SLAW EXAMPLE

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ABSTRACT. Abstract goes here :)

CONTENTS

1. SLAW

SLAW is a computer program that derives representative scaling laws of a process from sensitivity analysis experimental data.

The objective of the program algorithm is to determine the characteristic value of a variable $Y[X]$ (the *dependent variable*) of some process depending only on n problem parameters, $X_1, \ldots, X_n$ (the *independent variables*). The algorithm is based on an integration of dimensional analysis with a multivariate linear regression backward elimination procedure. In addition to the scaling laws, the program provides a set of dimensionless groups ranked by relevance.

1.1. **Assumptions and model.** The basic assumption is that the characteristic value is given by

$$Y = a \prod_{j=1}^{n} X_j^{\sum_{i=0}^{m} a_{ij}} .$$

SLAW identifies this power law with m dimensionless quantities ranked by their significance to the characteristic value by using the model

$$Y = a \prod_{j=1}^{n} X_j^{a0j} \prod_{j=1} m\Gamma_i ,$$

where a is a numeric constant and the dimensionless quantities Γ_i are given by

$$\Gamma_i = \prod_{j=1}^{n} X_j^{a_{ij}}$$

Additionally, the algorithm takes into account uncertainties by considering only n independent variables.

1.2. **Multivariate linear regression model.** Experimental data is fitted by means of a *multivariate linear regression model*; *i.e.*, a linearization of the power law model

$$\log[Y] = \beta_0 + \sum_{j=1}^{n} \beta_j \log[X_j] + \epsilon ,$$

where ϵ is the error term and the coefficients β_0 and β_j are given by

$$\beta_0 = \log[a] \qquad \text{and} \qquad \beta_j = \sum_{i=0}^{m} a_{ij} .$$

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Description	Symbol	Dimensions
pendulum period (dependent variable)	τ	T
bob mass	m	M
pendulum length	l	L
initial angle (deviation)	θ	1
bob characteristic dimension (diameter)	d	L
density of fluid surrounding the bob	ρ	M/L^3
acceleration of free fall	g	L/T^2

Table 1. Physical quantities assumed to affect the period of a pendulum swinging through a fluid medium

More properly, experimental data is used to find the numerical values of a, β_0 and β_j by minimizing the squared sums of errors subject to linear constrains.

The program source code can be downloaded at `http://www-bcf.usc.edu/~fordon/SLAW/`.

2. Example

As an example of using SLAW, consider a pendulum swinging through a fluid medium. The characteristic variable is the pendulum period, τ. It is assumed that the problem parameters are the pendulum length, mass, fluid density and so on. Table 1 summarizes all the assumptions.

2.1. Program input.

SLAW has to be fed by two files: one containing the dimensions of the physical quantities affecting the pendulum period in a formatted file, see appendix A.1. The other file must contain the experimental data also in a formatted file, see appendix A.2.

2.2. Program output.

SLAW output is the coefficients of the model in two main steps. The first step presents a model of the main effects affecting the phenomenon. In the present case, the first step output can be summarized by

$$\tau_{\text{slaw1}} = e^{1.872} l^{0.500} g^{-0.500} = e^{1.872} \sqrt{l/g}\,.$$

The second steps results in a model that refines the first step model by adding the most significant effects. In this case, the second step output is

$$\tau_{\text{slaw2}} = e^{0.125} m^{-0.016} d^{0.048} \rho^{0.016} = e^{0.125} \frac{d^{0.048}\rho^{0.016}}{m^{0.016}}\,.$$

Finally, the overall result is found by multiplying the results of both steps

$$\tau = \tau_{\text{slaw1}}\tau_{\text{slaw2}} = e^{1.997}\sqrt{\frac{l}{g}}\left(\frac{\rho d^3}{m}\right)^{0.016}\,. \tag{2.1}$$

2.3. Interpretation.

Equation (2.1) presents the complete model for the example. In the first term, it can be seen that the period scales to

$$\tau \sim \sqrt{\frac{l}{g}}\,,$$

which coincides with the case of the ideal pendulum; *i.e.*, a pendulum that swings in vacuum.

The second term, on the other hand, represents the effect of the fluid medium on the period:

$$\tau \sim \left(\frac{\rho d^3}{m}\right)^{0.016}\,.$$

One can see that ρd^3 is the mass of the fluid displaced by the bob; that is,

$$\tau \sim \left(\frac{m_{\text{fluid}}}{m_{\text{bob}}}\right)^{0.016}\,.$$

In other words, this term is important when the mass of the bob and the mass of the fluid it displaces are comparable. Or, in English fluid resistance is significant if the bob sweeps out a mass comparable to itself.

Appendix A. Source code

A.1. Problem physical quantities. Quantities, dimensional set and quantities dimensions source file for SLAW:

```
% period  m   l   theta  diam.  air density  gravity
% s       kg  m   rad    m      kg/m3        m/s2
  0       0   1   0      1      −3           1        % L
  0       1   0   0      0      1            0        % M
  1       0   0   0      0      0            −2       % T
```

A.2. Experimental data. Sensitivity analysis experimental data used in the pendulum example:

```
%    period  m       l      theta        diameter   fluid density  gravity
%    s       kg      m      rad          m          kg/m3          m/s2
     2.248   0.295   1.265  0.085953704  0.0399034  1.195056201    9.81
     2.26    0.295   1.265  0.171423406  0.0399034  1.195056201    9.81
     1.328   0.295   0.435  0.245519496  0.0399034  1.195056201    9.81
     1.337   0.295   0.435  0.466402423  0.0399034  1.195056201    9.81
     1.844   0.1475  0.84   0.129040859  0.0345948  1.195056201    9.81
     1.84    0.1475  0.84   0.255037     0.0345948  1.195056201    9.81
     0.912   0.1475  0.192  0.516337133  0.0345948  1.195056201    9.81
     0.891   0.1475  0.192  0.278987417  0.0345948  1.195056201    9.81
     1.093   0.1475  0.287  0.362964317  0.0345948  1.195056201    9.81
     1.084   0.1475  0.287  0.189342043  0.0345948  1.195056201    9.81
     1.121   0.1475  0.287  0.651811167  0.0345948  1.195056201    9.81
     1.821   0.05    0.816  0.067300169  0.0184658  1.195056201    9.81
     1.836   0.05    0.816  0.132792341  0.0184658  1.195056201    9.81
     1.84    0.05    0.816  0.262203488  0.0184658  1.195056201    9.81
     1.866   0.05    0.816  0.491673135  0.0184658  1.195056201    9.81
     1.197   0.05    0.354  0.154134896  0.0184658  1.195056201    9.81
     1.198   0.05    0.354  0.298697423  0.0184658  1.195056201    9.81
     1.234   0.05    0.354  0.554015696  0.0184658  1.195056201    9.81
     1.255   0.05    0.354  0.655149327  0.0184658  1.195056201    9.81
     0.823   0.05    0.158  0.334982296  0.0184658  1.195056201    9.81
     0.828   0.05    0.158  0.603897219  0.0184658  1.195056201    9.81
     0.558   0.05    0.085  0.307567108  0.0184658  1.195056201    9.81
     1.434   0.02    0.502  0.109126499  0.01397    1.195056201    9.81
     1.446   0.02    0.502  0.411364964  0.01397    1.195056201    9.81
     1.528   0.02    0.502  0.716285835  0.01397    1.195056201    9.81
     1.605   0.02    0.502  0.785398163  0.01397    1.195056201    9.81
     0.924   0.02    0.202  0.494826586  0.01397    1.195056201    9.81
     0.667   0.02    0.11   0.244978663  0.01397    1.195056201    9.81
     0.671   0.02    0.11   0.244978663  0.01397    1.195056201    9.81
     2.053   0.005   1.025  0.105943307  0.0085344  1.195056201    9.81
     2.044   0.005   1.025  0.210493613  0.0085344  1.195056201    9.81
     2.059   0.005   1.025  0.40300633   0.0085344  1.195056201    9.81
     0.668   0.005   0.106  0.253837789  0.0085344  1.195056201    9.81
     0.678   0.005   0.106  0.253837789  0.0085344  1.195056201    9.81
     0.4265  0.005   0.042  0.785398163  0.0085344  1.195056201    9.81
     0.4335  0.005   0.042  0.785398163  0.0085344  1.195056201    9.81
     2.044   0.001   1.037  0.104726344  0.0050038  1.195056201    9.81
     2.049   0.001   1.037  0.208127938  0.0050038  1.195056201    9.81
     2.066   0.001   1.037  0.568847858  0.0050038  1.195056201    9.81
     0.4795  0.001   0.052  0.785398163  0.0050038  1.195056201    9.81
     2.818   0.05    1.6    0.087277713  0.0184658  998            9.81
     2.868   0.05    1.6    0.112028962  0.0184658  998            9.81
     2.884   0.05    1.6    0.031239833  0.0184658  998            9.81
     2.918   0.02    1.6    0.112028962  0.01397    998            9.81
     2.944   0.02    1.6    0.06241881   0.01397    998            9.81
     2.896   0.02    1.6    0.031239833  0.01397    998            9.81
```