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# Quantum Notes

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Some notes for quantum



## INTRODUCTION

Some notes continued from the full theoretical physics notes are [here](#).





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### 2.1 Collections

#### 0. Fine Structure Constant

$\alpha = \frac{k_e e^2}{\hbar c} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{\hbar c} = \frac{e^2 c}{\mu_0 \hbar^2}$

In electrostatic cgs units,  $\alpha = \frac{e^2}{\hbar c}$ .

In natural units,  $\alpha = \frac{e^2}{4\pi}$ .

#### 1. Hydrogen Atom

Potential  $V(r) = -\frac{Ze^2}{4\pi\epsilon_0 r}$ .

Energy levels:  $E_n = -\left(\frac{Z^2 \mu e^4}{32\pi^2\epsilon_0 \hbar^2}\right) \frac{1}{n^2} = -\left(\frac{Z^2 \hbar^2}{2\mu a_\mu^2}\right) \frac{1}{n^2} = \frac{\mu c^2 Z^2 \alpha^2}{2n^2}$ .

Ground state of hydrogen atom  $\psi_{100}(r) = \frac{1}{\sqrt{\pi}} \frac{1}{a^{3/2}} e^{-Zr/a}$ .

### 2.2 Approximation Methods

#### 2.2.1 Variational Method

Why don't we just use a most general variational method to find out the ground state? Because we will eventually come back to the time-independent Shrodinger equation.

Suppose we have a functional form

$$E(\psi^*, \psi, \lambda) = \int dx \psi^* H \psi - \lambda \left( \int dx \psi^* \psi - 1 \right)$$

The reason we have this Lagrange multiplier method is that the wave function should be normalized and this multiplier provides the degree of freedom. We would only get a wrong result if we don't include this DoF.

Variation of  $\psi^*$ ,

$$\delta E = \int dx \delta \psi^* H \psi - \int dx \delta \psi^* \psi = 0$$

Now what?

$$H\psi - \lambda\psi = 0$$

Not helpful.

## 2.2.2 Variational Method and Virial Theorem

For a potential  $V(x) = bx^n$ , we can prove that virial theorem is valid for ground state if we use Gaussian trial function  $e^{-\alpha x^2/2}$ .

A MMA proof is here.

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This open source project is hosted on GitHub: [quantum](#) .

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