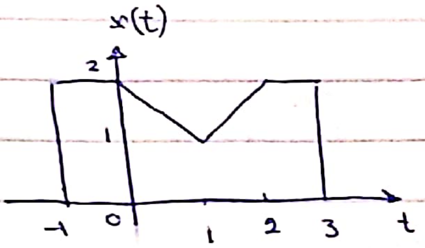
DSP Problem Session (PS-02) [Assembly]

4-25



~~$\angle X(\Omega) = ?$~~

$\boxed{a}$ $\angle X(\omega)$

* Note the computing $X(\omega)$ will be time consuming. Instead, we see that ~~shifting x(t) t~~ $x(t)$ is a shifted version of a real even signal.

$x(t+1) = y(t) \equiv$ real and even signal

$FT[x(t+1)] = FT(y(t)) = Y(\omega) \longrightarrow \angle Y(\omega) = 0$

$e^{j\omega} X(\omega) = Y(\omega)$

$X(\omega) = e^{-j\omega} Y(\omega)$

$\angle X(\omega) = (-\omega) + 0 = -\omega$

$\boxed{b}$ $X(0) = ?$

See that $X(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$. When substituting $\omega = 0$, $X(0) = \int_{-\infty}^{\infty} x(t)\, dt$ which is the area under the curve of the signal. $X(0) = 7$

[c] $\int_{-\infty}^{\infty} X(\omega)\, d\omega$

See that $x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, e^{J\omega t}\, d\omega$

when substituting $t=0$

$x(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega)\, e^{J\omega(0)}\, d\omega$

Therefore $\int_{-\infty}^{\infty} X(\omega)\, d\omega = 2\pi\, x(0) = 4\pi$

[d] $\int_{-\infty}^{\infty} X(\omega)\, \frac{2\sin(\omega)}{\omega}\, e^{2J\omega}\, d\omega$

Let $Z(\omega) = \frac{2\sin(\omega)}{\omega}\, e^{2J\omega}$

Then $\int_{-\infty}^{\infty} X(\omega)\, Z(\omega)\, e^{+J\omega t}\Big|_{t=0}\, d\omega$

If $D(\omega) = X(\omega)\, Z(\omega)$ then the integral is

$iFT(D(\omega))\Big|_{t=0}(2\pi)$

Remember that (from the table of properties)

$X(\omega) Z(\omega) \xrightarrow{iFT} x(t) * z(t)$

See that $Z(\omega) = 2\, \text{sinc}(\omega)\, e^{2J\omega}$

~~Rem~~ Remember that (From the table of transforms)

$\text{rect}(t/T) \xrightarrow{FT} |T|\, \text{sinc}(T\omega/2)$

Therefore $z(t) = \text{rect}((t+2)/T)$ (Note the shift from the properties to

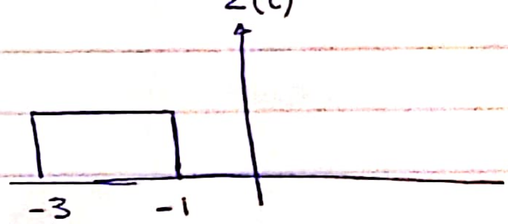
Recall that

$\text{rect}(t) = 1 \quad , \quad -\frac{1}{2} \leqslant t \leqslant \frac{1}{2}$

$\text{rect}\left[\frac{t+2}{2}\right] = 1 \quad , \quad -\frac{1}{2} \leqslant \frac{t+2}{2} \leqslant \frac{1}{2}$

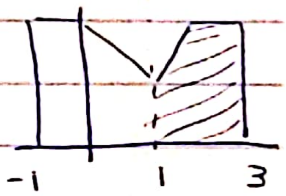
$z(t) = 1 \quad , \quad -1 \leqslant t+2 \leqslant 1$

$-3 \leqslant t \leqslant -1$



$x(t) * z(t) \Big|_{t=0} = \frac{1}{2} \times 7 \times 2\pi = 7\pi$

(Recall ~~integratio~~ convolution → reverse, shift by (t))



$\boxed{f}$ Sketch $iFT(Re(X(\omega)))$

~~Rec~~ See that $X(\omega) = Re + J\, Im$

We can express any signal as a summation of an even and an odd signal. The even part is

$\mathcal{E}v(x(t)) = \frac{1}{2}[x(t) + x(-t)]$

$FT(\mathcal{E}v(x(t))) = \frac{1}{2}[X(\omega) + \bar{X}(\omega)]$

$= Re$

$iFT[Re] = \mathcal{E}v(X(t)) = \frac{1}{2}[x(t) + x(-t)]$