The first derivative can be used to approximate the value of functions. The approximation is done by considering the *difference quotient*. Let $f[x]$ be a differentiable function in x. Then, the difference quotient is defined as

$$f'[x] \equiv \frac{f[x+\Delta x] - f[x]}{\Delta x} \,.$$

This approximation is called the *best linear approximation* to f near x.

From such an approximation, derive

$$f[x+\Delta x] \sim \Delta x f'[x] + f[x] \;,$$

which can be used in approximations, specially when Δx is small compared to x.

As an example, estimate the value of $\sin 0.1$. First, since sine is differentiable for all of reals, then $\sin' = \cos$. Next, note that $0.1 = 0.0 + 0.1$. thus, relate $x = 0.0$ and $\Delta x = 0.1$ and replace such values in the approx equation:

$$\sin[0.1] = \sin[0.0 + 0.1] \sim 0.1 \cdot \cos 0.0 + \sin 0.0 \sim 0.1 \,.$$

Using a calculator, one finds that $\sin 0.1 \sim 0.0998334166468 28$. So the approximation error is 0.17%.

As another example, estimate $\cos 0.1$. One can repeat the last calculation as for the sine, but it's easier to use the periodicity of the cosine in this case:

$$\cos[0.1] = \sin[0.1 + \tau/4] \sim \sin[\tau/4] \sim 1.0 \,,$$

where $\tau = 2\pi$. In geometric terms, we have translated the cosine function graph by $\tau/4$, which coincides with the sine function graph.

Using a calculator, one finds $\cos 0.1 \sim 0.995004165278026$.

[v.0.0.1 19:48 01/09/2014]