Transmission coefficient

$$T = \frac{j_t}{j_i}$$

For this square potential, we have

1. $0 < x < a$,

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V_0$$

Solution should be

$$\psi_2(x) = Ae^{k_2 x} + Be^{-k_2 x}$$

1. $x < 0, x > a$

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$$

Solutions should be

$$\psi_1(x) = e^{ik_1 x} + re^{-ik_1 x}$$

$$\psi_3(x) = te^{ik_1 x}$$

Boundary conditions are

$$\psi_1(0) = \psi_2(0), \psi_1'(0) = \psi_2'(0), \psi_2(a) = \psi_3(a), \psi_2'(a) = \psi_2'(a)$$

Put the waves functions in, we get

$$1 + r = A + B, ik_1 - rik_1 = Ak_2 - Bk_2$$

$$Ae^{k_2 a} + Be^{-k_2 a} = te^{ik_1 a}, k_2 Ae^{k_2 a} - k_2 Be^{k_2 a} = tik_1 e^{ik_1 a}$$

Use Solve function in Mathematica,

```
In[42]:= sol3 =
  Solve[1 + r == A + B && I k1 - r I k1 == A k2 - B k2 &&
    A Exp[k2 a] + B Exp[-k2 a] == t Exp[I k1 a] &&
    k2 A Exp[k2 a] - k2 B Exp[-k2 a] == t I k1 Exp[I k1 a], {r, A, B, t}]
```

$$\text{Out[42]=} \left\{\left\{r \to \frac{\left(-1+e^{2 a k2}\right)\left(k1^2+k2^2\right)}{-k1^2+e^{2 a k2} k1^2+2 i\, k1\, k2+2 i\, e^{2 a k2} k1\, k2+k2^2-e^{2 a k2} k2^2},\right.\right.$$

$$A \to -\frac{2\, k1\, (k1 - i\, k2)}{-k1^2+e^{2 a k2} k1^2+2 i\, k1\, k2+2 i\, e^{2 a k2} k1\, k2+k2^2-e^{2 a k2} k2^2},$$

$$B \to \frac{2\, e^{2 a k2}\, k1\, (k1 + i\, k2)}{-k1^2+e^{2 a k2} k1^2+2 i\, k1\, k2+2 i\, e^{2 a k2} k1\, k2+k2^2-e^{2 a k2} k2^2},$$

$$\left.\left. t \to -\frac{4\, e^{-i a k1+a k2}\, k1\, k2}{-i\, k1^2+i\, e^{2 a k2} k1^2-2\, k1\, k2-2\, e^{2 a k2} k1\, k2+i\, k2^2-i\, e^{2 a k2} k2^2}\right\}\right\}$$

So we can find $|t|^2$.

$$T = |t|^2 = \frac{4k_1^2 k_2^2}{(k_1^2 - k_2^2)^2 \sinh^2(k_2 a) + 4k_1^2 k_2^2 \cosh^2(k_2 a)}$$

We know

$$\cosh^2(k_2 a) - \sinh^2(k_2 a) = 1$$

So

$$T = \frac{4k_1^2 k_2^2}{1 + (k_1^2 + k_2^2)^2 \sinh^2(k_2 a)}$$

Take limits

At $V_0 \to \infty$, we have $k_2 \to 0$

$$T \to 0$$

This is the right case because infinity square potential allows no transmission.

At $E \ll V_0$, we have $k_2 \to 0$

$$T \to 0$$

This is also right because energy is so small to transmit through the square potential.