Considere o sistema:

$$\frac{dy}{dx} = (-y + z)e^{(1-x)} + 0,5y; \qquad y(0) = 3 \quad \leftarrow f_1(x,y,z)$$

$$\frac{dz}{dx} = y - z^2; \qquad z(0) = 0,2 \quad \leftarrow f_2(x,y,z)$$

no domínio $x = 0$ até $x = 3$.

$x_{i+1} = x_i + h$
$y_{i+1} = y_i + f_1(x_i, y_i, z_i)h$
$z_{i+1} = z_i + f_2(x_i, y_i, z_i)h$

a) $x_0 = 0$, $y(0) = 3$, $z(0) = 0,2$, $h = 0,25$

$f_1(x_0, y_0, z_0) = f(0, 3, 0,2) = (-3 + 0,2)e^1 + 0,5 \cdot 3 = -6,116$
$f_2(x_0, y_0, z_0) = 3 - 0,2^2 = 2,96$
$y_1 = y_0 + f(x_0, y_0, z_0)h = 1,471$
$z_1 = z_0 + f(x_0, y_0, z_0)h = 0,94$

$i = 1$, $x = 0,25$
$y(0,25) = 1,471$
$z(0,25) = 0,94$
$f_1(x_1, y_1, z_1) = (-1,471 + 0,94)e^{(1-0,25)} + 0,5 \cdot 1,471 = -0,388$
$f_2(x_1, y_1, z_1) = 1,471 - 0,94^2 = 0,5873$
$y_2 = y_1 + f_1(x_1, y_1, z_1)h = 1,374$
$z_2 = z_1, f_2(x_1, y_1, z_1)h = 1,086$

$i = 2$, $x = 0,50$
$y(0,5) = 1,374$
$z(0,5) = 1,086$

$f_1(x_2, y_2, z_2) = 0,213$

$f_2(x_2, y_2, z_2) = 0,191$

$y_3 = y_2 + f_1(x_2, y_2, z_2)\,h = 1,4273$

$z_3 = z_3 + f_1(x_2, y_2, z_2)\,h = 1,1347$

X	Y	z
0	3	0,2
0,25	1,471	0,94
0,50	1,374	1,086
0,75	1,427	1,134

b) Runge Kutta 2°O

Calcula o K_1

$K_{1,Y} = f_1(X_i, Y_i, Z_i)$

$K_{1,Z} = f_2(X_i, Y_i, Z_i)$

Calcular K_2 com K_1

$K_{2,Y} = f_1(x_i + h, Y_i + h \cdot K_{1,Y}, z_i + h \cdot K_{1,Z})$

$K_{2,Z} = f_2(x_i + h, Y_i + h \cdot K_{1,Y}, z_i + h \cdot K_{1,Z})$

Calcula Y_{i+1} e Z_{i+1}

$$Y_{i+1} = Y_i + \frac{h}{2}(K_{1Y} + K_{2Y})$$

$$Z_{i+1} = Z_i + \frac{h}{2}(K_{1Z} + K_{2Z})$$

Passo 1: $X_0 = 0$, $Y_0 = 3$, $Z_0 = 0,2$

$K_{1Y} = (-3 + 0,2)e^1 + 0,5 \times 3 = -6,11$

$K_{1Z} = 3 - (0,2)^2 = 2,96$

$K_{2y} = (-1,472 + 0,94)e^{0,75} + 0,1736 = -0,39$

$K_{2z} = [3 + 0,25(-6,11)] - (0,2 + 0,25 \cdot 2,96)^2 = 0,588$

$$y_1 = 3 + \frac{0,25}{2}(-6,11 - 0,39) = 2,187$$

$$z_1 = 0,2 + \frac{0,25}{2}(2,96 + 0,588) = 0,643$$

Passo 2: $x_1 = 0,25$, $y_1 = 2,187$, $z_1 = 0,643$

$K_{1y} = (-2,187 + 0,643)e^{0,75} + 0,5 \cdot 2,18 = -2,173$

$K_{1z} = 2,187 - (0,643)^2 = 1,773$

$K_{2y} = -0,096$

$K_{2z} = 0,462$

$$y_2 = 2,187 + \frac{0,25}{2}(-2,17 - 0,09) = 1,903$$

$$z_2 = 0,643 + \frac{0,25}{2}(1,77 + 0,462) = 0,923$$

i	X	Y	Z
0	0	3	0,2
1	0,25	2,187	0,643
2	0,50	1,903	0,923