# Quantum Notes

***Release 1.38***

**Lei Ma**

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Some notes for quantum

CHAPTER

ONE

# INTRODUCTION

Some notes continued from the full theoretical physics notes are here.

CHAPTER

TWO

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## 2.1 Vocabulary

Vocabulary of physics, the fountain of research ideas.

0. Fine Structure Constant

:math:‘ alpha = frac{k_mathrm{e} e^2}{hbar c} = frac{1}{(4 pi varepsilon_0)} frac{e^2}{hbar c} = frac{e^2 c mu_0}{2 h}‘

In electrostatic cgs units, :math‘alpha = frac{e^2}{hbar c}‘.

In natural units, :math:‘ alpha = frac{e^2}{4 pi}‘ .

1. Hydrogen Atom

Potential $V(r) = -fracZe^2 4\pi\epsilon_0 r$.

Energy levels: :math:‘ E_{n} = -left(frac{Z^2 mu e^4}{32 pi^2epsilon_0^2hbar^2}right)frac{1}{n^2} = -left(frac{Z^2hbar^2}{2mu a_{mu}^2}right)frac{1}{n^2} = frac{mu c^2Z^2alpha^2}{2n^2}.‘

Ground state of hydrogen atom $\psi_{100}(r) = \frac{1}{\sqrt{\pi}}\frac{1}{a^{3/2}}e^{-Zr/a}$.

## 2.2 Approximation Methods

### 2.2.1 Variational Method

#### Trial functions

1. $\psi(x) = \cos\alpha x$, for $|\alpha x| < \pi/2$, otherwise 0.
2. $\psi(x) = \alpha^2 - x^2$, for $|x| < \alpha$, otherwise 0.
3. $\psi(x) = C\exp(-\alpha x^2/2)$.
4. $\psi(x) = C(\alpha - |x|)$, for $|x| < \alpha$, otherwise 0.
5. $\psi(x) = C\sin\alpha x$, for $|\alpha x| < \pi$, otherwise 0.

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Why don’t we just use a most general variational method to find out the ground state? Because we will eventually come back to the time-independent Shrodinger equation.

Suppose we have a functional form

$$E(\psi^*, \psi, \lambda) = \int dx \psi^* H \psi - \lambda \left( \int dx \psi^* \psi - 1 \right)$$

The reason we have this Lagrange multiplier method is that the wave function should be normalized and this multiplier provides the degree of freedom. We would only get a wrong result if we don't include this DoF.

Variation of $\psi^*$,

$$\delta E = \int dx \delta\psi^* H \psi - \int dx \delta\psi^* \psi = 0$$

Now what?

$$H\psi - \lambda\psi = 0$$

Not helpful.

### 2.2.2 Variational Method and Virial Theorem

For a potential $V(x) = bx^n$, we can prove that virial theorem is valid for ground state if we use Gaussian trial function $e^{-\alpha x^2/2}$.

A MMA proof is here.

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This open source project is hosted on GitHub: quantum .

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