University of Jyväskylä - Course TIEJ6003
intro2QC Summer2024: ex1

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The exercises marked with ***** are considered important-to-solve; the remaining exercises are more advanced and meant to challenge you.

Exercise 1.1*: expectation

Suppose we prepare a quantum system in an **eigenstate** $|\psi\rangle$ of some observable M with corresponding eigenvalue m. What is the average observed value of M (that is, when repeatedly observing the same prepared state $|\psi\rangle$ by the operator M), and the standard deviation of this statistical process?

Exercise 1.2*: cascades

Suppose $\{L_\ell\}$ and $\{M_m\}$ are two sets of measurement operators. Show that a measurement defined by the measurement operators $\{L_\ell\}$ followed by a measurement defined by the measurement operators $\{M_m\}$ is physically equivalent to a single measurement defined by measurement operators $\{N_{lm}\}$ with the representation $N_{lm} \equiv M_m L_\ell$.

Exercise 1.3: operators

Consider a ket space spanned by the eigenkets $\{|a'\rangle\}$ of a Hermitian operator A. There is no degeneracy.

(a) Prove that

$$\prod_{a'}(A - a')$$

is a null operator – that is, applying it on any ket vector results in the zero vector.

(b) What is the significance of

$$\prod_{a'' \neq a'} \frac{(A - a'')}{a' - a''}?$$

(Note the double product notation, which is equivalent to $\prod_{a'} \prod_{a'' \neq a'}$

(c) Illustrate (a) and (b) by setting $A := S_z$ of a spin-$\frac{1}{2}$ system (Pauli's Z).