Usando Adams-Bashforth e Adams-Moulton, ambos de terceira ordem, como preditor-corretor, determinar $y(1)$ sendo:

$$\frac{dy}{dx} = x + y, \qquad y(0) = 1$$

em $[0, 1]$ com $h = 0,2$ e $\varepsilon < 10^{-3}$. $= 0,001$
Empregar Euler Explícito para calcular as soluções nos pontos iniciais da malha.

$x = [0, 1]$ com passo $h = 0,2$, erro $< 10^{-3}$

Euler explicito → achar segundo valor do adam

$Y_{i+1} = Y_i + f(x, y) \cdot h$ → $\left.\begin{matrix} x_i = 0 \\ Y_i = 1 \end{matrix}\right\} f(x,y) = \underline{1}$

$Y_1 = 1 + 1 \cdot 0,2 = \underline{1,2}$

$Y_0 = 1$, $Y_1 = 1,2$

$Y_2 = Y_1 + f(X_1, Y_1) \cdot h$ → $\left.\begin{matrix} X_i = 0,2 \\ Y_i = 1,2 \end{matrix}\right\} f(x,y) = 1,4$

$Y_2 = 1,2 + 1,4 \cdot 0,2$

$Y_2 = 1,48$ → $X_2 = 0,4$ → $f(x,y) = 1,88$

Adams Bashforth 3°:

$$Y_{i+1} = Y_i + \frac{h}{12} \cdot [23 \cdot f(x_i, y_i) - 16 \cdot f(x_{i-1}, y_{i-1}) + 5 \cdot f(x_{i-2}, y_{i-2})]$$

$$Y_3 = Y_2 + \frac{0,2}{12} [23 \cdot 1,88 - 16 \cdot 1,4 + 5 \cdot 1]$$

$Y_3 = 1,9106$ $X_3 = 0,6$ → $f(x,y) = 2,51$

Adams Moulton 3°:

$$Y_{i+1} = Y_i + \frac{h}{12} \cdot [5 f(x_{i+1}, y_{i+1}) + 8 f(x_i, y_i) - f(x_{i-1}, y_{i-1})]$$

$K = 1$

$$Y_3 = Y_2 + \frac{0{,}2}{12}\left[5 \cdot 2{,}51 + 8 \cdot 1{,}88 - 1{,}4\right]$$

$$Y_3 = 1{,}9165$$

$$\varepsilon = \left|\frac{1{,}9165 - 1{,}9106}{1{,}9106}\right| = 3 \cdot 10^{-3} > 1 \cdot 10^{-3}$$

Refazer Moulton com novo Y_3

$f(0{,}6,\ 1{,}9165) = 2{,}5165$

$K = 2$

$$Y_3 = Y_2 + \frac{0{,}2}{12}\left[5 \cdot 2{,}5165 + 8 \cdot 1{,}88 - 1{,}4\right]$$

$$Y_3 = 1{,}91704$$

$$\varepsilon = \left|\frac{Y_{K2} - Y_{K1}}{Y_{K1}}\right| = 2{,}82 \cdot 10^{-4} < 1 \cdot 10^{-3} \checkmark$$

$X_3 = 0{,}6 \qquad Y_3 = 1{,}9170 \qquad f(X_3, Y_3) = 2{,}517$

$i = 4 \quad x = 0{,}8$:

$$\underset{i=4}{Y_{i+1}} = Y_i + \frac{h}{12} \cdot \left[23 \cdot \underset{i=3}{f(X_i, Y_i)} - 16 \cdot \underset{i=2}{f(X_{i-1}, Y_{i-1})} + 5 \cdot \underset{i=1}{f(X_{i-2}, Y_{i-2})}\right]$$

$$Y_4 = 1{,}917 + \frac{0{,}2}{12}\left[23 \cdot 2{,}517 - 16 \cdot 1{,}88 + 5 \cdot 1{,}4\right]$$

$Y_4 = 2{,}4971 \qquad x = 0{,}8 \qquad f(X_4, Y_4) = 3{,}2971$

Aplicando Moulton

$$Y_4 = Y_3 + \frac{0{,}2}{12}\left[5 \cdot \underset{i=4}{3{,}2971} + 8 \cdot \underset{i=3}{2{,}517} - \underset{i=2}{1{,}88}\right]$$

$$Y_4 = 2{,}4960$$

$$\varepsilon = \left| \frac{Y_M - Y_B}{Y_B} \right| = 4{,}2 \cdot 10^{-4} < 1 \cdot 10^{-3} \quad \checkmark$$

$$\underbrace{X_4 = 0{,}8 \qquad Y_4 = 2{,}4960 \qquad f(X_4, Y_4) = 3{,}296}_{1}$$

$i = 5 \quad X = 1{,}0$:

$$\overset{i=5\;\;4}{Y_{i+1}} = Y_i + \frac{h}{12} \cdot \left[23 \cdot \overset{i=4}{f(X_i, Y_i)} - 16 \cdot \overset{i=3}{f(X_{i-1}, Y_{i-1})} + 5 \cdot \overset{i=2}{f(X_{i-2}, Y_{i-2})} \right]$$

$$Y_5 = 2{,}496 + \frac{0{,}2}{12} \left[23 \cdot 3{,}296 - 16 \cdot 2{,}517 + 5 \cdot 1{,}88 \right]$$

$$Y_5 = 3{,}2449 \qquad X = 1{,}0 \qquad f(X_4, Y_4) = 4{,}245$$

Aplicando Moulton

$$Y_5 = Y_4 + \frac{0{,}2}{12} \left[5 \cdot \overset{i=5}{4{,}2449} + 8 \cdot \overset{i=4}{3{,}296} - \overset{i=3}{2{,}517} \right]$$

$$Y_5 = 3{,}2472$$

$$\varepsilon = \left| \frac{Y_M - Y_B}{Y_B} \right| = 7 \cdot 10^{-4} < 1 \cdot 10^{-3} \quad \checkmark$$

$$X_5 = 1{,}0 \qquad Y_5 = 3{,}2472 \qquad f(X_5, Y_5) = 4{,}247$$

i	x	y	f(x,y)
0	0	1	1
1	0,2	1,2	1,4
2	0,4	1,48	1,88
3	0,6	1,917	2,517
4	0,8	2,496	3,296
5	1,0	3,247	4,247