Electrical Power and Energy Systems 106 (2019) 546–560

Contents lists available at ScienceDirect

Electrical Power and Energy Systems

journal homepage: www.elsevier.com/locate/ijepes

The effect of price responsive loads uncertainty on the risk-constrained optimal operation of a smart micro-grid

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ARTICLE INFO

Keywords:
Demand response
Robust optimization
Two-stage stochastic optimization
Conditional value at risk (CVaR)
Uncertainty

ABSTRACT

The financial and technical decisions of a smart micro-grid energy management system have always been affected by the existence of uncertainty in various parameters, such as electric load, electricity price, and renewable generation. Especially when the behavior of price and load is influenced by each other in a smart micro-grid. In this paper, we describe the uncertainty issue in the consumption behavior of price responsive loads and we utilize a model based on robust optimization to consider this uncertainty. The effects of considering the uncertainty of price responsive loads with the proposed model on a typical risk-constrained optimal operation of a smart micro-grid are illustrated through two case studies. The first case involves solving the considered optimal operation problem without considering the proposed uncertainty model while the second one shows the effects of proposed uncertainty model on the optimal solutions. The operation optimization problem is formed as a mixed-integer two-stage stochastic framework. Results show that with this uncertainty model the manager of smart micro-grid can increase its bids in day-ahead market at peak and mid-peak time slots while it can decrease them in off-peak time slots. Also, considering the impacts of this uncertainty model on profitability and system reliability, the smart micro-grid manager needs a trade-off between increasing its profit and increasing the system operation risk.

1. Introduction

Smart micro-grid (MG) underpins modernization of the MG by developing sensors, telecommunication, and digital technologies and serves as a solution for using renewable energies, improving system reliability and reducing electricity generation costs. It also adds new capabilities such as implementing demand response programs [1], using plug-in hybrid electric vehicles and storage devices [2]. Intelligent communication between the energy suppliers, consumers, energy storage devices and other smart MG elements will reduce the energy cost of customers and increase the desire of the system operator to use the renewable energy sources. Also, reducing the power plants generation due to the penetration of renewable energy sources will reduce environmental issues such as global warming and natural resource depletion. MG is a small energy network that includes distributed energy resources such as renewable (e.g. wind, solar) and non–renewable (e.g. micro-turbine (MT), diesel generator) energy sources, loads and energy storage devices. MGs even can be operated in the case of island mode, when the external faults occur [3]. The development of green technologies, such as renewable generations and plug-in hybrid electric vehicles, and the use of demand response programs in MGs, is dependent on the use of smart MGs technology. Therefore, it can be said the MGs and the smart MGs have common goals [4], so a smart MG is considered in this study. In the restructured electricity market, MGs can participate in electricity distribution market in interaction with distribution market operator which is known as an upstream market operator [5]. Therefore, optimal operation of an MG is important for its manager and also can be a challenge to it if the uncertainty modeling of some important parameters such as renewable generation, load and market price is ignored.

1.1. Related works

Many papers have been studied in the field of MGs optimal operation by considering uncertainties using robust optimization. In [6], a hybrid optimization model based on robust and stochastic approaches is proposed to minimize the expected MG's net cost. To consider the real-time (RT) market price uncertainty, the robust optimization is applied to limit the unbalanced power in RT market and reduce the economic risk. In [7], the robust energy management of an MG is formulated as a form of two-stage structure in which the social benefit cost minimization is modeled in the first stage while largest power exchange cost

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https://doi.org/10.1016/j.ijepes.2018.10.027
Received 27 April 2018; Received in revised form 9 October 2018; Accepted 27 October 2018
0142-0615/

Nomenclature

Indices

$(.)_{t,s}$	at time t in scenario s
b	index for battery
i	index for MT
s	index for scenario
t	index for time slot
w	index for WT

Parameters, constants and sets

a_i, b_i, c_i	generation cost coefficients of MT i
C_{CO_2}	externality cost of generation CO_2 (\$/kg)
$CO_{2,i,t}$	carbon dioxide pollutant of MTi(kg/kW)
$C_{w,t,s}^{gen}$	generation cost of WT w(\$/h)
$C_w^{o\&m}$	annual Operation and maintenance cost of WT w(\$/kW)
CF_w	capacity factor of WT w
$d_{t,s}^{e}$, $d_{t,s}^{ne}$	price responsive/normal loads (kW)
$d_{t,s}^{net}$	net load (kW)
E_b	capacity of battery b (kWh)
$e(t,t)$, $e(t,t')$	self and cross Elasticity
$e^R(t,t)$, $e^R(t,t')$	robust self and cross Elasticity
In_w	capital cost of WT w(\$)
int	interest rate
k_{start}	startup constant cost of MTs (\$)
MUT_i, MDT_i	minimum up/down time of MT i(h)
NS, NT	number of scenarios and time slots
NDG, NB	number of MTs and batteries
NW	number of WTs.
n_w	loan repayment term (yr)
P_b^C, P_b^D	maximum charging /discharging power of battery b (kW)
P_i^{max}, P_i^{min}	maximum/minimum power limits of MT i(kW)
P_t^{grid}	maximum exchangeable power (kW)
$P_{w,t,s}$	available output power of WT w (kW)
P_w^r	rated power of WT w(kW)
RU_i, RD_i	ramp up/down rate of MT i(kW/h)
SoC_b^{max}, SoC_b^{min}	maximum/minimum SoC of battery b
U	uncertainty set
$VOLL$	value of lost load (\$/kWh)
Γ	conservatism degree
λ_t^{pen}	penalty for bid deviations (\$/kWh)
η_b^C, η_b^D	charging/discharging efficiency of battery$b$
ΔT, ρ_s	duration of time slot (h) and probability of scenario$s$
$\lambda_{t,s}^{da}$, $\lambda_{t,s}^{real}$	day-ahead/real-time market prices (\$/kWh)
λ_t^{0da}, $d_{t,s}^{0e}$	initial price (\$/kW)/initial price responsive loads (kW)
β	risk aversion parameter

Decision variables

$b_{b,t,s}^C$, $b_{b,t,s}^D$	binary variable, 1 if charging/discharging
$C_{i,t,s}^{ope}$	operating costs of MTi(\$)
$C_{i,t,s}^{gen}$	generation cost of MTi(\$)
$C_{i,t,s}^{start}$	startup cost of MTi(\$)
$C_{i,t,s}^{em}$	emission cost of MTi(\$)
$ENNS$	expected energy not supplied (kWh)
$on_{i,t,s}$, $off_{i,t,s}$	startup and shutdown indicators of MT i
P_t^{da}	DA electricity market transactions (bids) (kW)
$P_{t,s}^{del}$	real power delivery (kW)
$P_{t,s}^{real}$	RT electricity market transactions (kW)
$P_{b,t,s}^C$, $P_{b,t,s}^D$	charging/discharging power of battery$b$ (kW)
$P_{t,s}^{pen}$	deviation power between day-ahead bids and real power delivery (kW)
$SoC_{b,t,s}$	state of charge battery b
$shed_{t,s}$	curtailed loads (kW)
$V_{i,t,s}$	commitment status of MT i

Abbreviations

Conditional value at risk CVaR
Day-ahead DA
Expected energy not supplied EENS
Micro-turbine MT
Probability density function PDF
Real-time RT
State of charge SoC
Wind turbine WT
Value of lost load VOLL

under worst-case scenario is obtained in the second stage. Due to integration of load and renewable generation uncertain sets in the proposed optimization framework and hence difficulty solving, the Taguchi's orthogonal array method is used to select the most probable scenario. In [8], a two-stage robust optimization is proposed to schedule the energy generation in an MG aimed at long-term operating cost minimization. The unit commitment problem is modeled in the first stage and solved as a mixed integer programming. Also, the economic dispatch and RT energy scheduling are modeled in the second stage and solved by Lyapunov optimization method. The most pessimistic effects of uncertainties in the load demand and renewable generation are considered in both stages using robust optimization. In [9], a distributed algorithm is presented based on dual decomposition for an MG energy manager aiming at minimizing social net cost. The worst-case amounts of multiple wind farms generations are considered using robust optimization technique. In Ref. [10], the uncertainties of renewable generation and forecasted loads in an MG energy management problem are modeled by point estimation method and robust optimization, respectively. In [11], the MG planning problem is decomposed into investment and operation problems using Benders decomposition method. The investment decisions are made in the master problem and then optimal operation decisions will be made in sub-problem in order to examine the optimality of investment decisions. The robust optimization method is used to calculate the worst-case scenarios of loads, price and renewable generation forecasting errors in the sub-problem.

Stochastic methods have been used in many papers to handle uncertainties in MGs and similar energy systems like virtual power plant as well. In [12], stochastic programming is employed for tackling the uncertainty of renewable generation in a risk minimization investment problem and Markowitz model is used for implementing this goal. In [13], in order to minimize net costs of an MG, a stochastic optimization model is proposed and renewable generation scenarios are generated using Latin Hypercube Sampling method. In this proposed model, both price responsive loads and distributed generation units are used for compensating the renewable generation forecasting errors. In [14], a two-stage stochastic programming is used to minimize the expected cost and power losses of an MG including plug-in electric vehicles. The optimal transaction power between MG and day-ahead (DA) market and the real-time operation decisions are modeled in the first and second stage, respectively. In [15], a two-stage stochastic optimization is used for an MG operation cost (energy and reserve) minimization modeling. In this regard, the price responsive loads and the plug-in hybrid electric vehicles are used for compensating the renewable generation uncertainty and load curve modification, respectively. In [16], a two-stage probabilistic model is presented for joint energy and reserve scheduling of a virtual power plant. The uncertainties of renewable generation, price and load are integrated by point estimation method. In [17], the stochastic scenarios are employed for modeling the uncertainties of pool market prices, loads and climate condition and in order to the determination of selling price for a retailer. In this regard, three cases of pricing including fixed pricing, time-of-use pricing, and RT pricing are compared to each other. Finally, it is shown the RT pricing leads to a higher profit.

In some studied research works, the MG management system has been considered as a profitable entity. In [18], the profit maximization of a smart MG with proposing a controlling algorithm for air-conditioning loads is investigated. Also, the uncertainties of loads, energy price, wind speed, and connection status with the upstream network are considered.

In [19], the authors propose a bi-level model to participate MG in power market. The bidding profit is optimized by interior point algorithm in the upper level while the unit commitment is coded and solved as a mixed integer nonlinear programming in the lower level of proposed optimization framework, respectively. Also, uncertainties of loads, wind speed and solar irradiance are considered. In [20], the optimal bidding problem for an MG aggregator is proposed in order to maximize its profit. Besides that, a novel demand response agreement is presented between MG aggregator and its customers. The optimization problem is formulated as a stochastic optimization for considering the uncertainties of loads, wind speed, and market price. In [21], a novel two-stage robust optimization is proposed for scheduling joint day-ahead demand response program and distributed generation while maximizing MG profits against uncertainties such as load demand and renewable generation. In [22], the profit maximization of a wind/solar/energy storage hybrid system is investigated under demand response program and frequency-based pricing by implementing the Availability Based Tariff mechanism. In [23], two formulations are proposed in order to maintenance scheduling and operation of an MG. In the first formulation, a cost minimization problem is proposed in a single-level form subject to MG' operation constraints and in the second formulation, a profit maximization problem is proposed in a bi-level form subject to scheduling the required preventive maintenance of MG' houses distributed energy systems with the aim of maximizing their profits. In [24], the offering problem for a virtual power plant aiming to expected profit maximization is presented as a mixed integer linear programming where the uncertainty of market prices and renewable generation are tackled by stochastic scenarios. In [25], the authors present a stochastic model to consider the uncertainties of electricity and gas prices as well as demand in a smart energy hub environment. The fully automated energy management system based on reinforcement learning algorithm is presented in [26] and also in [27] aiming at encouraging the consumers to reduce the peak load of both electricity and gas network in a multi-carrier distribution system. Also, in [28] an integrated demand response program based on an ordinal potential game is proposed for a multi-carrier energy system (gas and electricity). The goal of this model is maximizing payoffs of both consumers and utilities.

1.2. Contribution

According to a review of related works, there are no proper models for considering the effects of price responsive loads uncertainty on the optimal decisions such as buying and selling in markets, storage system performance, and system reliability. This paper suggests a method for considering this uncertainty and also shows its effects on the smart MG optimal decisions. Indeed, we claim there is a possibility that the consumption behavior of some price responsive loads does not change based on the price at the time of use. In other words, the amount of change can be different from the estimated values. So we utilize a model based on robust optimization to consider worst-case price responsive loads scenarios. Therefore, the novel contribution of this paper is as follows:

- The uncertainty of price responsive loads is described and a model based on robust optimization is used to take into account this

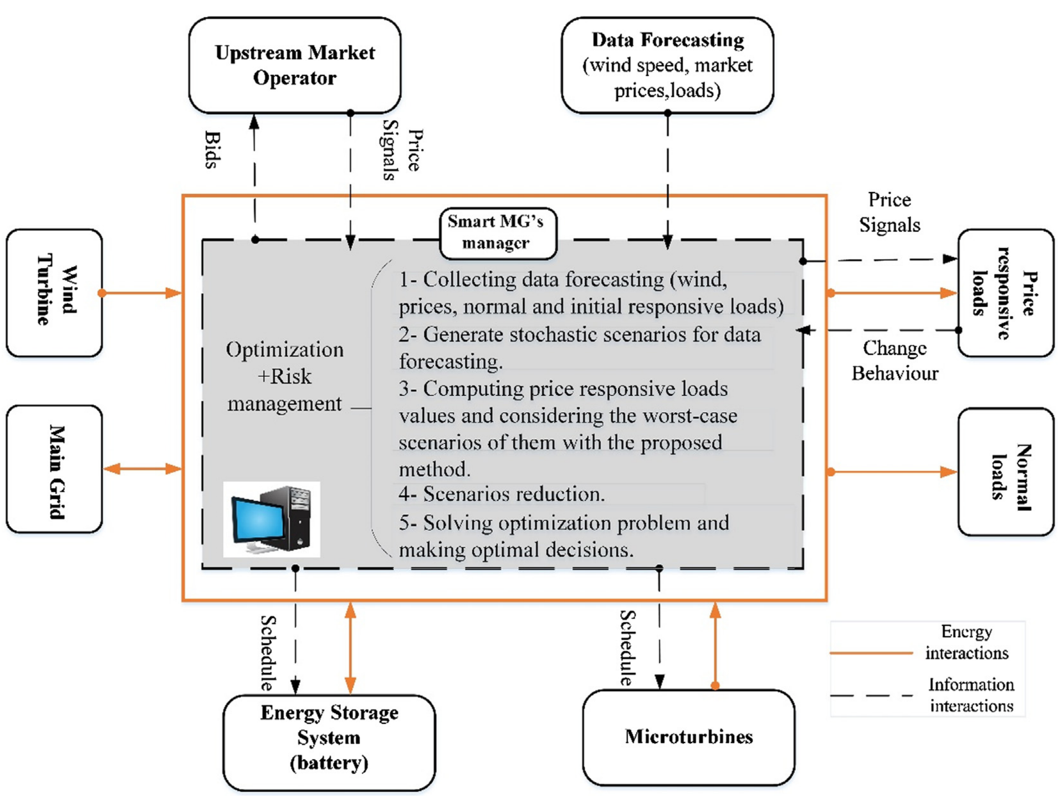

Fig. 1. Considered model for smart MG energy management.

uncertainty in the smart MG optimal operation problem.

1.3. Methodology

The smart MG considered in this paper trades energy in DA and RT electricity markets for procuring and balancing energy, respectively. This smart MG consists of renewable (wind turbine (WT)), and non-renewable (MT) energy systems, battery and some price responsive and normal loads. The goal is the maximization of the smart MG profit through submission of optimal bids in DA electricity market before a specified bidding deadline and the optimal operation of MG over the operating day. The proposed smart MG profit maximization is cast as a two-stage stochastic mixed-integer linear optimization model due to the existence of uncertainties in various parameters such as wind speed, market prices and system price responsive and normal loads. In this regard, various stochastic scenarios of the mentioned parameters are generated using Monte Carlo simulation and then they are reduced to a smaller number of scenarios based on Kantorovich distance algorithm. The reduced scenarios are applied directly in proposed stochastic optimization framework, except price responsive loads scenarios. In fact, firstly the worst scenarios of responsive loads are chosen by the robust used method, then they would be applied to the stochastic optimization framework as well.

Furthermore, the conditional value at risk (CVaR) method is used to control the system decisions risk. CvaR is one of the commonly used risk measurement methods and has been used for various energy management issues. In Ref. [29], the risk management is modeled using CvaR method for distribution companies due to uncertainties in RT prices and loads. In Ref. [30], risk control of low-profit scenarios for a generation company and in Ref. [31] for a retailer are modeled using CVaR. Also, Ref. [32] used CvaR for limiting the likelihood of high renewable generation shortage for the independent system operators.

1.4. Organization

The remainder of this paper is organized as follows. Section 2 describes system model. Uncertainty modeling of parameters and uncertainty of price responsive loads model are presented in Section 3. Problem formulation and numerical results are provided in Sections 4 and 5, respectively. Finally, Section 6 is dedicated to the conclusion.

2. Model description

The schematic overview of the whole paper is illustrated in Fig. 1. It is assumed that all generating units belong to smart MG' manager. The manager serves the loads by procuring the energy from electricity markets and its own distributed resources and sell electricity to them at the same DA market prices to avoid the exclusive sale of energy to the loads. Therefore, the manager is responsible for smart MG' optimal decisions seeking its own profit maximizations. To this end, the necessary information from all the elements involved in the optimal operation such as generation units, weather forecast, markets prices forecast, loads forecast, and storage system status are collected. Then, based on the modeling of the existing elements and equipment, optimal risk constrained decisions are made for the optimal bids submission in DA and for 24 h optimal scheduling over the operating day by the manager.

2.1. Market interactions and system operation description

The manager has to submit the power amounts that it needs to purchase/sell in the DA electricity market to the upstream market operator. These DA bids must be submitted before the beginning of the operating day and also before a defined deadline [20]. Moreover, they are determined based on the forecast data including wind speed, loads and market prices. On the other hand, the real operation conditions over the operating day may be different due to uncertainties of the forecast data. These uncertainties can lead to a deviation between the real interaction with upstream network and the submitted DA bids quantities. Therefore, the manager will need to balance its power deviations via RT electricity market over the operating day. Also, the manager needs to schedule the optimal operating points of MTs, battery and load curtailment amounts during the operating day.

3. Price responsive loads uncertainty

In this section, the uncertainty modeling of market prices, normal loads and wind power is firstly explained by the stochastic scenarios. Then, the author focuses on the main contribution of this paper by modeling the uncertainty of price responsive loads.

3.1. Stochastic optimization methodology

The stochastic optimization is a technique that deals with modeling of a number of optimization problems in which some of the problem data are assumed to be non-deterministic. This technique, in contrast to the deterministic models, usually brings more realistic answers and directly covers the lack of deterministic models. In stochastic optimization models, the probability distributions for uncertain parameters are considered instead of the constant assumption of them. The goal is to make a decision that is feasible for all states and to optimize the expected values of stochastic variables and decisions [33]. Due to existence various uncertainties in smart MG responsive and normal loads, DA and RT market prices and WT generations, two-stage stochastic optimization is employed where these uncertainties are considered with probable stochastic scenarios. Each system scenario contains the hourly price responsive and normal loads, the hourly DA and RT market prices and the hourly WT generation. In other words, each scenario represents a possible operation state of smart MG. The scenario generation procedures for the all uncertain smart MG' parameters are explained below.

3.1.1. Normal loads, DA and RT market prices scenarios

The normal loads are the smart MG's loads without any price elasticity. The hourly DA market price and normal loads forecasts are available for manager based on historical data over a long period (e.g., one year). It is assumed that both normal loads forecasting errors and DA market price forecasting errors follow normal distributions [34]. The normal loads scenario $d_{t,s}^{ne}$ at hour t can be computed as the sum of the forecasted normal loads $\bar{d}_t^{ne}$ and its error scenario $\Delta d_{t,s}^{ne}$[19]:

$$d_{t,s}^{ne} = \bar{d}_t^{ne} + \Delta d_{t,s}^{ne} \tag{1}$$

where $\Delta d_{t,s}^{ne}$ is supposed to follow a normal probability distribution. As its average is zero and its standard deviation is 10% of its hourly forecasted values. The DA price scenario $\lambda_{t,s}^{da}$ at hour t can be computed as the sum of the forecasted DA price $\bar{\lambda}_t^{da}$ and its error scenario $\Delta\lambda_{t,s}^{da}$:

$$\lambda_{t,s}^{da} = \bar{\lambda}_t^{da} + \Delta\lambda_{t,s}^{da} \tag{2}$$

where $\Delta\lambda_{t,s}^{da}$ is supposed to follow a normal probability distribution. As its average is zero and its standard deviation is 10% of its hourly forecasted values, too. The RT price scenario $\lambda_{t,s}^{real}$ at hour t is generated with the expectation of DA price scenario at hour t but with the standard deviation of 15%. Therefore, the RT price scenarios are generated with higher standard deviation to demonstrate much variability of RT electricity markets [6].

3.1.2. WT power generation scenarios

Due to stochastic wind speed behavior, WT power generation will be affected by uncertain wind speed. The shape of the Weibull distribution function depends on its parameters and can provide a suitable fitting for almost any distribution of information and observation results [35]. The Weibull probability density function (PDF) is formulated in Eq. (3):

$$f_w(WS) = \frac{\beta' \ WS^{\beta'-1}}{\alpha^{\beta'}} \exp\left[-\left(\frac{WS}{\alpha'}\right)^{\beta'}\right] \tag{3}$$

where β' and α' are called the shape and the scale parameter, respectively. These two parameters can be obtained from Weibull distribution fitting into historical wind speed data. Therefore, the Weibull PDF for each time slots can be known and the stochastic wind speed scenarios can be calculated using Eq. (4).

$$WS_{t,\,s} = \bar{W}S_t + \Delta WS_{t,\,s} \tag{4}$$

where $WS_{t,\,s}$is the wind speed at hour $t$in scenario s, $\bar{W}S_t$ is wind speed forecast at hour $t$and $\Delta WS_{t,\,s}$is a random variable obtained using fitted Weibull PDF at hour t. These wind speed scenarios are generated with the standard deviation 5% of their hourly forecasted values. Then, the generated wind speed scenarios convert into corresponding WT power scenarios using the WT power curve. Here the 4-parameters logistic function is used to model the WT power curve due to its high degree of accuracy. The 4-parameters logistic model is formulated as follows [36]:

$$P_w(WS) = \frac{P_w^r}{1 + \exp(2s_w(WS_{ip,w} - \ WS)/(P_w^r - P_{ip,w}))} \tag{5}$$

where P_w is the power generation function of WT w. P_w^r and $P_{ip,w}$ are the rated power and power amount at the inflection point $WS_{ip,s}$ in the power curve of WT w, respectively.

$$s_w = \frac{dP_w(WS)}{dWS}|_{\ ws=ws_{ip,w}} \tag{6}$$

$s_w$in Eq. (6) is the slop of the power curve at the inflection point and equals to the first derivative of the WT power generation function with respect to the wind speed. The second derivative of the power generation function with respect to the wind speed is 0 at the inflection point as shown in Eq. (7).

$$\frac{d^2P_w(WS)}{dWS^2}|_{\ WS=\ WS_{ip,w}} = 0 \tag{7}$$

3.2. Concept and modeling of price responsive loads uncertainty

The responsive loads in the price-based demand response programs are expected to change their consumption behavior by responding to the electricity price variations through reducing their consumption or shifting it from the expensive price time slots to the cheap price time slots [37]. In this paper, the real-time pricing is used as the price-based DR program. In this proposed model the sale price in the smart MG is considered to be equal to DA market price. Therefore, the DA prices are used as real-time pricing of the operating day. We claim that there is uncertainty in the consumption behavior of price responsive loads. It can be justified for the following reasons:

- Some consumers may be reluctant to reduce their consumption or shift it to low-price hours due to the necessary need for power consumption or the occurrence of an unexpected event.
- The elasticity coefficients in the elasticity matrix are often estimated by historical load and price data analysis [38].

Therefore, the elasticity coefficients confront uncertainty which can affect the hourly price responsive load quantities. Accordingly, we are interested in showing the effects of considering the price responsive loads uncertainty on the optimal bidding and operation problem of a smart MG. The following describes the uncertainty modeling of the price responsive loads.

The elasticity matrix demonstrates the sensitivity of loads to the price variations and is a 24 × 24 dimension matrix for whole operation day periods. This is represented in the following [39]:

$$\begin{bmatrix} \frac{\Delta d_1^e}{d_1^{0e}} \\ \frac{\Delta d_2^e}{d_2^{0e}} \\ \cdots\cdots\cdots \\ \frac{\Delta d_{24}^e}{d_{24}^{0e}} \end{bmatrix} = \begin{bmatrix} e(1,\,1) & e(1,\,2) & \cdots & e(1,\,24) \\ e(2,\,1) & e(2,\,2) & \cdots & \cdots \\ \cdots & \cdots & e(t,\,t) & \cdots \\ e(24,\,1) & \cdots & \cdots & e(24,\,24) \end{bmatrix} \begin{bmatrix} \frac{\Delta\lambda_1^{da}}{\lambda_1^{0da}} \\ \frac{\Delta\lambda_2^{da}}{\lambda_2^{0da}} \\ \cdots\cdots\cdots \\ \frac{\Delta\lambda_{24}^{da}}{\lambda_{24}^{0da}} \end{bmatrix} \tag{8}$$

The 24 by 24 elasticity matrix, according to Eq. (8), consist of self and cross-elasticity coefficients in the diagonal and off-diagonal elements respectively. The self-elasticity coefficient $e(t,\,t)$, represents the price responsive loads variation during time slot t to the price at that time slot. The cross-elasticity coefficient $e(t,\,t')$, represents the price responsive loads variation during time slot t to the price during time slot t'. Using Eq. (8), the optimal consumption of price responsive loads which participate in the real-time pricing programs in order to maximize their benefits, is modeled as follows [39]:

$$d_{t,s}^e = d_{t,s}^{0e} \times \left\{1 + e(t,\,t) \times \frac{[\lambda_{t,s}^{da} - \lambda_t^{0da}]}{\lambda_t^{0da}} + \sum_{\substack{t'=1 \\ t' \neq t}}^{NT} e(t,\,t') \times \frac{[\lambda_{t',s}^{da} - \lambda_{t'}^{0da}]}{\lambda_{t'}^{0da}}\right\}, \quad \forall\ t,\,s \tag{9}$$

where $d_{t,s}^{0e}$ is initial price responsive loads under each scenario. Their scenarios are computed using the method described in Eq. (1). In fact, the initial price responsive loads are the initial status of price responsive loads before responding to the market price changes and their behavior is like the behavior of normal loads. So the system manager can forecast them based on historical data. In this paper, we used a model based on robust optimization to consider responsive load uncertainty. Robust optimization method through considering the worst-case probabilities of input variables will lead to the appropriate and optimal decisions. The main application of this method is when there is not adequate data for the input variable PDF [10]. Thus, the most important reason for using the robust optimization method for modeling the uncertainty of responsive load is that there is not appropriate information of elasticity coefficients' PDF.

The cross-elasticity coefficient $e(t,\,t')$ in each time period (i.e. peak, mid-peak and off-peak) is supposed as a random variable which can vary with an interval $[\underline{\hat{e}}(t,\,t'), \bar{\hat{e}}(t,\,t')]$with the estimated value to be $e^e(t,\,t')$. $\bar{\hat{e}}(t,\,t')$ and $\underline{\hat{e}}(t,\,t')$ are upper and lower bound of the cross-elasticity coefficient in the elasticity matrix, respectively. It is assumed that the manager can obtain these upper and lower bounds based on historical data [40]. In this method, the elasticity matrix converts to a robust elasticity matrix due to consider the worst-case scenario of elasticity coefficients at their upper or lower bounds. Therefore, an integer conservatism degree (i.e. Γ, Γ∈[0, T]) is employed in order to limit the number of time slots that $e(t,\,t')$ is allowed to deviate from its estimated value $e^e(t,\,t')$. Elasticity coefficients must be equal to their estimated values for $\Gamma = 0$. If $\Gamma = \text{n}$, then in n arbitrary hours in the operating time slots, the value of real elasticity coefficient is different from the estimated value [41]. It can be observed that the elasticity matrix become more conservatism with increasing Γ. In practice, the manager can adjust the elasticity matrix robustness by selecting the value of conservatism degree Γ. To create an uncertainty set, it is necessary to define several axillary variables such as z_t^+ and z_t^- to determine that the elasticity coefficients are equal to their boundary values or equal to their estimated ones. So, $z_t^+ = 1$ indicates that the values of elasticity coefficients are equal to their upper bound values. $z_t^- = 1$ indicates that the values of elasticity coefficients are equal to their lower bound values and finally $z_t^+ = 0$ or $z_t^- = 0$ indicates that the values of elasticity coefficients are equal to their estimated values. The uncertainty set can be defined as Eq. (10).

$$U = \left\{ e^R(t,t') \in \mathbb{R}^{NT}: \sum_{t=1}^{NT} (z_t^+ + z_t^-) \leqslant \Gamma,\ \ e^R(t,t') = e^e(t,t') + z_t^+ \bar{e}(t,t') - z_t^- \underline{e}(t,t'),\ \forall\, t,t' \in NT \right\} \tag{10}$$

$$\underline{e}(t,t') = e^e(t,t') - \underline{\hat{e}}(t,t') \tag{11}$$

$$\bar{e}(t,t') = \bar{\hat{e}}(t,t') - e^e(t,t') \tag{12}$$

Eqs. (11) and (12) are used for notation brevity in the uncertainty set. The axillary variables will be obtained by a random variable (rv_t) subject to a uniform distribution over [0 1] like Eq. (13) [40]. Then a robust elasticity matrix will be formed using the defined uncertainty set in Eq. (10). Finally, the price responsive loads will be determined by robust elasticity matrix as is mentioned in Eq. (14). This process that is shown in Fig. 2 will continue until the average of the obtained profiles in the repetitions converge to a constant profile, and do not change with the continuation of the repetitions. The average of all profiles obtained in repetitions is the output of the flowchart. The explained method will be implemented on each initial price responsive loads scenarios. It is worth mentioning that, the explained procedure is applied to self-elasticity coefficients in the elasticity matrix as well.

$$\begin{cases} e^R(t,t') = e^e(t,t') - \underline{e}(t,t') & , z_t^+ = 0,\ z_t^- = 1,\ \text{if } rv_t \leqslant \Gamma/2NT \\ e^R(t,t') = e^e(t,t') + \bar{e}(t,t') & , z_t^+ = 1,\ z_t^- = 0,\ \text{if } rv_t \geqslant 1 - \Gamma/2NT \\ e^R(t,t') = e^e(t,t') & , z_t^+ = 0,\ z_t^- = 1,\ \text{otherwise} \end{cases} \tag{13}$$

$$d_{t,s}^e = d_{t,s}^{0e} \times \left\{ 1 + e^R(t,t) \times \frac{[\lambda_{t,s}^{da} - \lambda_t^{0da}]}{\lambda_t^{0da}} + \sum_{\substack{t'=1 \\ t' \neq t}}^{NT} e^R(t,t') \times \frac{[\lambda_{t',s}^{da} - \lambda_{t'}^{0da}]}{\lambda_{t'}^{0da}} \right\} \forall\, s,t \tag{14}$$

3.3. Model implementation

The proposed price responsive uncertainty model can be implemented on the optimal operation of a smart MG as follows. At first, the manager collects all the forecasts information before the day of operation. This information includes the data of the operating day, i.e. wind speed, loads and RT prices and the DA electricity market data, i.e. DA prices. The Monte Carlo simulation is used to generate stochastic scenarios for initial responsive and normal loads, DA and RT prices and wind speed based on their PDFs that have mentioned in previous sections. In this paper, 500 stochastic scenarios are generated for each uncertain parameter. In order to show the uncertainty of parameters in stochastic optimization, a great number of scenarios are required and this leads to increasing computational time. By using a mathematical method, this high number of scenarios can be reduced. These methods usually are based on calculating the possible distance between the main sample and scenario sample. In stochastic optimization problems, one of the most common scenario reduction methods is the Kantorovich distance. In [42], fast forward selection algorithm which is based on Kantorovich distance is proposed. Here, this algorithm is used for diminishing the number of scenarios.

This algorithm is a repetitive one, in which an empty scenario tree is formed, then the scenario that minimizes the Kantorovich distance between initial and selected set is chosen one by one. When the needed number of scenarios is selected, this algorithm is terminated. Then the probability of each scenario that is not selected is transferred to the closest selected scenario. Finally, a reduced scenario tree with determined possibility is obtained. Accordingly, 500 stochastic scenarios for each parameter (i.e. initial price responsive loads, normal loads, DA price, RT price and wind speed) are reduced to 50 stochastic scenarios for each parameter using the mentioned algorithm. Then, the worst-case scenarios of price responsive loads are calculated based on Section 3.2. finally, the system scenarios set is formed.

After these data pre-process, the manager computes the optimal DA purchasing/selling bids and submit them to the upstream market operator before a defined deadline. Therefore, the optimal DA bidding computation is the first-stage decision variable in the proposed two-stage stochastic optimization. This decision must be made before the smart MG uncertainties are disclosed [20].

The second-stage decision variables are made after the uncertainties become known and they depend on the first-stage decisions. So, each stochastic scenario represents the real operation state of the smart MG over the operating day. The number of optimal solutions of the second stage decision variables is equal to the number of stochastic scenarios. Depending on which probable scenario will occur during the operating day, the manager will make optimal decisions according to the second-stage optimal solutions of the proposed optimization problem. The second stage decision variables include the MTs dispatching, the real power delivery between smart MG and upstream network, battery charging/discharging scheduling and load shedding if needed. The outputs of the proposed optimization framework consist of both first stage and second stage decisions [20]. Fig. 3 shows the procedure of the model implementation.

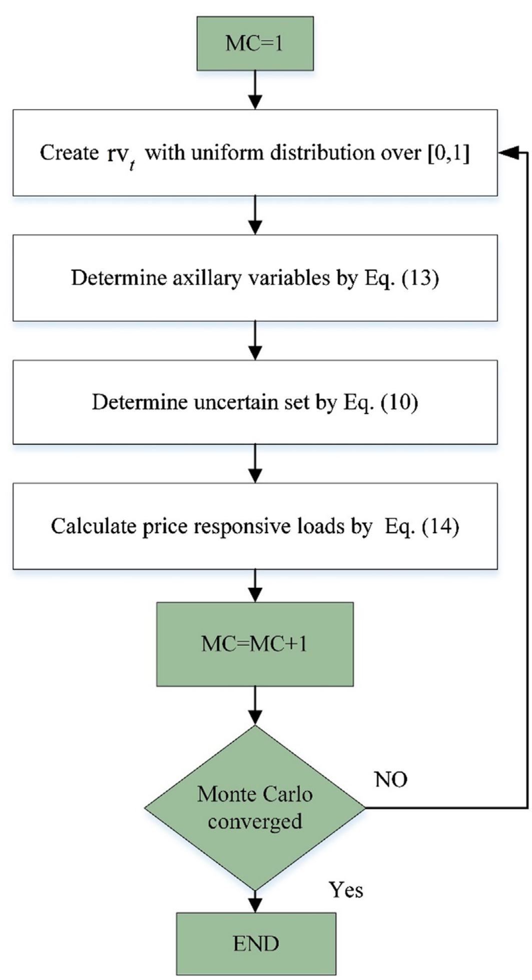

Fig. 2. The flowchart of the proposed uncertainty modeling of price responsive loads.

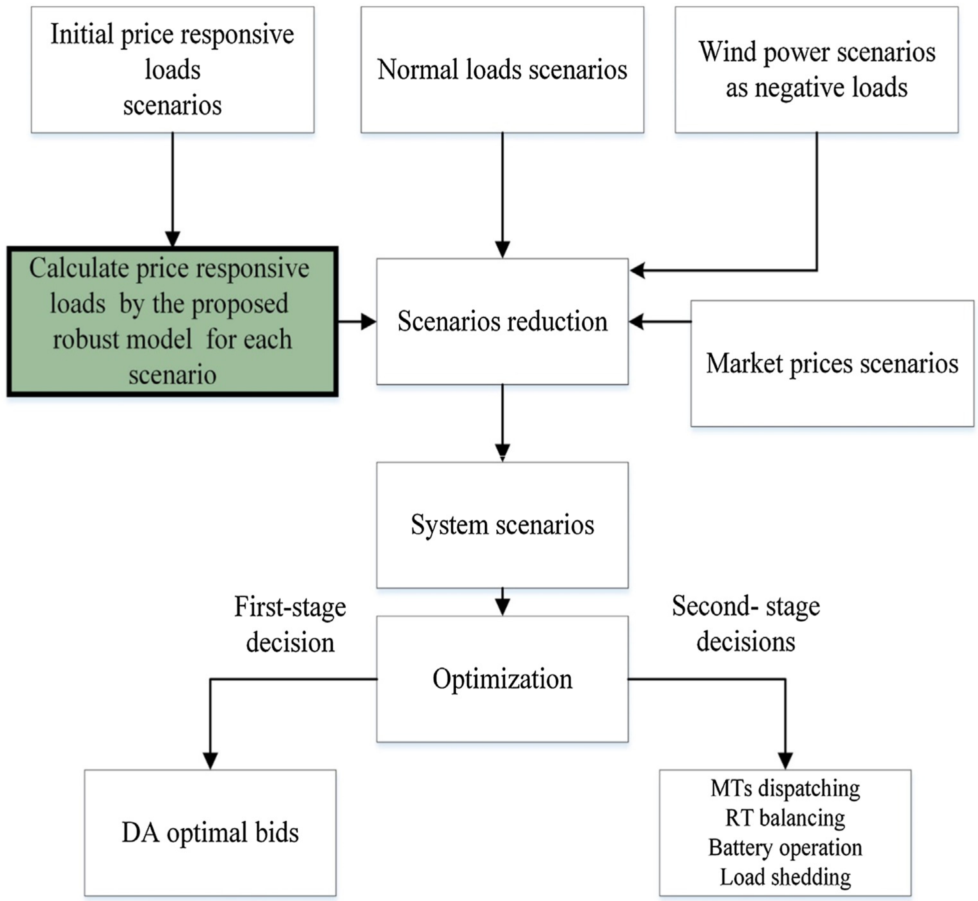

Fig. 3. Model implementation.

4. Problem formulation

4.1. Risk measure

The most common way of the risk management is to add a term to the objective function which the risk associated with profit distribution is measured [33]. In this paper, risk-averse decisions of the system are modeled using the CVaR. The CVaR at α confidence level is defined as the expected profit of the $(1-\alpha)100\%$ worst profit scenarios and can be jointed in the objective function as follows [31]:

$$\max \sum_{s=1}^{NS} \rho_s Profit_s + \beta Risk \tag{15}$$

Subject to:

$$Risk = \eta - \frac{1}{1-\alpha} \sum_{s=1}^{NS} \rho_s \chi_s \tag{16}$$

$$\chi_s \geqslant \eta - Profit_s , \quad \forall s \tag{17}$$

$$\chi_s \geqslant 0 , \quad \forall s \tag{18}$$

where α is the confidence level, $\mathrm{Profit_s}$ is the profit in the scenario $s$and ρ_s is the probability of that scenario. β is the risk aversion parameter and decisions become more robust as β increases. Also η is value-at-risk which represents the greatest level ensuring that probability of receiving a profit less than that level is lower than $1-\alpha$ [20]. χ_s is a non-negative auxiliary variable and can be determined as Eq. (19).

$$\chi_s = \begin{cases} \eta - Profit_s & Profit_s \leqslant \eta \\ 0 & otherwise \end{cases} \tag{19}$$

4.2. Objective function

The objective function without risk control is maximizing the expected profit as follows:

$$\max \sum_{s=1}^{NS} \rho_s Profit_s \tag{20}$$

where the profit in each scenario equals to:

$$Profit_s = \sum_{t=1}^{NT} \{\Delta T(\lambda^{da}_{t,s} d^{net}_{t,s} - \lambda^{da}_{t,s} P^{da}_{t} - \lambda^{real}_{t,s} P^{real}_{t,s} - \sum_{w=1}^{Nw} C^{gen}_{w,t,s} - \lambda^{pen}_{t} |P^{pen}_{t,s}| - VOLLshed_{t,s}) - \sum_{i=1}^{NDG} C^{ope}_{i,t,s}\} \tag{21}$$

The profit in each scenario as mentioned in Eq. (21) is the sum of incomes obtained from selling electricity to smart MG loads and trading in electricity markets, minus the cost of trading in electricity markets, WT and MTs operating costs, cost of bids deviation and cost of load shedding. The following Eq. (22), represents the power penalty quantities that smart MG must be traded in the RT electricity market over the operating day for balancing them. This penalty is for DA bids deviations and equals to the difference between the real power delivery and DA bids. The real power delivery is traded between the smart MG and the upstream network over the operating day. So, the equation for the $P^{real}_{t,s}$in the Eq. (21) is like Eq. (22). The DA electricity market transactions P^{da}_{t} , the RT electricity market transactions $P^{real}_{t,s}$and the real power delivery $P^{del}_{t,s}$would be negative if smart MG is selling power to the upstream network.

$$P^{pen}_{t,s} = P^{del}_{t,s} - P^{da}_{t} , \quad \forall t,s \tag{22}$$

4.2.1. Constraints

The smart MG constraints are given in the following.

4.2.1.1. Mts operation constraints. The MTs operating cost consists of generation, startup and pollution costs for $\forall\ i,\ t,\ s$ and can be modeled as follows:

$$C_{i,t,s}^{ope} = C_{i,t,s}^{gen} + C_{i,t,s}^{start} + C_{i,t,s}^{em} \tag{23}$$

$$C_{i,t,s}^{gen} = a_i V_{i,t,s} + \Delta T \sum_{m=1}^{Ni} \lambda_{i,m} P_{i,m,t,s} \tag{24}$$

$$P_{i,1,t,s} \leqslant P_{i,1}^{\max} - P_i^{\min} \tag{25}$$

$$P_{i,m,t,s} \leqslant P_{i,m}^{\max} - P_{i,m-1}^{\max}\ \ ,\quad \forall\ m = 2 \ldots NL_i - 1 \tag{26}$$

$$P_{i,N_i,t,s} \leqslant P_i^{\max} - P_{i,N_i-1}^{\max} \tag{27}$$

$$P_{i,t,s} = P_i^{\min} V_{i,t,s} + \sum_{m=1}^{N_i} P_{i,m,t,s} \tag{28}$$

$$C_{i,t,s}^{start} = k_{start}\, on_{i,t,s} \tag{29}$$

$$C_{i,t,s}^{em} = C_{CO_2}\, CO_{2,i,t} P_{i,t,s} \tag{30}$$

The generation cost of MTs in Eq. (24), is modeled by piecewise linear function [43,19] where a_i, m, N_i and $\lambda_{i,m}$are generation cost of MT i at its minimum power, the indices of parts in the cost function of MT i, the number of parts in the cost function of MT i and the marginal cost of segment m in the cost function of MT i($/kWh), respectively [37]. $P_{i,m}^{\max}$ and $P_{i,m,t,s}$ in Eqs. (25)–(28) are the maximum limit of power generation in the m-th part of MT i cost function (kW) and power generation of MT $i$from the m-th part at time t in scenario s, respectively [19]. Startup cost and pollution cost are given in Eqs. (29) and (30), respectively. The operation constraints for MTs are as follows:

$$P_i^{min} V_{i,t,s} \leqslant P_{i,t,s} \leqslant P_i^{max} V_{i,t,s} \tag{31}$$

$$P_{i,t,s} - P_{i,t-1,s} \leqslant RU_i(1 - on_{i,t,s}) + P_i^{min} on_{i,t,s} \tag{32}$$

$$P_{i,t-1,s} - P_{i,t,s} \leqslant RD_i(1 - off_{i,t,s}) + P_i^{min} off_{i,t,s} \tag{33}$$

$$\sum_{\ell=t}^{t+MUT_i-1} V_{i,\ell,s} \geqslant MUT_i on_{i,t,s} \tag{34}$$

$$\sum_{\ell=t}^{t+MDT_i-1} (1 - V_{i,\ell,s}) \geqslant MDT_i off_{i,t,s} \tag{35}$$

$$on_{i,t,s} - off_{i,t,s} = V_{i,t,s} - V_{i,t-1,s} \tag{36}$$

$$on_{i,t,s} + off_{i,t,s} \leqslant 1 \tag{37}$$

The boundary capacity limit of MTs power generation is modeled in Eq. (31). The MTs' ramping rates are limited in Eqs. (32) and (33). The minimum up/down time constraints for MTs are represented in Eqs. (34) and (35) and finally, the binary variables relationships are formulated as Eqs. (36) and (37) [37].

4.2.1.2. generation cost. The generation cost of WT w at time t in scenario $s$is equal to Eq. (38).

$$C_{w,t,s}^{gen} = C_w \times P_w(WS_{t,s}) \tag{38}$$

$$C_w = \frac{In_w \times \frac{\text{int}(1+\text{int})^{n_w}}{(1+\text{int})^{n_w}-1} + C_w^{o\&m}}{P_w^r \times CF_w \times 8760} \tag{39}$$

The per kilowatt hour generating cost of WT $w$in Eq. (39), consists of investment, operation and maintenance costs [10].

4.2.1.3. Battery constraints. The battery model For $\forall\ b,\ t,\ s$ is as follows [20]:

$$0 \leqslant P_{b,t,s}^C \leqslant b_{b,t,s}^C P_b^C;\ \ 0 \leqslant P_{b,t,s}^D \leqslant b_{b,t,s}^D P_b^D \tag{40}$$

$$SoC_{b,NT,s} = SoC_{b,1,s};\ \ \text{SoC}_b^{min} \leqslant SoC_{b,t,s} \leqslant SoC_b^{max} \tag{41}$$

$$SoC_{b,t+1,s} = SoC_{b,t,s} + \Delta T\left(\frac{\eta_b^C P_{b,t,s}^C}{E_b} - \frac{P_{b,t,s}^D}{\eta_b^D E_b}\right) \tag{42}$$

$$b_{t,s}^C + b_{t,s}^D = 1;\ \ b_{b,t,s}^C,\ b_{b,t,s}^D \in \{0,\ 1\} \tag{43}$$

Constraints (40) and (41) are charging and discharging power and State of Charge (SoC) limits, respectively. SoC at each time slot must equal to SoC at previous time slot plus the energy stored at the current time slot which is formulated in Eq. (42). Also, Eq. (43) prevents battery charging and discharging at the same time.

4.2.1.4. Exchangeable power constraint. Exchangeable power at each time with the upstream network is constrained like Eq. (44) due to technical constraints such as transformer's capacity.

$$|P_{t,s}^{del}| \leqslant P_t^{grid}\ ,\ |P_t^{da}| \leqslant P_t^{grid}\quad \forall\ \text{t},\ s \tag{44}$$

4.2.1.5. Reliability constraints. In this paper, the expected energy not supplied (EENS) is used as a smart MG reliability index like Eq. (45). The manager is penalized in the objective function for curtailing the loads based on the value of lost load (VOLL). VOLL is the energy price for compensating curtailed loads [11]. Additionally, to avoid decreasing the smart MG reliability, EENS is limited to below 100 KWh as Eq. (46).

$$EENS = \sum_{t=1}^{NT} \sum_{s=1}^{NS} \rho_s shed_{t,s} \tag{45}$$

$$EENS \leqslant 100 \tag{46}$$

4.2.1.6. Power balance. The sum of total power generation of MTs and WT, the real power delivery and total net charging/discharging power from battery should be equal to net load as follows:

$$\textstyle\sum_{i=1}^{NDG} P_{i,t,s} + P_{t,s}^{del} + \sum_{b=1}^{Nb} (P_{b,t,s}^D - P_{b,t,s}^C) = d_{t,s}^{net}\ \ \forall \text{t,s} \tag{47}$$

where

$$d_{t,s}^{net} = d_{t,s}^{ne} + d_{t,s}^{e} - shed_{t,s} - \sum_{w=1}^{Nw} P_{w,t,s}\ \ \forall \text{t,s} \tag{48}$$

5. Numerical results

5.1. Smart MG data

The proposed optimization problem is applied to a typical MG including two similar MTs, one WT, one battery and some price responsive and normal loads. The MTs' parameters are shown in Table 1 [6]. The carbon dioxide (CO_2) produced by an MT is considered to be

Table 1
Parameters of MTs.

Unit	a ($)	b ($/kW)	c ($/kW^2)	P^{min} (kW)	P^{max} (kW)
MT 1	0.4	0.0397	0.00051	20	60
MT 2	0.4	0.0397	0.00051	20	60
Unit	MUT (h)	MDT (h)	RU (kW/h)	RD (kW/h)	CO_2 (kg/kWh)
MT 1	1	1	40	40	0.7
MT 2	1	1	40	40	0.7

0.7 kg/kWh and its emission cost is 0.001 \$/kg [44]. The other environmental pollutants like sulfur dioxide (SO_2) and nitrogen dioxide (NO_2) are neglected for MTs due to their few productions. The MTs' quadratic generation cost is approximated by a three-piece wise linear function. The 60 kW WT data are taken from [5]. The WT' unitary capital cost due to [45] is considered 1170 \$/kW and it is paid with a 6%, 20 year-loan. The installation and the operation and maintenance costs of WT are considered 30% and 1.5% of its capital cost, respectively [45]. The battery capacity is 50 kWh and its maximum charging/discharging power is 25 kW. The charging/discharging efficiency are 0.9 and 0.85, respectively [6].

The scheduling horizon is 24-h and each time slot is considered to be 1 h. The forecast initial price responsive loads and normal loads and DA market prices are shown in Fig. 4. The elasticity matrix coefficients and their considered upper and lower bounds for price responsive loads are shown in Table 2 [38]. The initial prices of different time slots, i.e. off-peak, mid-peak and peak are assumed to be 0.1, 0.13 and 0.17 \$/kWh, respectively. These prices are chosen by sensitivity analysis method so that changing the behavior of price responsive loads in response to DA market price variations does not lead to the reduction of smart MG's load factor. 144 Wind speed data are retrieved from [46]. The sampling rate is equal to one sample per ten minutes and we have 6 samples per hour. So the Weibull parameters can be fitted in every hour and its hourly PDF is obtained. Fig. 5 shows the hourly average of wind data. The wind scenario in each hour is obtained from its average value as a forecasted value and its random value obtained from Weibull PDF at that hour. The VOLL, bid deviations penalty and confidence level α are set equal to 0.25 \$/kWh, 0.05 \$/kWh and 0.9, respectively.

The hourly expected values of DA and RT market price scenarios with their standard deviation are presented in Table 3. Also, all 50 DA and RT stochastic scenarios are shown in Figs. 6 and 7. The bolded curves are the expected values of prices. The proposed uncertainty model of price responsive loads and scenarios generation and reduction process are performed by MATLAB while the optimization problem is solved by GAMS software [47] using CPLEX solver. It is worthwhile to note that the absolute value function given in Eq. (21) leads to the nonlinearity of optimization problem. Therefore, a natural convex relaxation method described in [41] is used for solving this problem.

5.2. Simulation results

In this section, the simulation results of proposed optimization model are shown and the effectiveness of proposed contribution of this paper is analyzed using sensitivity analysis. Fig. 8, shows the simulation results of the proposed uncertainty modeling of price responsive loads. According to this figure, the initial price responsive loads (i.e. blue line in Fig. 8) are converted into price responsive loads (i.e. black line in Fig. 8) by responding to DA market price variations. Therefore, the energy consumption reduces at peak time slots and shifts to off-peak time slots. On the other hand, we can see with increasingΓ, the elasticity of price responsive loads decreases. It means with considering the worst-case elasticity of price responsive loads scenario, the consumption of price responsive loads at peak time slots will increase and at the off-peak time slots will decrease compared to their uncertain quantities (i.e. black line in Fig. 8). In this simulation result, the maximum price responsive loads growth at peak and off-peak periods are 4.6% and 3%, respectively. To observe the effect of considering the uncertainty of price responsive loads on the optimal decisions, two different case studies are examined as follows:

- Case 1: Solving the proposed optimization model without considering the uncertainty of price responsive loads.

In this case, the DA optimal bids and the optimal decisions of manager over the operating day compute without considering the uncertainty of price responsive loads. Fig. 9 shows all the smart MG's component behavior under $\Gamma = 0$ and $\beta = 1$ in first possible scenario on the operating day. As can be seen, the MTs are operated at their full power capacity and at the whole time slots. Also, the surplus energy at time slots 1, 2, 4, 9 and 17 are balanced due to charging the battery at these time slots. Also, the battery is discharged at time slots 6, 11, 12, 23 and 24. Therefore, the battery often is charged at low electricity price time slots and it is discharged at high electricity price time slots.

The DA optimal bids and the real power delivery in the first scenario are shown in Fig. 10. As can be seen, the manager has to balance the smart MG energy in the RT electricity market due to small deviations between DA optimal bids and real power delivery quantities. Thus, it sells surplus power at time slots 11–23 in the RT electricity market while paying \$ 9 to the upstream market operator for DA bidding small deviation. The expected profit under whole stochastic scenarios is \$ 277 and the expected profit in the 10% worst scenarios is \$ 268.

The sensitivity analysis results of the risk aversion parameter (i.e. β) on the optimal solutions are shown in Figs. 11–15. According to Figs. 11 and 12, the power procurement from DA electricity market increases at peak time slots as the risk aversion parameter increases. It can be said more in detail that, the smart MG tends to purchase more power in the DA electricity market and sell more power for balancing in the RT electricity market during time slots when the expected DA price is lower than the expected RT price (e.g. the hours 9,10,12,15 and 18 in Table. 3). Moreover, the smart MG tends to purchase more power in the DA electricity market during time slots when the expected DA price is higher than the expected RT price, too (e.g. the hours,16, 20, 21 and 22 in Table 3). Because in several stochastic scenarios, the RT price is higher than DA price. Hence, in order to reduce the risk of receiving lower profits in some bad RT scenarios, the smart MG should buy more power from the DA electricity market. In order to have more energy at peak times for selling in RT electricity market, in addition to purchasing power more in DA electricity market, the production of MTs are fixed in their full capacity and the discharged energy has also risen. This is shown in Figs. 13 and 14.

On the other hand at off-peak time slots with increasing β, in order to avoid losing profit in some bad price scenarios , the smart MG prefers to use non-market elements such as MTs and battery. It means the smart MG purchases less power in DA market and instead increases the production of MTs and battery charging power. This is shown in Figs. 13 and 14 at hours 1–5. Fig. 15 illustrates the impact of risk aversion parameter on the expected profit. As can be seen, the expected profit decreases and the CVaR increases as β increases. increasing the risk aversion parameter, the expected return of the smart MG operation strategy decreases, while its associated risk decreases as well.

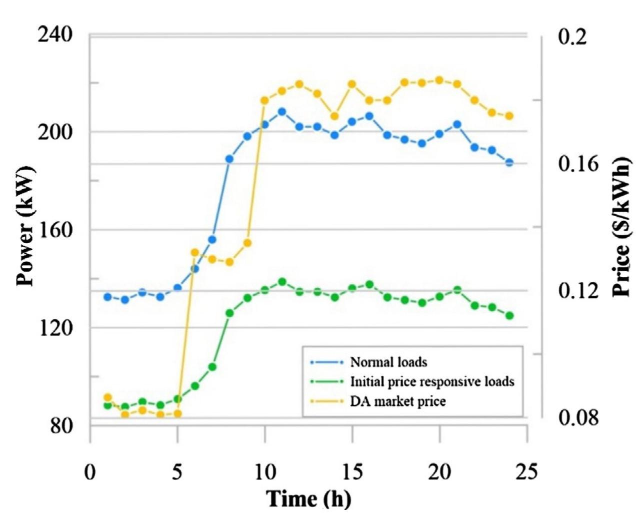

Fig. 4. Forecast values of loads and DA market price.

Table 2
Estimated values of elasticity matrix coefficients with their upper and lower bounds.

Time slots	Off-peak (1-6)			Mid-peak (6-7)			Peak (7-24)		
	Estimated	Lower bound	Upper bound	Estimated	Lower bound	Upper bound	Estimated	Lower bound	Upper bound
Off-peak (1-6)	−0.15	−0.16	−0.05	0.1	0.05	0.15	0.08	0.04	0.12
Mid-peak (6-7)	0.1	0.05	0.15	−0.24	−0.36	−0.12	0.08	0.04	0.12
Peak (7-24)	0.08	0.04	0.12	0.08	0.04	0.12	−0.22	−0.23	−0.1

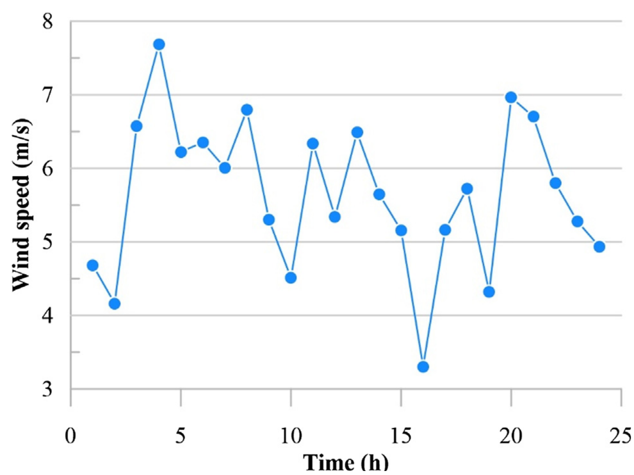

Fig. 5. Hourly forecasted wind speed.

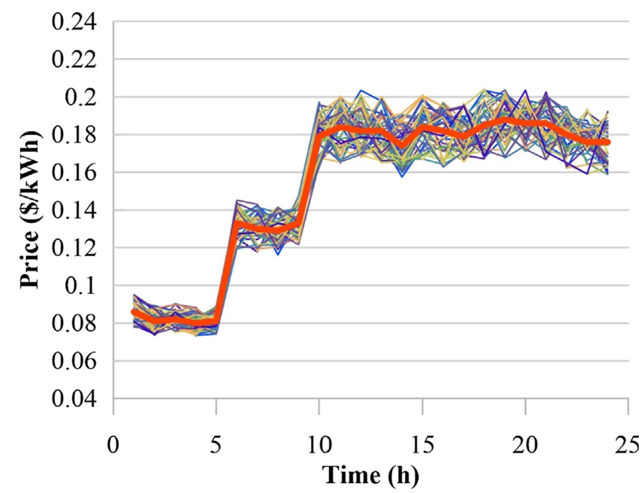

Fig. 6. The DA price scenarios with their bolded expected values.

- Case 2: Solving the proposed optimization model with considering the uncertainty of price responsive loads.

Figs. 16–19 illustrate the effect of considering the uncertainty of price responsive loads on the optimal solutions. As can be seen, it affects more on the DA optimal bids and charging/discharging of the battery. According to Fig. 16, the DA optimal bids decrease during off-peak time slots (i.e. time slots 1–6) and increase during mid-peak and peak time slots (i.e. time slots 6–24) as the conservatism degree increases. In fact, the smart MG has to submit less power during off-peak time slots and more power during mid-peak and peak time slots in the DA electricity market. The reason is that a higher value of Γrepresents fewer values of price responsive loads during off-peak time slots and higher values of price responsive loads during mid-peak and peak time slots. In an ideal demand response program, all the price responsive consumers are expected to shift their demand from peak periods to off-peak periods. The property of robust proposed model is that it does not take risks and chooses the worst kind of demand response. That means that a smaller percentage of consumers will shift their consumption. Then the peak load is increased and the off-peak load is decreased relative to the ideal mode (see Fig. 8). It worth to mentioning that the risk is controlled in case 2 with value of 1 for the risk aversion parameter β.

Variations of DA optimal bids with increasing $\Gamma$are obtained in Table 4. According to these results, the price responsive loads uncertainty affects DA bids values. The manager of the smart MG by taking into account the price responsive loads behavior and possibility of not responding to the prices, may not risk and increase its conservatism degree for sending the optimal bids. Therefore, the solutions in the third column are more robust than the first one.

Also, with increasing Γ the charging/discharging level of battery increases during different time slots. This is shown in Fig. 17. The *SoC* in low price hour 2 is 0.6 and has increased to 0.62 and 0.68 for $\Gamma = 12$ and $\Gamma = 24$, respectively. Also, *SoC* in high price hour 19 is 0.71 and has decreased to 0.68 and 0.64 for $\Gamma = 12$ and $\Gamma = 24$, respectively. The

Table 3
Statistics information of DA and RT scenarios.

Time (h)	DA market price ($/kWh) Normal distribution		RT market price ($/kWh) Normal distribution		Time (h)	DA market price ($/kWh) Normal distribution		RT market price ($/kWh) Normal distribution	
	Expected values	Standard deviation	Expected values	Standard deviation		Expected values	Standard deviation	Expected values	Standard deviation
1	0.086	0.005	0.085	0.006	13	0.182	0.01	0.181	0.014
2	0.081	0.005	0.081	0.007	14	0.174	0.01	0.173	0.014
3	0.082	0.004	0.083	0.007	15	0.184	0.009	0.186	0.016
4	0.080	0.005	0.081	0.007	16	0.182	0.01	0.180	0.015
5	0.081	0.004	0.080	0.007	17	0.179	0.009	0.178	0.014
6	0.133	0.007	0.134	0.008	18	0.185	0.011	0.186	0.013
7	0.130	0.006	0.130	0.01	19	0.188	0.01	0.183	0.017
8	0.129	0.007	0.126	0.01	20	0.186	0.009	0.184	0.015
9	0.133	0.007	0.134	0.01	21	0.186	0.011	0.183	0.014
10	0.179	0.010	0.181	0.015	22	0.180	0.01	0.179	0.014
11	0.184	0.011	0.182	0.016	23	0.176	0.007	0.176	0.014
12	0.182	0.011	0.183	0.013	24	0.176	0.011	0.175	0.016

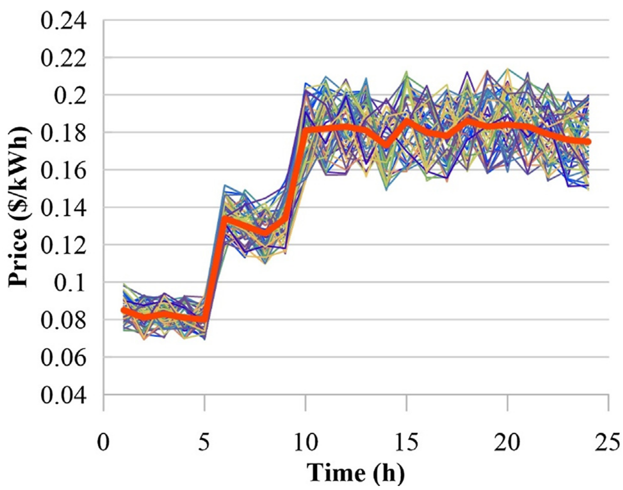

Fig. 7. The RT price scenarios with their bolded expected values.

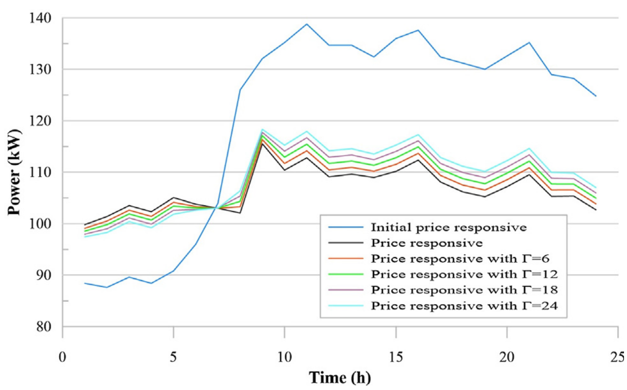

Fig. 8. Price responsive loads variation with different conservatism degree.

reason is the increase in price responsive loads. So, considering the uncertainty of price responsive loads, the battery will charge more energy at lower price hours and discharge more energy in high price hours. Therefore, battery plays an important role in the profitability of the system's manager as shown in Table 5. In this Table, the sensitivity of smart MG's expected profit over conservatism degree Γis shown with and without consideration of battery. According to this table, the expected profit of the smart MG is increased slightly by considering the different conservatism degrees of price responsive loads. The reason is that the probability of more consumption of price responsive loads has increased at high price peak time slots. According to Eq. (21), the consumption of loads is the source of revenue for the system's manager.

Fig. 18 depicts the MTs production with increasing Γ. As can be seen, their production decrease at hour 3 due to the decrease of price responsive load at this hour. However, they work at full capacity for the rest of the time slots. Also, the balancing power interaction with RT market increase with increasing Γas shown in Fig. 19. The reason is the increase in price responsive loads.

The manager would cost more when a contingency occurs and smart MG has to be operated in island mode. Table 6 shows the EENS values when smart MG has to operate in island mode. As can be seen, with increasingΓ, the EENS values increase in island mode. Because fewer price responsive consumers are considered in the proposed uncertainty model and therefore the system's demand is increased. Hence, with increasingΓ, the smart MG operation risk increases.

6. Conclusion

In this paper, the uncertainty related to the price responsive loads is tackled by employing a robust method and tested in a risk-constrained stochastic optimization framework for a smart MG optimal operation problem. To demonstrate the efficiency of proposed uncertainty model, two cases including solving a risk-constrained optimal operation with and without price responsive loads uncertainty are considered.

The following conclusions are in order:

1. Using the utilized robust uncertainty model, the elasticity of price responsive loads decreased by 4.6% in comparison with its certain value.
2. Applying the proposed uncertainty robust model, affected the DA optimal bids and optimal decisions over the operating day such as MT's dispatch, battery operating points, and RT interactions.
3. The smart MG should purchase more power in DA electricity market during mid-peak and peak time slots and should purchase less power at the off-peak time slots due to uncertain behaviors of price responsive loads. This is especially important when the smart MG is price-maker in the electricity market. As the simulation results showed, the optimal DA bids values were deviated from their main

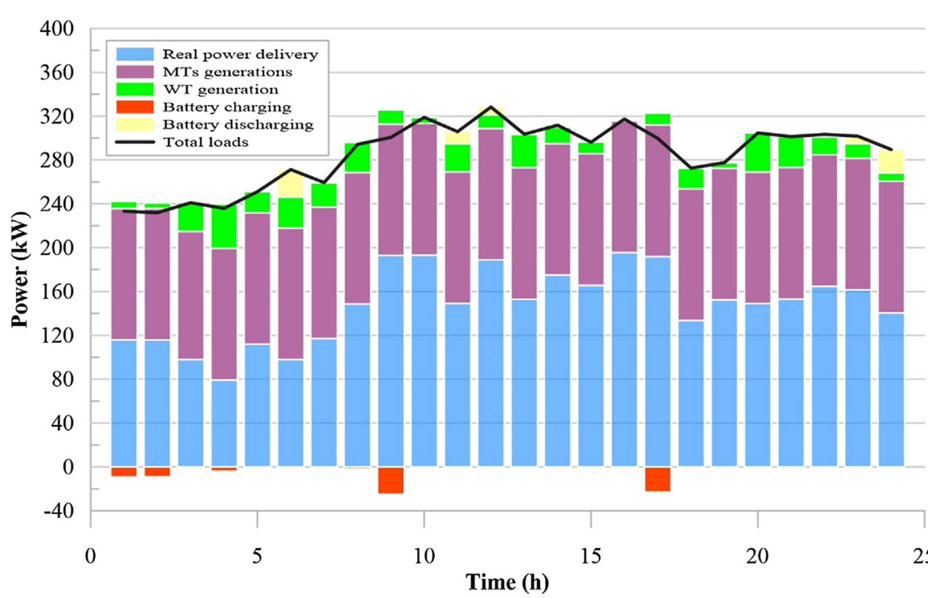

Fig. 9. Component behavior under Γ = 0 and $\beta = 1$ in the first possible scenario.

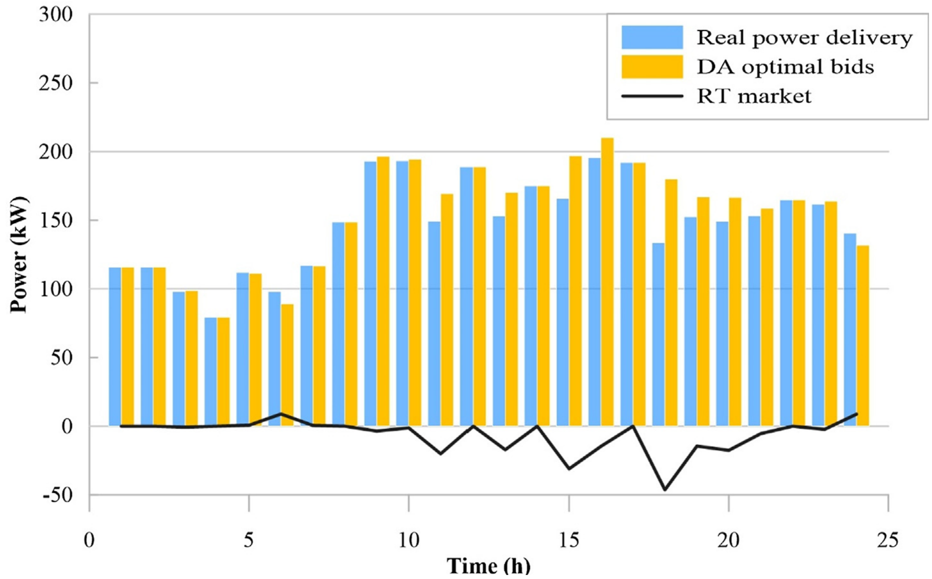

Fig. 10. Deviations between DA optimal bids and real power delivery quantities with $\beta = 1$in first scenario.

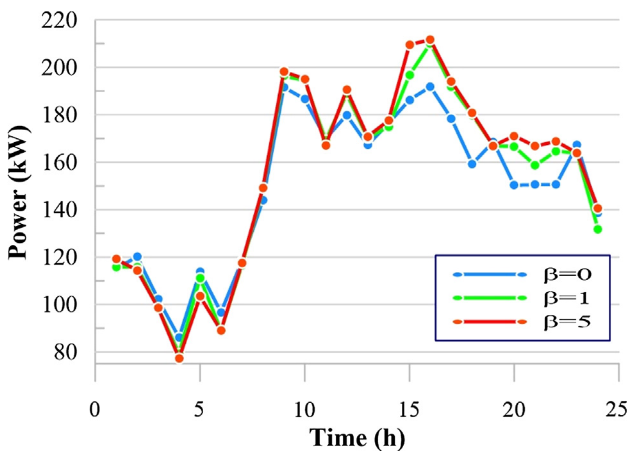

Fig. 11. Impact of risk aversion parameter on the DA bids.

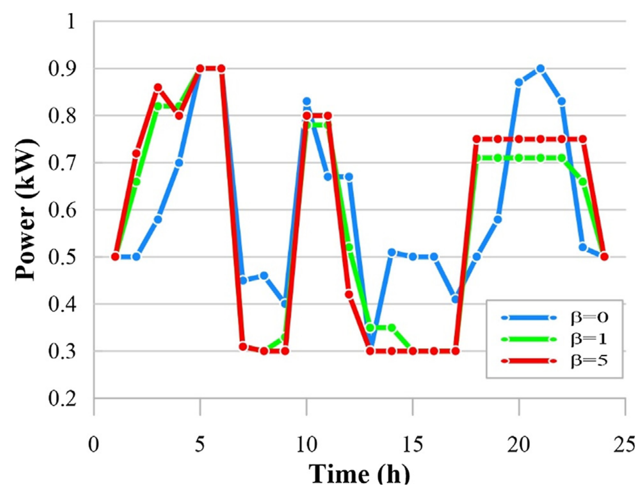

Fig. 13. Impact of risk aversion parameter on the SoC.

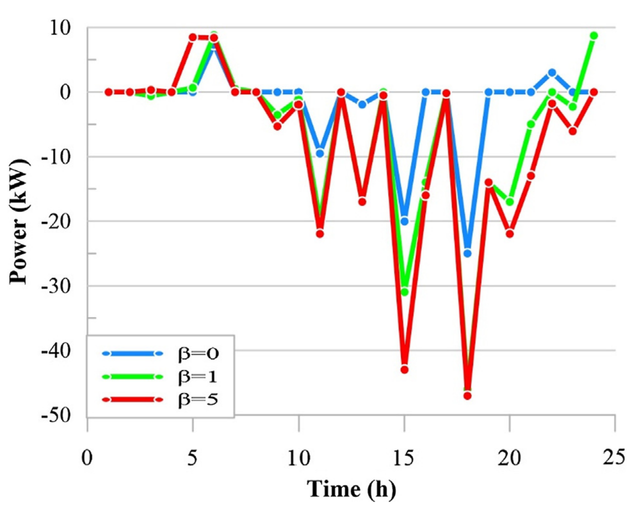

Fig. 12. Impact of risk aversion parameter on the RT bids.

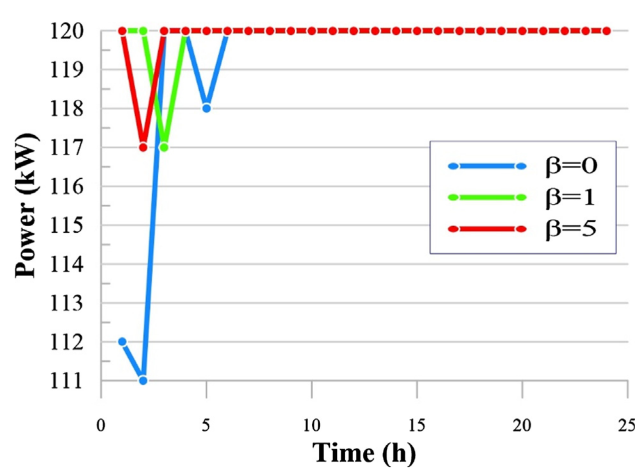

Fig. 14. Impact of risk aversion parameter on the MTs operation.

calculated values by increasing the conservatism degree. This can affect the system's profit.

4. Sensitivity analysis of smart MG's profits and EENS showed that reducing price responsive loads elasticity will increase smart MG expected profit while it will decrease the system reliability.
5. The optimal operating points of battery changed. It was more charged at lower price time slots and more discharged at high price

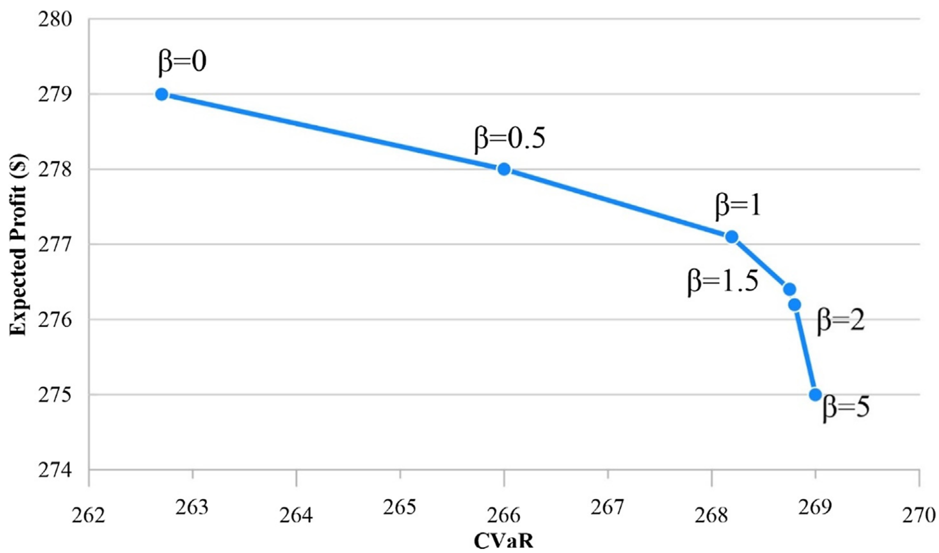

Fig. 15. Impact of risk aversion parameter on the expected profit.

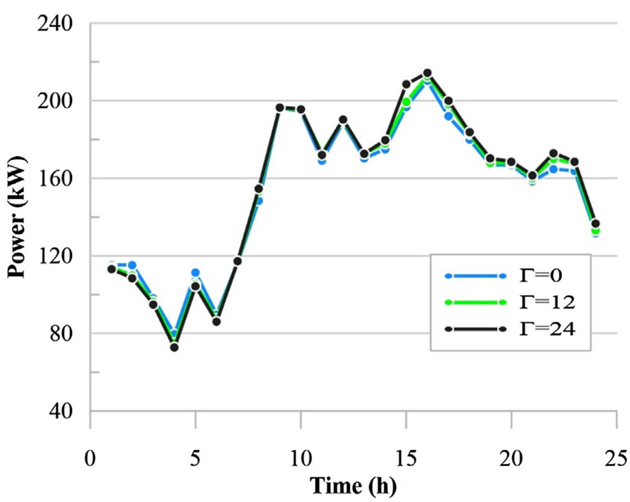

Fig. 16. Impact of conservatism degree on the DA bids.

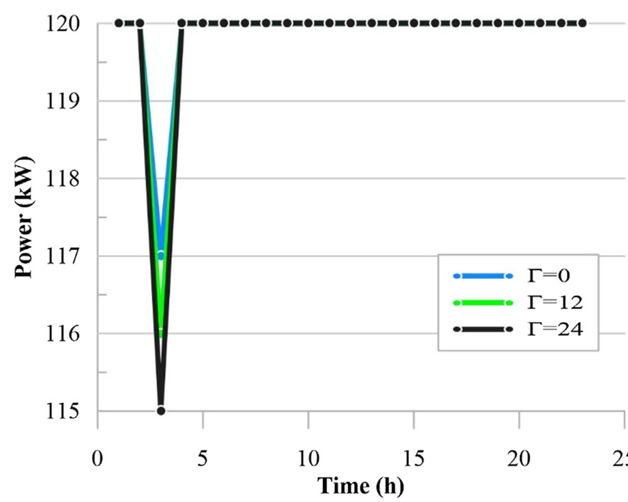

Fig. 18. Impact of conservatism degree on the MTs operation.

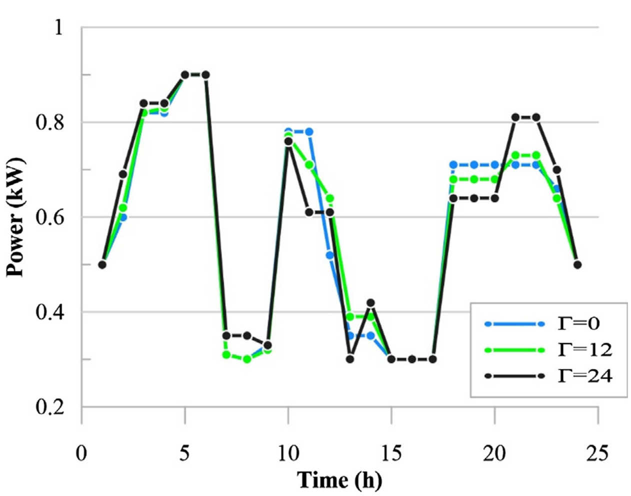

Fig. 17. Impact of conservatism degree on the SoC.

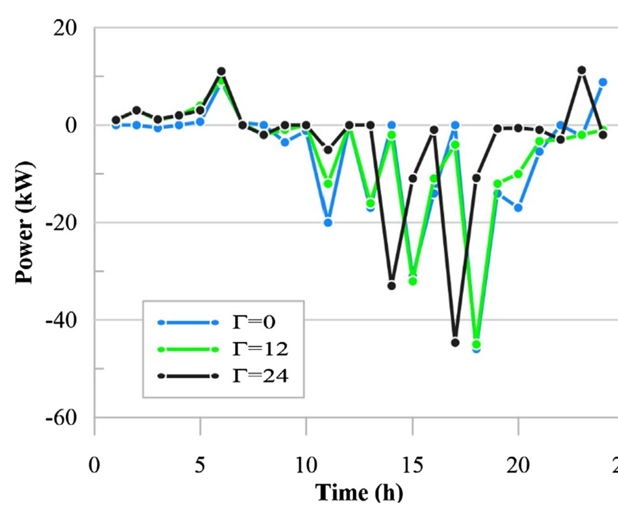

Fig. 19. Impact of conservatism degree on the RT bids.

Table 4
Variations of DA optimal bids.

Time (h)	Γ = 0	Γ = 12		Γ = 24	
	DA bids	DA bids	Variations (%)	DA bids	Variations (%)
1	115.3	115.1	−0.17	114.5	−0.69
2	115.3	113.0	−1.99	111.5	−3.29
3	98.1	97.4	−0.71	96.0	−2.14
4	79.5	77.2	−2.89	74.7	−6.03
5	111.3	108.6	−2.42	107.4	−3.5
6	89.7	88.5	−1.33	87.5	−2.45
7	116.5	116.6	0.08	117.3	0.68
8	148.5	151.2	3.83	152.6	2.7
9	196.3	196.4	0.05	196.5	0.1
10	194.4	195.1	0.36	195.7	0.66
11	169.2	171.0	0.71	171.6	1.41
12	188.8	189.7	1.06	190.4	0.84
13	170.2	172.7	1.58	172.6	1.41
14	175.0	176.1	0.6	178.6	2.05
15	196.8	198.3	0.7	204.6	3.96
16	210.2	211.7	0.3	213.4	1.52
17	192.0	194.2	0.3	196.1	2.13
18	179.9	181.8	0.71	182.8	1.6
19	166.9	167.9	0.59	170.2	1.97
20	166.7	167.0	0.17	167.6	0.54
21	158.6	159.2	0.37	160.5	1.2
22	164.7	167.0	1.39	170.0	3.2
23	163.8	165.5	1.03	166.3	1.52
24	131.7	132.3	0.45	134.8	2.35

Table 5
Expected profit.

Conservatism degree	Γ = 0	Γ = 12	Γ = 24
Expected profit with battery ($)	268	268.8	269.4
Expected profit without battery ($)	251	251.8	252.4

Table 6
EENS values in the islanded mode.

Conservatism degree	Γ = 0	Γ = 12	Γ = 24
EENS	3578	3600	3650

time slots with increasing the conservatism degree.

6. Finally, we conclude that considering the price responsive loads uncertainty can effect on the system's component and optimal profit and decisions.

Lastly, using other models that do not depend on input variables, such as information gap decision theory (IGDT) for modeling the price responsive load uncertainty is a good topic for future work.

Acknowledgements

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

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