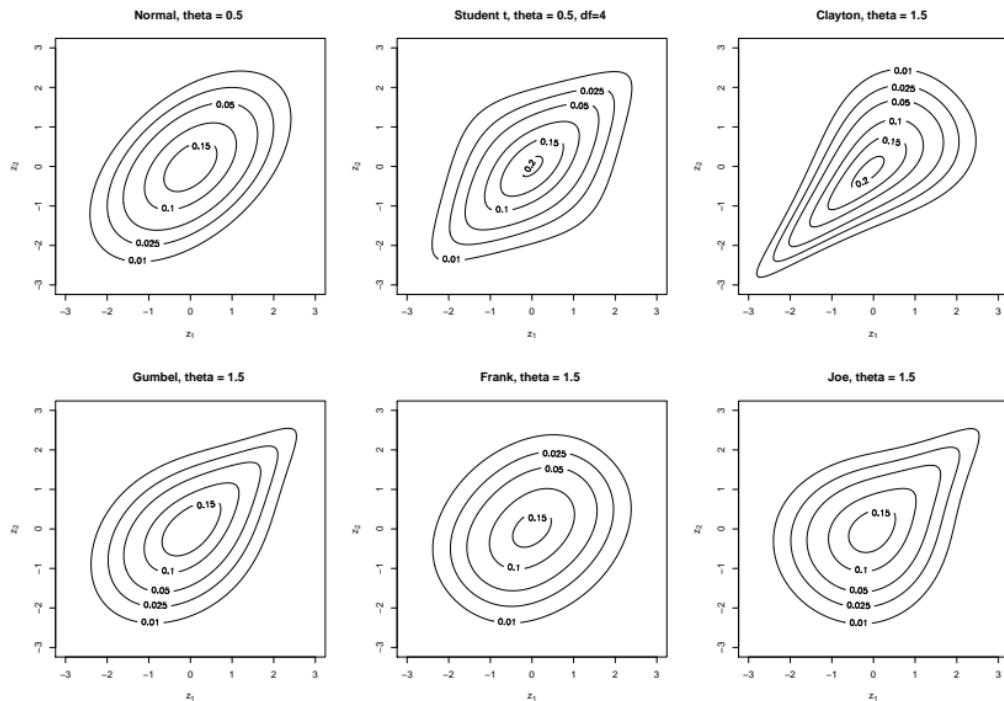
$$F(x_1, x_2) = C(F_1(x_1), F_2(x_2)) \quad (1)$$

If F_1 and F_2 are continuous, the copula C is unique.

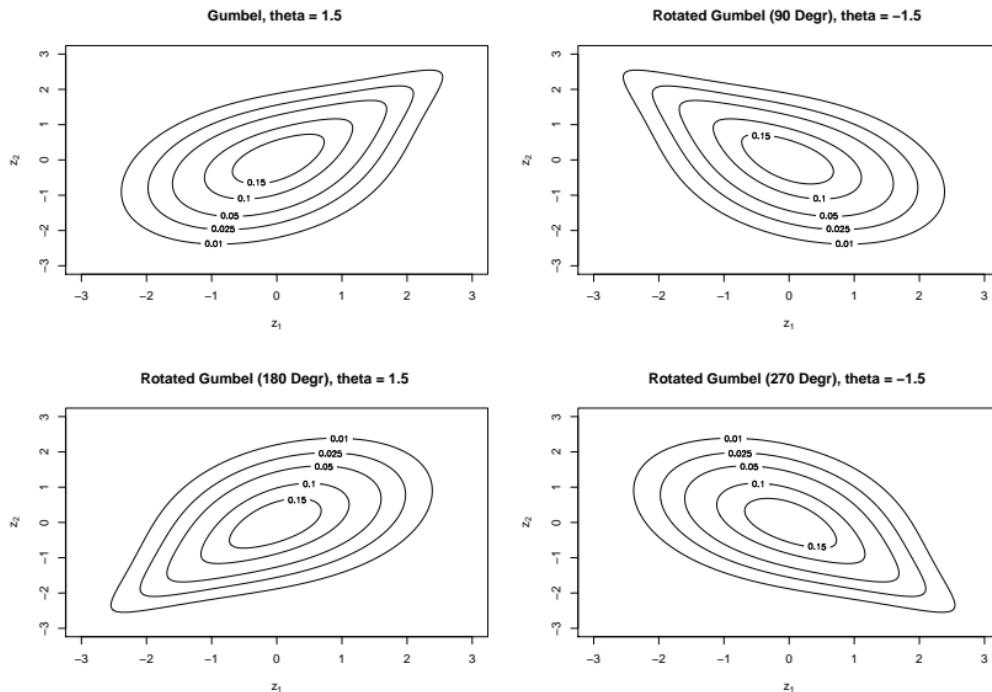
- ① Elliptical Copula Families, derived from elliptical distributions
- ② Archimedean Copula Families, based on archimedean generators

Various applications in; Financial Risk Management, Hydrology, Data Mining, Medical and Environmental studies.

Different Tails



Rotated Families



Vine Copula:R-vine Copula

Definition 1: Regular Vine Distribution

The joint distribution F for the d dimensional random vector $X = (X_1, X_2, \dots, X_d)$ has a regular vine distribution, if we can specify the triplet (F, v, B) such that:

- ① Marginal distributions: $F = (F_1, F_2, \dots, F_d)$ is a vector of continuous invertible marginal distribution functions, representing the marginal distribution functions of the random variable $X_i, i = 1, 2, \dots, d$
- ② Regular vine tree sequence: v is an R-vine tree sequence on d elements.
- ③ Bivariate Copulas: The set $B = \{C_e | e \in E_i, i = 1, 2, \dots, d - 1\}$ where C_e is a symmetric bivariate copula with density. Here E_i is the edge set of tree T_i in the R-vine tree sequence v .
- ④ Relationship between R-vine tree sequence v and the set B of bivariate copulas: For each $e \in E_i, i = 1, 2, \dots, d - 1, e = \{a, b\}$, C_e is the copula associated with the conditional distribution of $X_{C_{e,a}}$ and $X_{C_{e,b}}$ given $X_{D_e} = x_{D_e}$. Further, $C_e(\cdot, \cdot)$ does not depend on the specific value of x_{D_e} .

Definition 2: Pair copula and copula density associated with edge e

We will denote the copula C_e corresponding to edge e by $C_{C_{e,a}, C_{e,b}; D_e}$ and the corresponding density by $c_{C_{e,a}, C_{e,b}; D_e}$, respectively. This copula is also called a pair copula.

Vine Copula:R-vine Copula

R-Vine Copula

- Every vine copula has a Regular Vine structure (R-vine).
- 2 specific subgenres of R-vine:Canonical Vine and Drawable Vine.

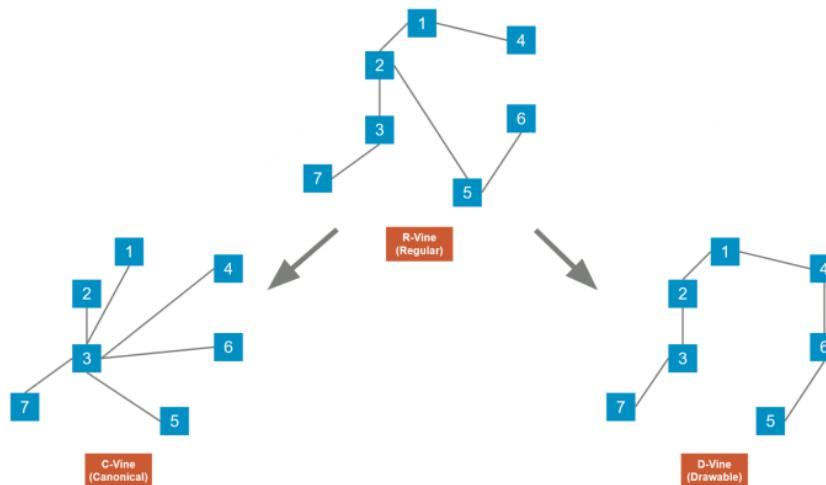


Figure: R-vine,C-vine,D-vine

Pioneering Works

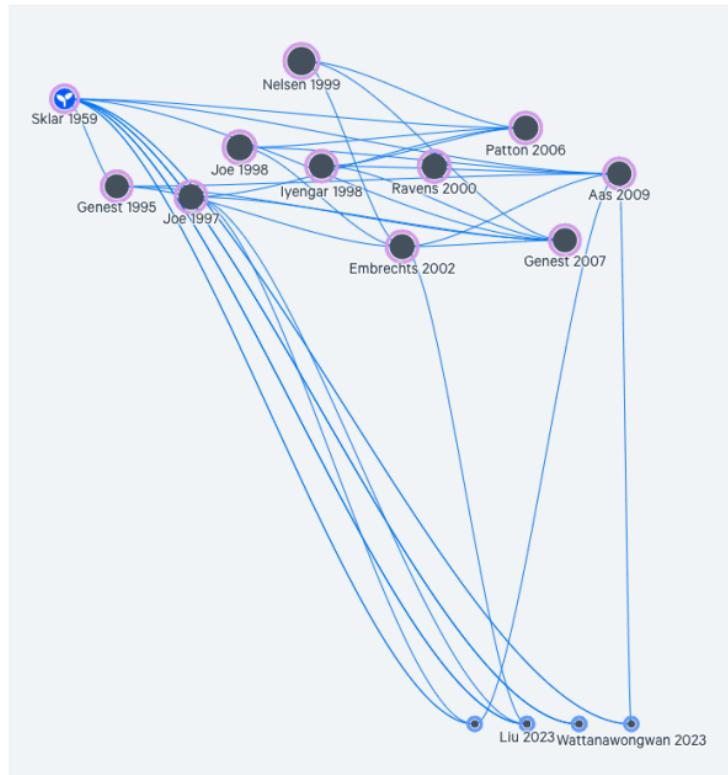
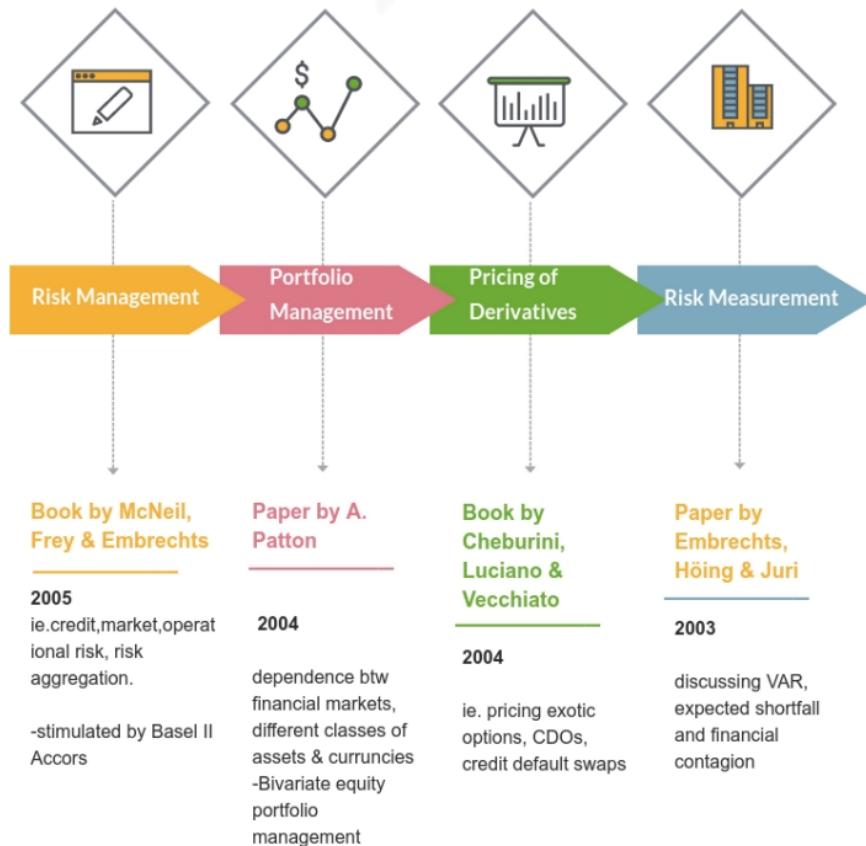


Image Credit: Litmaps <https://app.litmaps.com/>

Major Areas of Appl. in Finance

-the lack of normality & dependence btw extreme values of assets



My Research Papers

PhD Thesis

Worked on finite mixture model with vine copulas.

COMMUNICATIONS IN STATISTICS: CASE STUDIES, DATA ANALYSIS AND APPLICATIONS
2019, VOL. 8, NO. 2, 249–260
https://doi.org/10.1080/23788708.2018.1405823



Check for updates

Drought analysis using copula approach: a case study for Turkey

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ABSTRACT

Drought is one of the most drastic natural phenomena, leaving devastating economic and environmental damages. Most studies related to drought monitoring consider quantitative indices, primarily based on rainfall reduction. Being drought characteristics are important to monitor the impacts of drought on any specific area. The objective of this study is to introduce a new approach to monitor drought characteristics. Duration, severity and peak intensity are analyzed based on various drought indices. The dependence among characteristics are analyzed by using copula approach. The results show that for that purpose, monthly meteorological data for Aksheir Station in Konya are investigated.

ARTICLE HISTORY

Received: 10 January 2018
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KEYWORDS
drought analysis, copula, duration, severity, peak intensity, rainfall period

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Sectoral electricity consumption modeling with D-vine quantile regression: The US electricity market case

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ABSTRACT

Efficient electricity demand planning is crucial for energy market actors. However, it is also a concern of climate change. We aim at investigating how climate variables (heating and cooling degree days) may affect electricity demand. By examining electricity consumption in various US sectors, we propose a D-vine quantile regression models that exploits the dependence between covariates. The proposed model is compared with the linear quantile regression against the classical linear quantile regression. We find a positive effect of the climatic variables on electricity consumption that is as heating and cooling degree days. The results show that residential and commercial sectors' cooling need has a greater impact than heating need. Evidence suggests that residential and commercial electricity consumption increased the most during the winter and transition sectors' consumption are less sensitive. The D-vine quantile regression performs better than the linear quantile regression for almost all sectors.

KEYWORDS
Climate change, cooling degree days, D-vine quantile regression, electricity consumption, heating degree days (HDD), linear quantile regression

1. Introduction

Droughts, common climatic extremes, which results from a lack of precipitation as compared with the normal amount and often spans over large time and spatial scales, is the common natural hazard that causes damage leading to its corresponding damage (Wilhite 2000). For each year, some part of the world experiences dry periods and suffers from the huge economical results from drought. Historical droughts have large impacts over the population, often leading to significant damages (50% of the mortality due to natural hazards), whereas almost 7% of economic losses have been linked to their occurrence worldwide (Nunez et al. 2011). Monitoring and understanding the effect of drought and its properties have vital importance in developing drought management plans.

As a 7% of the total area of Turkey, Konya Closed Basin is located in the Central Anatolia region and covers an area of 53,850 km². Aksheir station, located in the south-west of the basin, is one of the stations faced with various dry periods (Selim 2013). Besides, Konya Closed Basin is an important area

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Color version of one or more of the figures in this article can be found online at www.tandfonline.com/tcas.

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1. Introduction

Efficient operation of an electricity market requires a forecast of future electricity demand to prepare short-, medium-, and long-term production plans. A critical component to these plans is an understanding of the customer demand to predict future usage in a way to not over or under supply power needs of the system. Factors that influence electricity consumption are numerous and varied in various studies (see, e.g., Elgamal et al. 2020; Zhang et al. 2020). These factors comprise population, income, urbanization, demographic structure, price of fuel, and technological advances as well as those associated with climate change identified as having most significant influence (Fan et al. 2020).

Climate change poses many challenges to electricity markets. On the supply side, the growing sector of renewable means of generating electricity depends on a variety of weather conditions (precipitation, wind, etc.) and thus faces significant challenges in meeting electricity demand. Furthermore, changes in these variables also cause considerable variations in electricity demand. For example, it is known that energy demand versus air temperature follows a "U" shaped relationship due to heating buildings when the air is cold in the winter and the use of air conditioning units in the summer. This relationship is also observed in electricity consumption. The increase in cooling increases in summer, whilst also reducing heating needs in winter (De Canio and Wing 2019). Hos et al. (2014) predicted this significant effect on electricity demand due to global warming when forecasting electricity consumption for Shanghai between 2011 and 2050. Therefore, it is essential to distinguish how demand in electricity markets is likely to shift due to changes in the temperature.

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RESEARCH ARTICLE

Analysis of asymmetric financial data with directional dependence measures

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Abstract

The increase of the product variety in the financial markets requires a clear understanding of the dependence structure with inherent meaning for the decision-makers. For a few decades, such dependence structures are often measured by correlation coefficients. However, financial data may reveal an asymmetric structure, which can be determined via directional dependence measures in the context of copulas. Previously, some symmetric copulae families were used to model the dependence structure of financial data. But they are merely used for financial time series data in a broader sense. In this study, a new set of asymmetric copulae were defined by using one parameter of Archimedean copulae families. For the first time, the directional dependence measures and the asymmetric directional dependence measures were analyzed. To illustrate the efficacy of the parameter estimation method, a small simulation scenario consisting of an asymmetric dependence pattern was considered. Then, the directional dependence measures and the asymmetric directional dependence coefficients were investigated for two different stock market data. The study's primary findings suggested that the newly generated asymmetric models might be useful for financial time series data. The asymmetric directional dependence coefficients can serve as an indicator to explain the variability of one stock in terms of the other.

Mathematics Subject Classification (2020): G1205, G1210, G1205

Keywords: Asymmetric models, directional dependence, Khondrajli copula, stock indices

1. Introduction

In the past three decades, copulas have gained popularity for modeling dependence structures between random variables, especially in finance and risk management. The main advantage of copulas relies on the fact that the joint distribution of the variables and their distributions are separated by the copula function. Moreover, copulas, also called copula functions, which arise from Sklar's theorem, are really flexible modeling tools for multivariate data in various research fields. The widely used normal distribution

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My Research Papers

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Check for updates

CD-vine model for capturing complex dependence

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Abstract

Copula based finite mixture models allow us to capture the dependence between random variables more flexibly. Although some cases of finite mixture models have been studied, limited efforts have been spent on finite mixture of vines. Instead of using classical mixture models, it is possible to use vine copulas into the (D-vine) or CD-vine models both for the dependence among the variables over different time points. The aim of this study is to create a new model, called CD-vine model, which captures the dependence variables in temporal order. To achieve this, cumulative distribution function values generated within the time components are tied together by the vine copula. The dependence structure among the dependence structure between variables at each time point is explained by a vine and the dependence among the time points is captured by the dependence structure between the vines. The proposed model is validated using simulated data and applied on four stock market indices.

1. Introduction

Over the last decades, copulae became a very popular tool to understand the dependence between random variables in different research fields such as finance [1], actuarial science [12], and weather related researches[13]. Under such multi-dimensionality, vine copulas are proposed to detect the complex dependence in multivariate setting by exploiting pair copulas [2,6,7,19,24]. Simply, vine copulas are probabilistic graphical tools, which allow us to overcome the limitation of standard copulae in higher dimensions. It is possible to express the joint density function by using both parametric and non-parametric vine copulae. In terms of a large number of possible decompositions for conditional dependency function, there are numerous ways to generate vines. Among those constructions, two popular types of vine copulas, widely used by researchers, are the Canonical (C) and Drawable (D) vines [23]. For the recent review on vines, interested readers are referred to [16].

The interest in studies based on finite mixture models on copulas is increased to reveal the hidden and complex dependence patterns among the variables in a more flexible manner. In that respect, the fruitful marriage of the finite mixture model and vines is also

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KEYWORDS
Copula-based model; C-vine;
D-vine; CD-vine mixture;
stock market indices

Dependence of Drought Characteristics:
Parametric and Non-parametric Copula Approach

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Abstract

Drought, which has harmful impacts both environmentally and economically, is one of the most devastating natural phenomena. Therefore, understanding the characteristics of drought is important for decision making. In this paper, we focus on the short-term droughts to quantify drought characteristics, with a primary focus on intra-seasonal drought indices. Understanding drought characteristics is essential for conducting an in-depth examination of its impacts on a specific area. That is why, in this study, we propose a new approach to measure the dependence structure between the variables reflecting the association between these characteristics. In that respect, it is crucial to model the joint behavior of these drought characteristics. For this purpose, we propose a new approach to measure the dependence structure between these characteristics by using parametric and non-parametric copula techniques. For that purpose, drought characteristics, such as duration, severity, mean intensity and frequency are analyzed relying on time series data. The dependence structure between these characteristics is evaluated and constructed using various period selection methods. The data set used in this study is retrieved from monthly meteorological observations collected at five different stations in Korea, located in the Central Asian Plateau. The results show that the proposed approach is more suitable than the traditional approach for droughts as the extreme drought cases. In that respect, the findings of the study examines the suitability of both parametric and non-parametric copula models for drought characteristics. The results obtained from the proposed approach are compared with the results obtained by the traditional approach. The results of the comparative study indicate the importance of using multiple drought indices for different geographical regions for extreme dry periods.

Key Words: Drought Analysis, Parametric Copula, Non-parametric Copula, Drought Characteristics, Ensemble Prediction

Mathematical Subject Classification: 62F07, 62H20, 62P10

1. Background

Drought is an inevitable and long-lasting natural phenomenon that often leads to environmental, societal and economic damages. It mostly arises from an insufficient amount of precipitation compared to the normal level, resulting in significant water scarcity. Droughts are considered as one of the most severe natural disasters that can cause severe damage to agriculture, environment and economy. Droughts have caused significant damages and have encouraged dry periods, leading to tremendous worldwide economic losses. To illustrate, droughts have had huge impacts on the population, often leading to significant damages (50% of the mortality due to natural hazards). Moreover, the effects of droughts are more serious in developing countries. According to the World Bank (Gholz et al., 2011), In that respect, monitoring and understanding drought requires increased attention and it is vital to design

Dependence of Drought Characteristics: Parametric and Non-parametric Copula Approach

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Dependence everywhere: Copulas and its applications

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Vine Copula Approach to Understand the Financial Dependence of the Istanbul Stock Exchange Index

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Abstract

Recently, the complex dependence patterns among various stocks gained more importance. Monitoring the dependency structure is critical for investors to manage their portfolio risks. Since the global financial crisis, researchers have become more interested in studying the dynamics of dependency within stock markets by using novel methodologies. This study aims to investigate a Regular-Vine copula approach to estimate the dependency structure of the Istanbul Stock Exchange index (ISE100). For this purpose, we consider 32 stocks related to 6 sectors according to ISE100. To reflect the time-varying impacts of the 2008–2009 global financial crisis, the dependence analysis is conducted over pre-, during-, and post-global financial crisis periods. Portfolio analysis is considered via a rolling window approach to capture the changes in the dependency structure. We compare the Regular-Vine-based generalized autoregressive conditional heteroskedasticity (GARCH) model with the traditional GARCH model with different innovations. Value at risk and expected shortfall risk measures are used to validate the models. Additionally, for the constructed portfolios, return performance is summarized with both Sharpe and Sortino ratios. To test the ability of the proposed Regular-Vine approach on the ISE100, another evaluation is conducted during the COVID-19 pandemic with various parameter settings. The main findings across different risky periods illustrate the suitability of using the Regular-Vine GARCH approach to model the complex dependence among stocks in emerging market conditions.

Keywords: R-Vine copula - Global financial crisis - Istanbul stock exchange - Value-at-risk - Expected shortfall

1 Introduction

In the past decade, financial markets experienced many crises due to underestimation of risk (MacKenzie & Spears, 2014; Jickling, 2009). Since the global financial crisis (GFC), researchers and practitioners have increasingly sought to develop new

Extended author information available on the last page of the article



Ongoing Research

- Cluster-specific ranking and variable importance for Scottish regional deprivation via vine mixtures (joint work-under publication)
- Semi-parametric approach for copula directed acyclic graphs (joint work with Gerda C. and Irene G. - under preparation)
- Dependency Modeling of Global Liquidity Conditions on Stock Markets During the Covid-19 Pandemic (joint work-under preparation)

Student Projects

- Exploiting clustering with vine copulas to examine the Scottish Multiple Deprivation Index 2020, 2025-2026
- Drought Analysis of UK by exploiting the benefits of copulas, 2025-2026
- Clustering Dementia Characteristics Using Vine Copula Mixture Models, 2024-2025
- Turkey Stock market Dependencies: Study based on wavelet-vine copula approach, 2023-2024
- Wavelet based copula approach for modeling market risk: Turkey ISE-100 Case Study, SoR MSc Thesis, Summer 2024

The End - New Start



How about the computational aspect?