# Higher Education Teaching and Learning Series 2024/25

**Incorporating A.I. for Continuous Assessment**

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# Layout

1. Teaching Mathematics to non-mathematicians.
2. Assessment with A.I.
3. Student Results & Reflections.
4. Next Steps.

**Question.** How do we analyse a collection of random variables when their distributions are unknown?

**1. First Order Second Moment (FOSM).**

$$E[Y] = \mu_Y = g(\mu_X) \quad \text{and} \quad \mathrm{Var}[Y] = \sigma_Y^2 \approx \sigma_X^2 \left(\frac{dg}{dX}\right)^2_{x=\mu_Y}$$

**2. Second Order Second Moment (SOSM).**

For further accuracy, we would include the second order term

$$E[Y] = \mu_Y + \frac{1}{2}\sigma_X^2 \left(\frac{d^2g}{dX^2}\right)_{x=\mu_Y}$$

**3. Monte Carlo Simulations.**

**4. A.I.**

# Example - FOSM, SOSM & Monte Carlo

Analyse the probability of failure of

$$X = \frac{X_1 X_3}{X_2}$$

given

$$X_1 \sim N(500, 75^2) \quad X_2 \sim N(600, 120^2) \quad \text{and} \quad X_3 \sim N(700, 210^2).$$

# Solutions - FOSM and SOSM

**FOSM Method.**

$$\mu_X \approx \frac{\mu_{X_1}\mu_{X_3}}{\mu_{X_2}} = \frac{500 \times 700}{600} = 583.$$

$$\begin{aligned}
\sigma_X^2 &= \sigma_{X_1}^2 \left(\frac{\partial X}{\partial X_1}\right)^2 + \sigma_{X_2}^2 \left(\frac{\partial X}{\partial X_2}\right)^2 + \sigma_{X_3}^2 \left(\frac{\partial X}{\partial X_3}\right)^2 \\
&= \sigma_{X_1}^2 \left(\frac{X_3}{X_2}\right)^2 + \sigma_{X_2}^2 \left(-\frac{X_1 X_3}{X_2^2}\right)^2 + \sigma_{X_3}^2 \left(\frac{X_1}{X_2}\right)^2 \\
&= 51892.36 \Rightarrow \sigma_X \approx 227.8.
\end{aligned}$$

**SOSM method.**

$$E[X] = \mu_X + \frac{1}{2}\sigma_X^2 \left(\frac{d^2X}{dX_i^2}\right)_{x=\mu_X} = 583 + \frac{1}{2}\sigma_{X_2}^2 \left(\frac{\partial^2 X}{\partial X_2^2}\right) = 606.3.$$

# Solutions - Monte Carlo

```
>> %This code is to run MC simulations to solve X=(X1X3)/(X2) where each Xi is
normally distributed;
>> %First we declare the means and standard deviations;
>> mu_1=500;
>> sd_1=75;
>> mu_2=600;
>> sd_2=120;
>> mu_3=700;
>> sd_3=210;
>> %We define how many simulations we are going to run (at least 10,000);
>> n=1000000;
>> %Now we define the normal distributions;
>> x1=normrnd(mu_1,sd_1,n,1);
>> x2=normrnd(mu_2,sd_2,n,1);
>> x3=normrnd(mu_3,sd_3,n,1);
>> %Finally, we will define our problem function;
>> X=(x1.*x3)./(x2);
>> %Results;
>> x_bar=mean(X);
>> x_sd=std(X);
```

| Name | Value |
|---|---|
| mu_1 | 500 |
| mu_2 | 600 |
| mu_3 | 700 |
| n | 1000000 |
| sd_1 | 75 |
| sd_2 | 120 |
| sd_3 | 210 |
| X | *1000000x1 double* |
| x1 | *1000000x1 double* |
| x2 | *1000000x1 double* |
| x3 | *1000000x1 double* |
| x_bar | 610.3309 |
| x_sd | 263.1841 |

# Reliability Index $\beta$

The reliability index, $\beta$, is the number of standard deviations the mean is away from failure. That is

$$\beta = \frac{\mu}{\sigma}$$

For exact normally distributed variables, the probability of failure, $F$, can be approximated as

$$\mathbb{P}(F) \approx 1 - \phi(\beta)$$

Notice that as $\beta \to \infty$, $\phi(\beta) \to 1$, which would imply that the probability of failure is equal to 0. This would be the ideal scenario.

# Student Results & Reflections

A summary of the 2023/24 cohort (72 students) and the 2024/25 cohort (38 students) along with a percentage of how many students correctly answered each section.

| Weighting | Method | 23/24 % | 24/25 % |
|---|---|---|---|
| 20% | FOSM | 83 | 92 |
| 20% | SOSM | 68 | 79 |
| 20% | Monte Carlo | 90 | 100 |
| 10% | A.I. | 100 | 100 |
| 30% | Conclusion | 17 | 47 |

# Next Steps

1. Continue to review areas A.I. is being used and assess the accuracy and limitations.
2. Attempt to incorporate A.I. into additional modules (ENGG198 - first year Mathematical Engineering).
3. Continue to research how students are using A.I. for continuous assessment, focussing on whether it helps them learn or simply answers the question.

Thank you!