Metodo de Runge Kutta 4°

formula geral: $Y_{i+1} = Y_i + (c_1 K_1 + c_2 K_2)h$

com: $K_1 = f(x_i, y_i)$ e $K_2 = f(x_i + a_2 h, Y_i + b_{21} K_1 \cdot h)$

ex] $c_1 = \frac{1}{2}$ $c_2 = \frac{1}{2}$ $a_2 = 1$ $b_{21} = 1 \to$ coeficientes fazem com que a expressão pareça com Euler modificado

$Y_{i+1} = Y_i + \frac{1}{2}(K_1 + K_2)h$

$K_1 = f(x_i, y_i)$ $K_2 = f(x_i + h, y_i + k_1 h)$

Dados p/ o problema:

$\frac{dy}{dx} = -1,2y + 7e^{-0,3x}$ $0 \leq x \leq 2$

$y(0) = 3$ $h = 0,5$

i	x	y
0	0	3
1	0,5	
2	1	
3	1,5	
4	2	

$i = 0$

$K_1 = f(x_0, y_0) = -1,2 \cdot 3 + 7e^0 = 3,4$

$K_2 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}K_1 h)$
$= -1,2 \cdot (3 + 0,5 \cdot 3,4 \cdot 0,5) + 7e^{-0,3(0 \cdot 0,5 \cdot 0,5)}$
$= 1,874$

$K_3 = f(x_i + \frac{1}{2}h, y_i + \frac{1}{2}K_2 \cdot h)$
$= -1,2 \cdot (3 + 0,5 \cdot 1,874 \cdot 0,5) + 7e^{-0,3(0 \cdot 0,5 \cdot 0,5)}$
$= 2,332$

$K_4 = f(x_i + h, y_i + k_3 h)$
$= -1,2 \cdot (3 + 2,332 \cdot 0,5) + 7e^{-0,3(0 + 0,5)}$
$= 1,026$

$$Y_1 = Y_0 + \frac{1}{6}(K_1 + 2K_2 + 2k_3 + k_4)h$$
$$Y_1 = 3 + \frac{1}{6}(3.4 + 2 \cdot 1.874 + 2 \cdot 2.332 + 1.026) \, 0.5$$
$$Y_1 = 4.070$$

$i = 1 \quad x = 0.5$

$K_1 = -1.2 \cdot 4.07 + 7 \cdot e^{-0.3 \cdot 0.5} = 1.141$

$R_2 = -1.2 (4.07 + 0.5 \cdot 0.141 \cdot 0.5) + 7 e^{-0.3(0.5 + 0.5^2)}$

$K_2 = 0.363$

$R_3 = -1.2 (4.07 + 0.5 \cdot 0.363 \cdot 0.5) + 7 e^{-0.3(0.5 + 0.5^2)}$

$K_3 = 0.597$

$R_4 = -1.2 (4.07 + 0.597 \cdot 0.5) + 7 e^{-0.3(0.5 + 0.5)}$

$K_4 = -0.056$

$Y_2 = Y_1 + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)h = \underline{4.320} \, h$

$i = 2 \quad x = 1$

$K_1 = -1.2 \cdot 4.32 + 7 \cdot e^{-0.3 \cdot 1} = 0.001$

$R_2 = -1.2 (4.32 + 0.5 \cdot 0.001 \cdot 0.5) + 7 e^{-0.3(1 + 0.5^2)}$

$K_2 = -0.373$

$R_3 = -1.2 (4.32 + 0.5 \cdot (-0.373) \cdot 0.5) + 7 e^{-0.3(1 + 0.5^2)}$

$K_3 = -0.261$

$R_4 = -1.2 (4.32 + (-0.261) \cdot 0.5) + 7 e^{-0.3(1 + 0.5)}$

$K_4 = -0.564$

$Y_3 = Y_2 + \frac{1}{12}(K_1 + 2K_2 + 2K_3 + K_4)h = \underline{4.167} \, h$

$i = 3 \quad x = 1.5$

$K_1 = -1.2 \cdot 4.167 + 7 \cdot e^{-0.3 \cdot 1.5} = -0.537$

$R_2 = -1.2 (4.167 + 0.5 \cdot (-0.537) \cdot 0.5) + 7 e^{-0.3(1.5 + 0.5^2)}$

$K_2 = -0.698$

$R_3 = -1.2 (4.167 + 0.5 \cdot (-0.698) \cdot 0.5) + 7 e^{-0.3(1.5 + 0.5^2)}$

$K_3 = -0.650$

$$K_4 = -1{,}2\left(4{,}167 + 0{,}5(-0{,}65)\right) + 7e^{-0{,}3(1{,}5+0{,}5)}$$

$$K_4 = -0{,}769$$

$$Y_4 = 4{,}167 \cdot \frac{1}{12}\left(-0{,}537 - 2\cdot 0{,}698 - 2\cdot 0{,}650 - 0{,}769\right)$$

$$\underline{Y_4 = 3{,}834} \, h$$

Exata: $Y_i = \dfrac{70}{9}e^{-0{,}3x_i} - \dfrac{43}{9}e^{-1{,}2x_i}$

$$Y_0 = \frac{70}{9}e^{-0{,}3\cdot 0} - \frac{43}{9}e^{-1{,}2\cdot 0} = 3$$

$$Y_1 = \frac{70}{9}e^{-0{,}3\cdot 0{,}5} - \frac{43}{9}e^{-1{,}2\cdot 0{,}5} = 4{,}073$$

$$Y_2 = \frac{70}{9}e^{-0{,}3\cdot 1} - \frac{43}{9}e^{-1{,}2\cdot 1} = 4{,}323$$

$$Y_3 = \frac{70}{9}e^{-0{,}3\cdot 1{,}5} - \frac{43}{9}e^{-1{,}2\cdot 1{,}5} = 4{,}170$$

$$Y_4 = \frac{70}{9}e^{-0{,}3\cdot 2} - \frac{43}{9}e^{-1{,}2\cdot 2} = 3{,}835$$

i	X	Y (metodo)	Y(Exato)	Eme
1	0	3	3	0
2	0,5	4,070	4,073	0,003
3	1	4,320	4,323	0,003
4	1,5	4,167	4,170	0,003
5	2	3,834	3,835	0,001