University of Jyväskylä - Course TIEJ6003

INTRODUCTION TO QUANTUM COMPUTING

Precept-05

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ex4

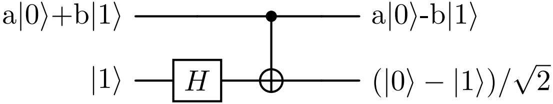
...processing solutions to ex4...

...in-class practice...

Grover Practice

We would like to explore Grover's algorithm over the 4 states of the 2-qubit system in the computational basis.

Let's first examine the following circuit:

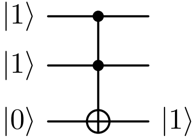


Explanation (up to $\sqrt{2}$):

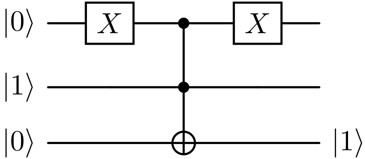
$$
\begin{aligned}
& a\,|0\rangle\,(|0\rangle - |1\rangle) + b\,|1\rangle\,(|0\rangle - |1\rangle) \\
\text{CNOT} \longrightarrow & \\
& = a\,|0\rangle\,(|0\rangle - |1\rangle) + b\,|1\rangle\,(|1\rangle - |0\rangle) \\
& = (a\,|0\rangle - b\,|1\rangle)(|0\rangle - |1\rangle)
\end{aligned}
$$

2-qubit oracles

- Marking the $|11\rangle$ state:



- Marking the $|01\rangle$ state:



2-qubit Grover

The Diffusion operator:

$$\begin{aligned}
\mathcal{D} &= H^{\otimes 2}(2\left|00\right\rangle\left\langle 00\right| - I)H^{\otimes 2} \\
&= H^{\otimes 2}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} H^{\otimes 2} \\
&= \frac{1}{2}\begin{pmatrix} -2 & 2 & 2 & 2 \\ 2 & -2 & 2 & 2 \\ 2 & 2 & -2 & 2 \\ 2 & 2 & 2 & -2 \end{pmatrix}
\end{aligned}$$

We set the winner to $|w\rangle{:=}|10\rangle$.

Given an initial state of the uniform superposition (informal “amplitudes’ perspective”),

$$(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4})$$

Since the winner is $|w\rangle{=}|10\rangle$ - the oracle flips its sign:

$$(\frac{1}{4}, \frac{1}{4}, -\mathbf{\frac{1}{4}}, \frac{1}{4})$$

and after diffusion we get

$$(0, 0, \mathbf{1}, 0)$$

Grover 2-qubit conclusion

A measurement and we are done, only in 1 iteration!

Remark: such a configuration has a period of 6 - after which it will return to its starting position.