SIMURGH

CURVE FITTING

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[Harold Cohen. Numerical approximation methods, p. 25.]

Theory predicts that the data in table 1 decreases with increasing x as a quadratic in $1/x$. To find the best fit of these data, define

$$f[x] = \alpha + \beta\frac{1}{x} + \gamma\frac{1}{x^2}\,.$$

Minimize the last equation by

$$e = \sum_k \left(\alpha + \beta\frac{1}{x} + \gamma\frac{1}{x^2} - f_k\right)^2 [1 \le k \le 4]_{\text{iv}}$$

resulting in

$$\partial_\alpha e = 2\sum_k \left(\alpha + \beta\frac{1}{x_k} + \gamma\frac{1}{x_k^2} - f_k\right)[1 \le k \le 4]_{\text{iv}} = 0\,,$$
$$\partial_\beta e = 2\frac{1}{x_k}\sum_k \left(\alpha + \beta\frac{1}{x_k} + \gamma\frac{1}{x_k^2} - f_k\right)[1 \le k \le 4]_{\text{iv}} = 0\,,$$
$$\partial_\gamma e = 2\frac{1}{x_k^2}\sum_k \left(\alpha + \beta\frac{1}{x_k} + \gamma\frac{1}{x_k^2} - f_k\right)[1 \le k \le 4]_{\text{iv}} = 0\,.$$

Then, obtain

$$\alpha + \beta\sum_{k=1}^{4}\frac{1}{x_k} + \gamma\sum_{k=1}^{4}\frac{1}{x_k^2} = \sum_{k=1}^{4} f_k\,,$$
$$\alpha\sum k = 1^4\frac{1}{x_k} + \beta\sum_{k=1}^{4}\frac{1}{x_k^2} + \gamma\sum_{k=1}^{4}\frac{1}{x_k^3} = \sum_{k=1}^{4} f_k\frac{1}{x_k}\,,$$
$$\alpha\sum k = 1^4\frac{1}{x_k^2} + \beta\sum_{k=1}^{4}\frac{1}{x_k^3} + \gamma\sum_{k=1}^{4}\frac{1}{x_k^4} = \sum_{k=1}^{4} f_k\frac{1}{x_k^2}\,.$$

x	$f[x]$
1.3	5.42
2.2	4.28
3.7	3.81
4.9	3.62

Table 1 Data with inverse power of x decrease

This becomes into

$$\begin{aligned}
4.000\alpha + 1.698\beta + 0.913\gamma &= 17.130\\
1.698\alpha + 0.913\beta + 0.577\gamma &= 7.883\\
0.913\alpha + 0.577\beta + 0.400\gamma &= 4.52\,.
\end{aligned}$$

from which $\alpha = 3.261$, $\beta = 1.480$, and $\gamma = 1.722$. With these values, the best fitting equation is

$$f[x] = 3.261 + 1.480\frac{1}{x} + 1.722\frac{1}{x^2}$$

and the error in fitting

$$e = \sum_{k=1}^{4}\left(3.261 + 1.480\frac{1}{x} + 1.722\frac{1}{x^2} - f_k\right)^2 = 0.001\,. \qquad \square$$