University of Jyväskylä - Course TIEJ6003
intro2QC Summer2024: ex1

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The exercises marked with * are considered important-to-solve; the remaining exercises are more advanced and meant to challenge you.

Exercise 1.1*: expectation

Suppose we prepare a quantum system in an **eigenstate** $|\psi\rangle$ of some observable M with corresponding eigenvalue m. What is the average observed value of M (that is, when repeatedly observing the same prepared state $|\psi\rangle$ by the operator M), and the standard deviation of this statistical process?

Exercise 1.2*: the measurement postulate and cascades

Quantum measurements can be described by a collection $\{M_m\}$ of *measurement operators* (m refers to the measurement outcomes).
Given a quantum state $|\psi\rangle$, then the probability that result m occurs is given by

$$p(m) = \langle\psi|M_m^\dagger M_m|\psi\rangle, \tag{1}$$

and the state of the system after the measurement is

$$\frac{M_m|\psi\rangle}{\sqrt{\langle\psi|M_m^\dagger M_m|\psi\rangle}}. \tag{2}$$

The measurement operators satisfy the *completeness equation*,

$$\sum_m M_m^\dagger M_m = I. \tag{3}$$

Suppose $\{L_\ell\}$ and $\{M_m\}$ are two sets of measurement operators. Show that a measurement defined by the measurement operators $\{L_\ell\}$ followed by a measurement defined by the measurement operators $\{M_m\}$ is physically equivalent to a single measurement defined by measurement operators $\{N_{lm}\}$ with the representation $N_{lm} \equiv M_m L_\ell$.

Exercise 1.3: operators

Consider a ket space spanned by the eigenkets $\{|a'\rangle\}$ of a Hermitian operator A. There is no degeneracy.

(a) Prove that
$$\prod_{a'}(A - a')$$
is a null operator – that is, applying it on any ket vector results in the zero vector.

(b) What is the significance of
$$\prod_{a'' \neq a'} \frac{(A - a'')}{a' - a''}?$$
(Note the double product notation, which is equivalent to $\prod_{a'} \prod_{a'' \neq a'}$

(c) Illustrate (a) and (b) by setting $A := S_z$ of a spin-$\frac{1}{2}$ system (Pauli's Z).