
Quantum Notes

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Some notes for quantum

INTRODUCTION

Some notes continued from the full theoretical physics notes are [here](#).

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2.1 Vocabulary

Vocabulary of physics, the fountain of research ideas.

0. Fine Structure Constant

$\alpha = \frac{k_e e^2}{\hbar c} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{\hbar c} = \frac{e^2}{4\pi\epsilon_0 \hbar c}$

In electrostatic cgs units, $\alpha = \frac{e^2}{\hbar c}$.

In natural units, $\alpha = \frac{e^2}{4\pi}$.

1. Hydrogen Atom

Potential $V(r) = -\frac{Ze^2}{4\pi\epsilon_0 r}$.

Energy levels: $E_n = -\left(\frac{Z^2 \mu e^4}{32\pi^2\epsilon_0^2\hbar^2}\right) \frac{1}{n^2} = -\left(\frac{Z^2\hbar^2}{2\mu a_{\mu}^2}\right) \frac{1}{n^2} = \frac{\mu c^2 Z^2 \alpha^2}{2n^2}$.

Ground state of hydrogen atom $\psi_{100}(r) = \frac{1}{\sqrt{\pi}} \frac{1}{a^{3/2}} e^{-Zr/a}$.

2.2 Approximation Methods

2.2.1 Variational Method

Trial functions

1. $\psi(x) = \cos \alpha x$, for $|\alpha x| < \pi/2$, otherwise 0.
2. $\psi(x) = \alpha^2 - x^2$, for $|x| < \alpha$, otherwise 0.
3. $\psi(x) = C \exp(-\alpha x^2/2)$.
4. $\psi(x) = C(\alpha - |x|)$, for $|x| < \alpha$, otherwise 0.
5. $\psi(x) = C \sin \alpha x$, for $|\alpha x| < \pi$, otherwise 0.

Why don't we just use a most general variational method to find out the ground state? Because we will eventually come back to the time-independent Shrodinger equation.

Suppose we have a functional form

$$E(\psi^*, \psi, \lambda) = \int dx \psi^* H \psi - \lambda \left(\int dx \psi^* \psi - 1 \right)$$

The reason we have this Lagrange multiplier method is that the wave function should be normalized and this multiplier provides the degree of freedom. We would only get a wrong result if we don't include this DoF.

Variation of ψ^* ,

$$\delta E = \int dx \delta \psi^* H \psi - \int dx \delta \psi^* \psi = 0$$

Now what?

$$H\psi - \lambda\psi = 0$$

Not helpful.

2.2.2 Variational Method and Virial Theorem

For a potential $V(x) = bx^n$, we can prove that virial theorem is valid for ground state if we use Gaussian trial function $e^{-\alpha x^2/2}$.

A MMA proof is here.



This open source project is hosted on GitHub: [quantum](#) .

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