University of Jyväskylä - Course TIEJ6003

Introduction to Quantum Computing

Precept-03

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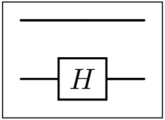
ex2

...processing solutions to ex2...

...in-class practice...

2-qubit circuit

What is the 4×4 unitary matrix for the following circuit?



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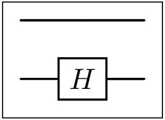

Solution:

$$I_1 \otimes H_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \otimes \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & -1 \end{pmatrix}$$

Deutsch's problem revisited

Suppose that we restate Deutsch's problem to probabilistically determine whether f is either *constant* or *balanced* with an error $\epsilon < \frac{1}{2}$. How many evaluations are needed on a classical computer to solve this problem?

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Solution:
A single evaluation is insufficient – an output would be a random guess with an error of $\epsilon = \frac{1}{2}$.

We claim that two evaluations are sufficient -

- If f is balanced, the probability that the first two evaluations are identical is the following:

$$\frac{1}{2} \cdot \frac{2^n/2 - 1}{2^n - 1}$$

- This probability is lower than $\frac{1}{2}$.

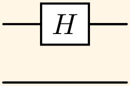
reviewing the take-out exercises

take-out problem-set

We now review the exercises that appear in the third problem-set ("take-out"). We shall go over the solution in tomorrow's Precept.

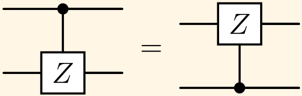
Exercise 3.1: scaling-up

What is the 4×4 unitary matrix for this circuit?



Exercise 3.2: equivalence

Show that



Exercise 3.3: CNOT from controlled-Z gates

Construct a CNOT gate from one controlled-Z gate, that is, the gate whose action in the computational basis is specified by the unitary matrix

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Exercise 3.4: Bell states are orthonormal basis

Verify that the Bell states form an *orthonormal basis* for the *2-qubit state space*.