Scars of the Transverse Field Ising Model on Discrete Geometries (Polyhedra)

September 17, 2025

Introduction

We are studying scars of the simple Ising model on discrete geometries (polyhedra). Here, *scars* are identified as special, sparser eigenstates of the Hamiltonian which are simultaneously eigenstates of the Ising term and of the transverse-field (TF) term separately; in addition, each such state is annihilated by exactly one of the two terms. In all of the following examples, the Hamiltonian is

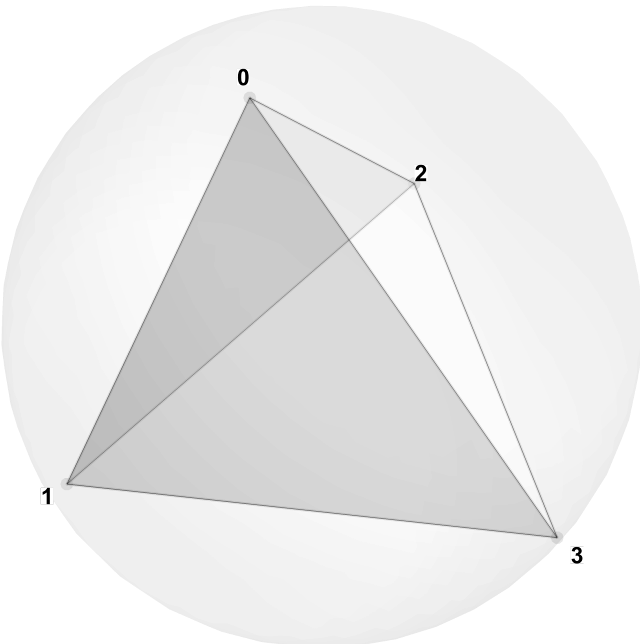
$$H = H_{\text{Ising}} + H_{\text{TF}}, \quad H_{\text{Ising}} = J \sum_{\langle i,j \rangle} \sigma_i^x \sigma_j^x, \quad H_{\text{TF}} = h \sum_i \sigma_i^z \tag{1}$$

where $J = 1, h = 3$ (antiferromagnetic).

Platonic Solids

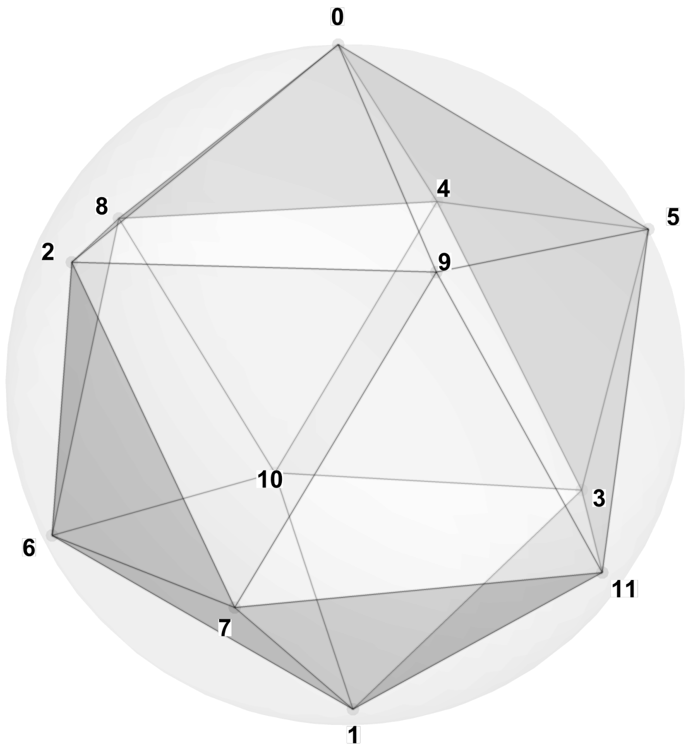
Tetrahedron

Overview and data.



- **Duality / paired solid:** self-dual, tetrahedron
- **Vertices (V), Faces (F), Edges (E):** $V = 4$, $F = 3$ (equilateral triangles), $E = 6$.
- **Solid point group:** $T_d \cong S_4$
  Vertex stabilizer subgroup: C_3 for rotations only, D_3 for full symmetry group.
- **Hilbert space:** $\dim \mathcal{H} = 2^4 = 16$ (spin-$\frac{1}{2}$ on each vertex).
- **Eigenvalue range:** $[-12.37, 12.71]$.

Scar structure: sets and multiplets.

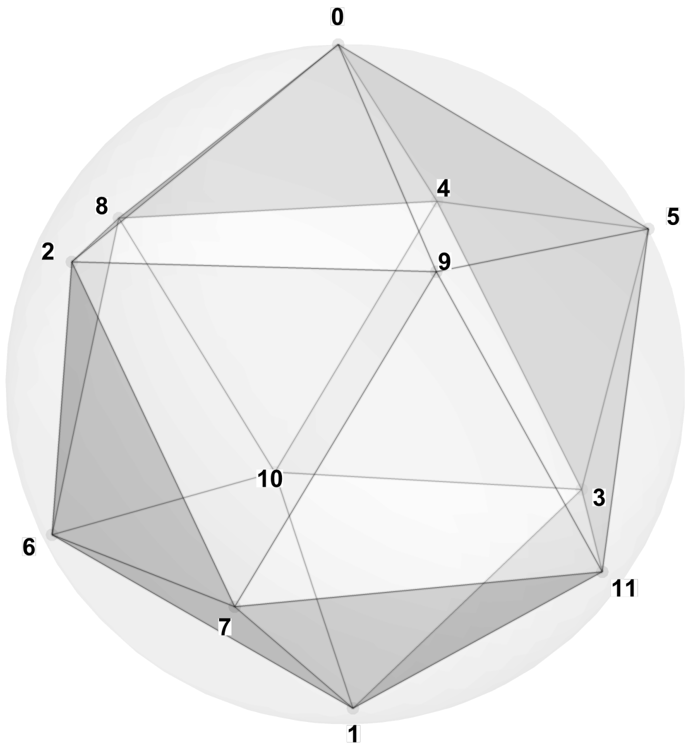


- **Number of scar sets:** `<1 or 2>`.
- For each scar set S_k, fill one table per set:

Scar set $S_{\texttt{<k>}}$:

Multiplet label	Energy E	Degeneracy	Annihilated by	Non-zero components (vs. 2^V)
`<m1>`	$\pm$ `<int>`	`<deg>`	H_{Ising} / H_{TF} / both	`<# non-zero>` / $2^{\texttt{<V>}}$

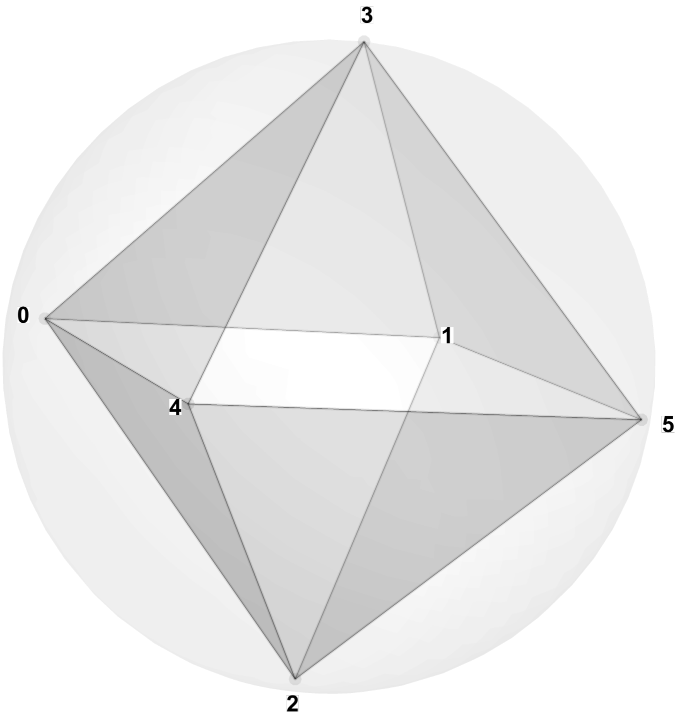
Local properties (RDMs).



- **Local RDM definition:** $\rho_A = \mathrm{Tr}_{\bar{A}}(|\psi\rangle\langle\psi|)$ on compact subsets of $n = 2, 3, 4, 5, 6$ sites, with $n < V/2$.
- **Compactness criterion:** `<how subsets chosen>`.
- **Diagnostics:** `<observables, entropies, purity, etc.>`.

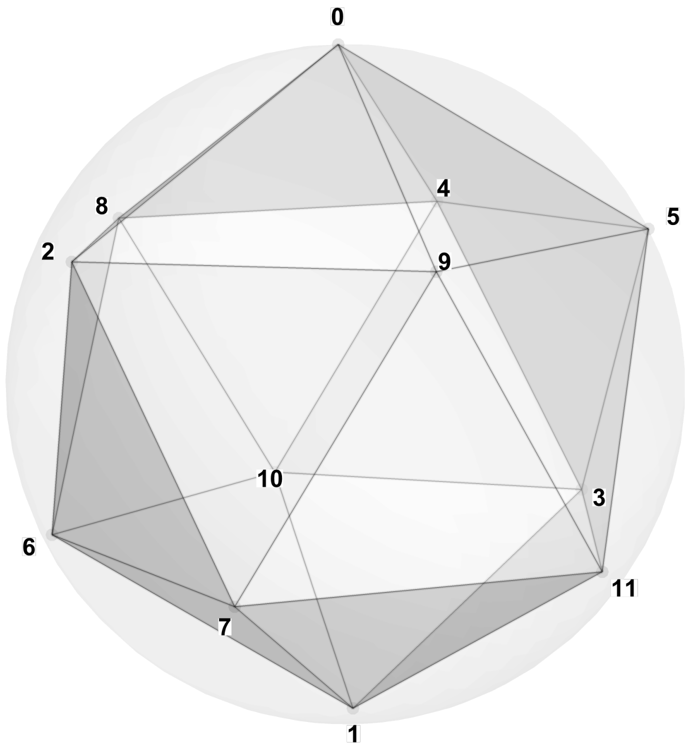
Octahedron

Overview and data.



- **Duality / paired solid:** `<DUALITY or "self-dual">`.
- **Vertices (V), Faces (F), Edges (E):** $V =$ `<V>`, $F =$ `<F>`, $E =$ `<E>`.
- **Solid point group:** `<POINT-GROUP>`.
  Vertex stabilizer subgroup: `<STABILIZER>`.
- **Hilbert space:** $\dim \mathcal{H} = 2^V$ (spin-$\frac{1}{2}$ on each vertex).
- **Eigenvalue range:** `<Specify conventions>`.

Scar structure: sets and multiplets.

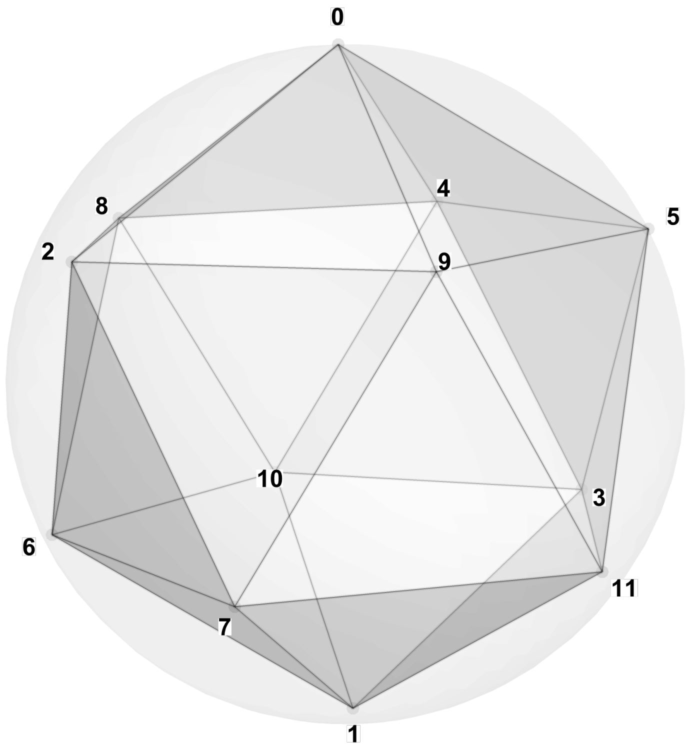


- **Number of scar sets:** `<1 or 2>`.
- For each scar set S_k, fill one table per set:

Scar set $S_{\texttt{<k>}}$:

Multiplet label	Energy E	Degeneracy	Annihilated by	Non-zero components (vs. 2^V)
<m1>	$\pm$ <int>	<deg>	H_{Ising} / H_{TF} / both	<# non-zero> / $2^{\texttt{<V>}}$

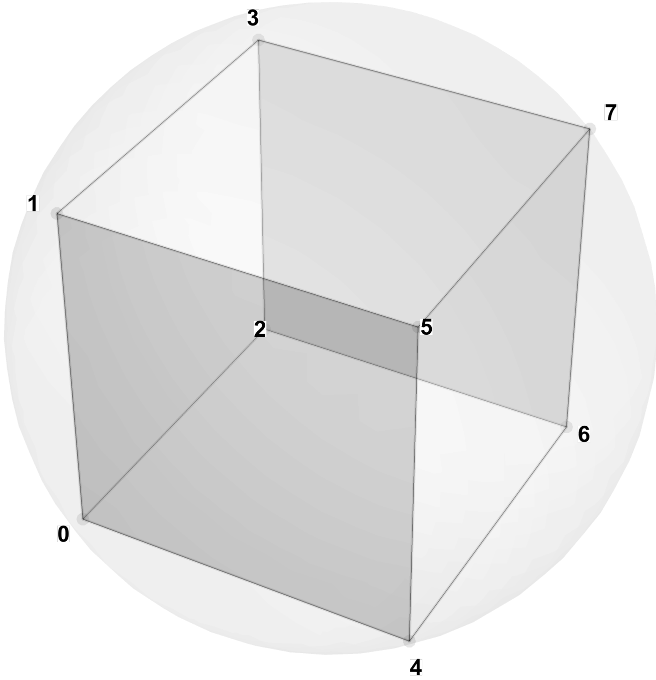
Local properties (RDMs).



- **Local RDM definition:** $\rho_A = \mathrm{Tr}_{\bar{A}}(|\psi\rangle\langle\psi|)$ on compact subsets of $n = 2, 3, 4, 5, 6$ sites, with $n < V/2$.
- **Compactness criterion:** `<how subsets chosen>`.
- **Diagnostics:** `<observables, entropies, purity, etc.>`.

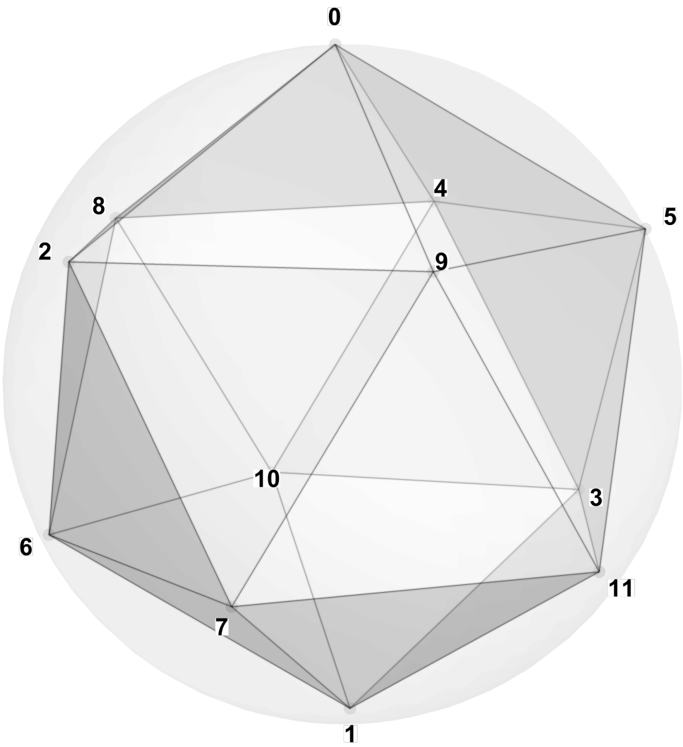
Cube

Overview and data.



- **Duality / paired solid:** `<DUALITY or "self-dual">`.
- **Vertices (V), Faces (F), Edges (E):** $V =$ `<V>`, $F =$ `<F>`, $E =$ `<E>`.
- **Solid point group:** `<POINT-GROUP>`.
  Vertex stabilizer subgroup: `<STABILIZER>`.
- **Hilbert space:** $\dim \mathcal{H} = 2^V$ (spin-$\frac{1}{2}$ on each vertex).
- **Eigenvalue range:** `<Specify conventions>`.

Scar structure: sets and multiplets.

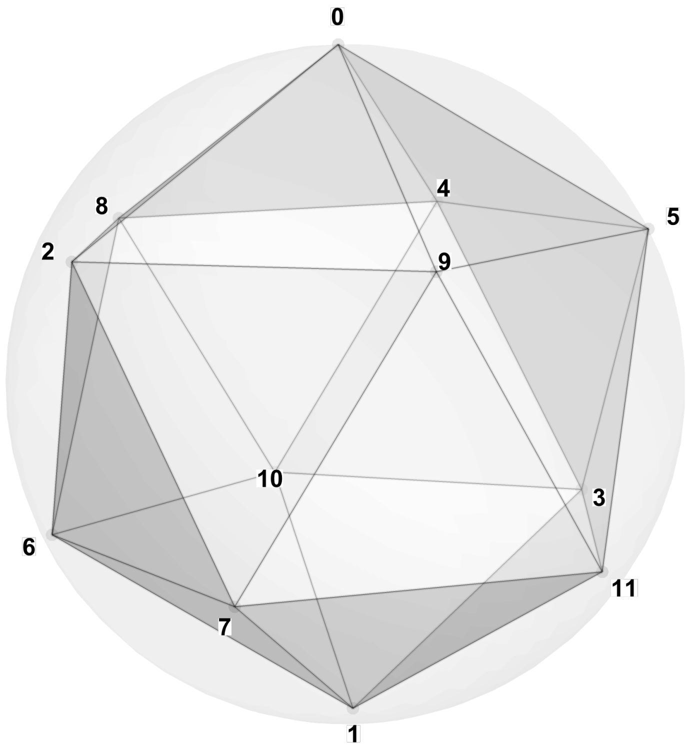


- **Number of scar sets:** `<1 or 2>`.
- For each scar set S_k, fill one table per set:

Scar set $S_{\texttt{<k>}}$:

Multiplet label	Energy E	Degeneracy	Annihilated by	Non-zero components (vs. 2^V)
`<m1>`	$\pm$ `<int>`	`<deg>`	H_{Ising} / H_{TF} / both	`<# non-zero>` / $2^{\texttt{<V>}}$

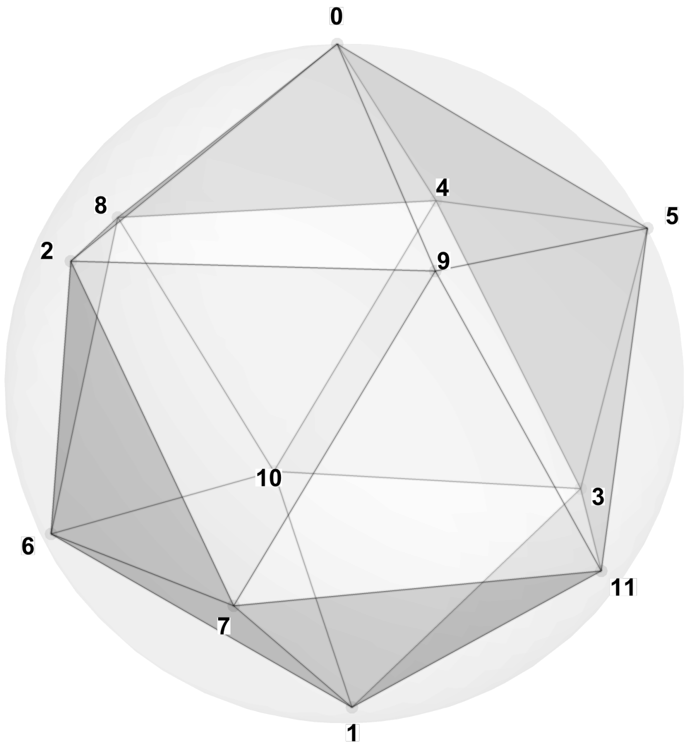
Local properties (RDMs).



- **Local RDM definition:** $\rho_A = \mathrm{Tr}_{\bar{A}}(|\psi\rangle\langle\psi|)$ on compact subsets of $n = 2, 3, 4, 5, 6$ sites, with $n < V/2$.
- **Compactness criterion:** `<how subsets chosen>`.
- **Diagnostics:** `<observables, entropies, purity, etc.>`.

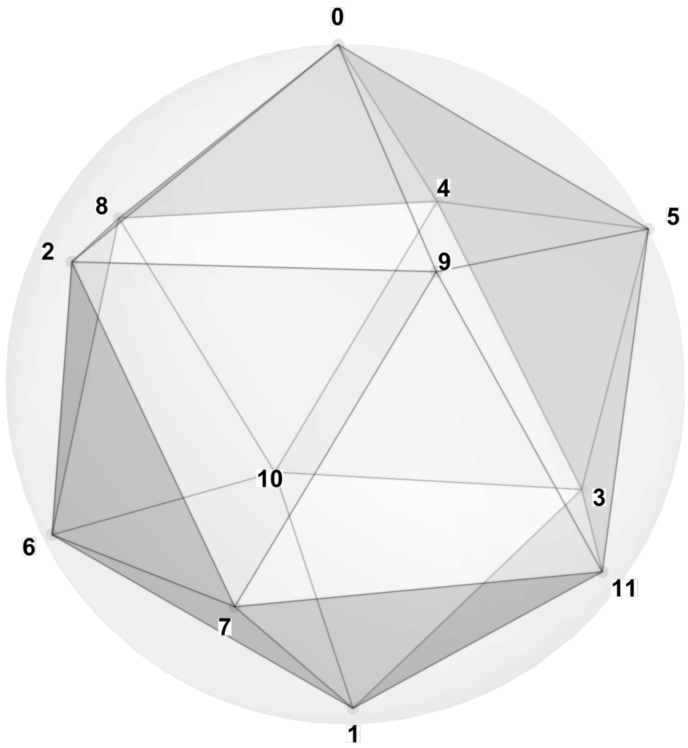
Icosahedron

Overview and data.



- **Duality / paired solid:** `<DUALITY or "self-dual">`.
- **Vertices (V), Faces (F), Edges (E):** $V =$ `<V>`, $F =$ `<F>`, $E =$ `<E>`.
- **Solid point group:** `<POINT-GROUP>`.
  Vertex stabilizer subgroup: `<STABILIZER>`.
- **Hilbert space:** $\dim \mathcal{H} = 2^V$ (spin-$\frac{1}{2}$ on each vertex).
- **Eigenvalue range:** `<Specify conventions>`.

Scar structure: sets and multiplets.

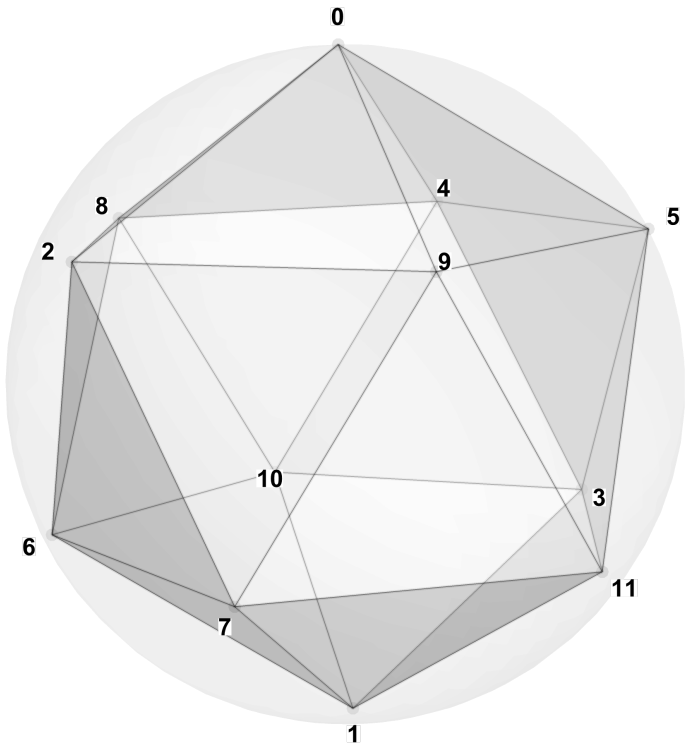


- **Number of scar sets:** <1 or 2>.
- For each scar set S_k, fill one table per set:

Scar set $S_{\texttt{<k>}}$:

Multiplet label	Energy E	Degeneracy	Annihilated by	Non-zero components (vs. 2^V)
<m1>	$\pm$ <int>	<deg>	H_{Ising} / H_{TF} / both	<# non-zero> / $2^{\texttt{<V>}}$

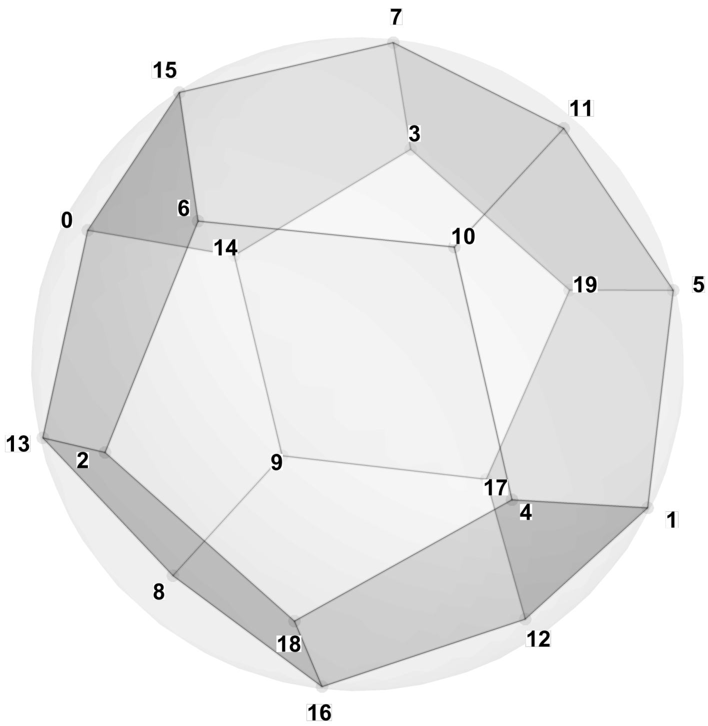
Local properties (RDMs).



- **Local RDM definition:** $\rho_A = \mathrm{Tr}_{\bar{A}}(|\psi\rangle\langle\psi|)$ on compact subsets of $n = 2, 3, 4, 5, 6$ sites, with $n < V/2$.
- **Compactness criterion:** `<how subsets chosen>`.
- **Diagnostics:** `<observables, entropies, purity, etc.>`.

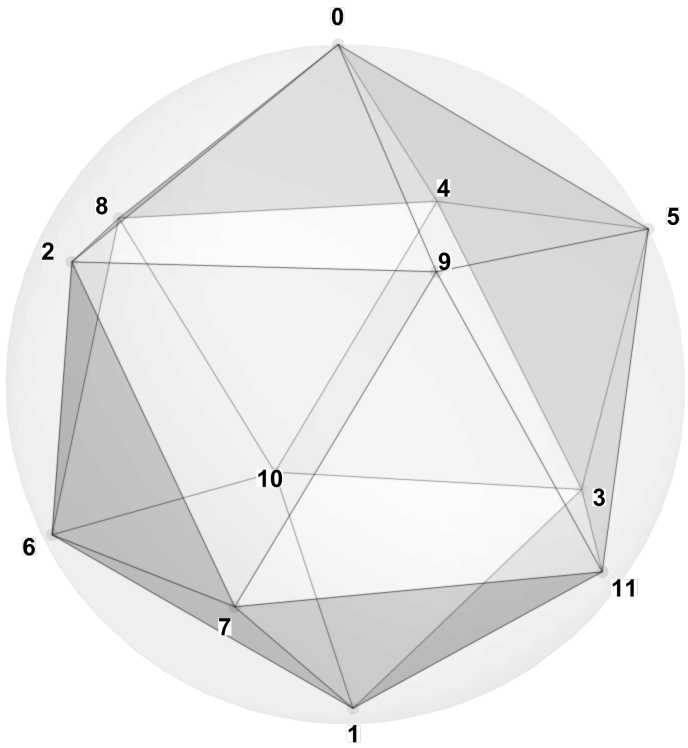
Dodecahedron

Overview and data.



- **Duality / paired solid:** `<DUALITY or "self-dual">`.
- **Vertices (V), Faces (F), Edges (E):** $V =$ `<V>`, $F =$ `<F>`, $E =$ `<E>`.
- **Solid point group:** `<POINT-GROUP>`.
  Vertex stabilizer subgroup: `<STABILIZER>`.
- **Hilbert space:** $\dim \mathcal{H} = 2^V$ (spin-$\frac{1}{2}$ on each vertex).
- **Eigenvalue range:** `<Specify conventions>`.

Scar structure: sets and multiplets.

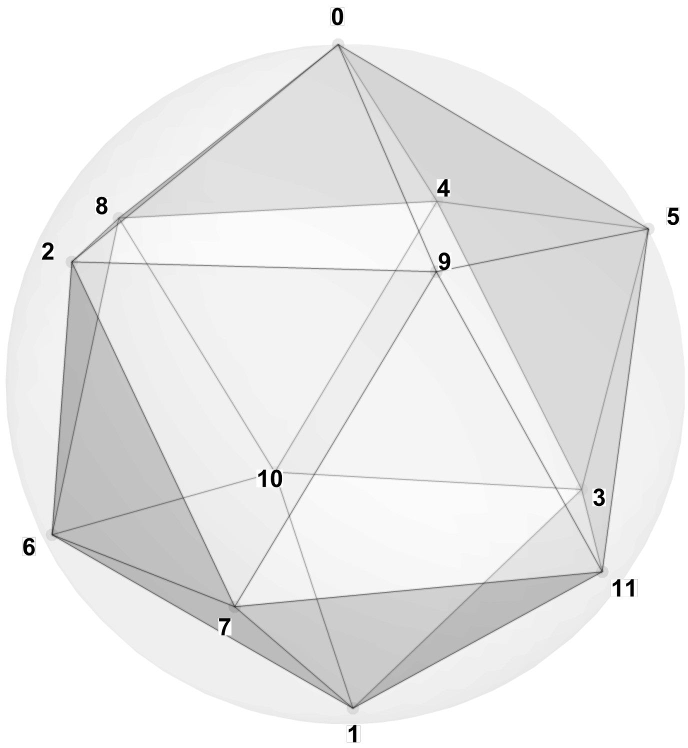


- **Number of scar sets:** `<1 or 2>`.
- For each scar set S_k, fill one table per set:

Scar set $S_{\texttt{<k>}}$:

Multiplet label	Energy E	Degeneracy	Annihilated by	Non-zero components (vs. 2^V)
`<m1>`	$\pm$ `<int>`	`<deg>`	H_{Ising} / H_{TF} / both	`<# non-zero> /` $2^{\texttt{<V>}}$

Local properties (RDMs).

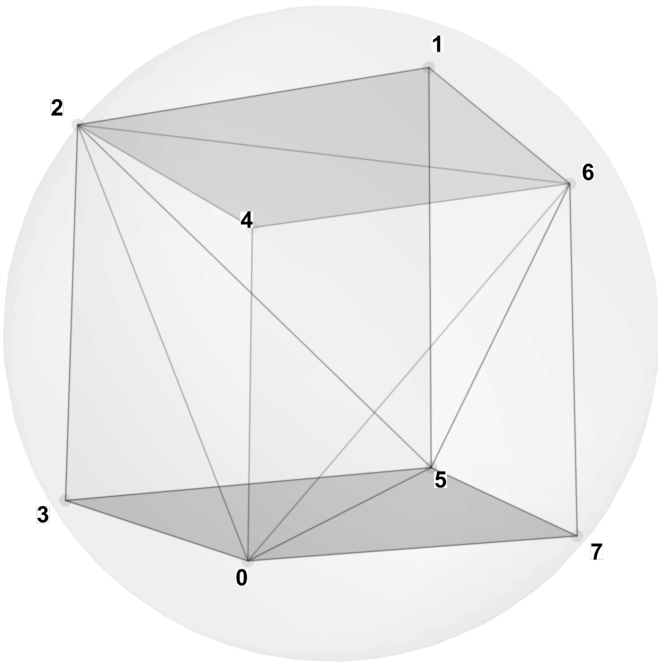


- **Local RDM definition:** $\rho_A = \mathrm{Tr}_{\bar{A}}(|\psi\rangle\langle\psi|)$ on compact subsets of $n = 2, 3, 4, 5, 6$ sites, with $n < V/2$.
- **Compactness criterion:** `<how subsets chosen>`.
- **Diagnostics:** `<observables, entropies, purity, etc.>`.

Catalan Solids

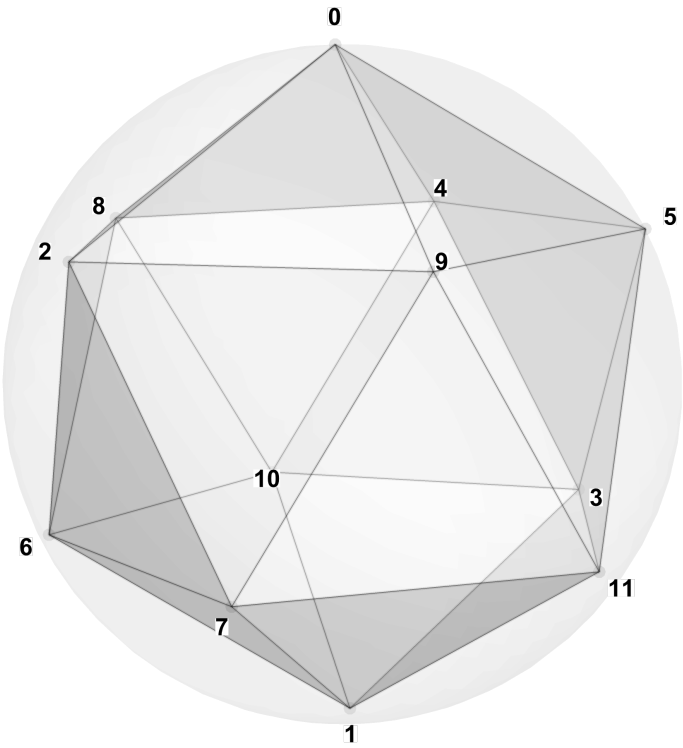
Triakis Tetrahedron

Overview and data.



- **Duality / paired solid:** <DUALITY or "self-dual">.
- **Vertices (V), Faces (F), Edges (E):** $V =$ <V>, $F =$ <F>, $E =$ <E>.
- **Solid point group:** <POINT-GROUP>.
  Vertex stabilizer subgroup: <STABILIZER>.
- **Hilbert space:** $\dim \mathcal{H} = 2^V$ (spin-$\frac{1}{2}$ on each vertex).
- **Eigenvalue range:** <Specify conventions>.

Scar structure: sets and multiplets.



- **Number of scar sets:** `<1 or 2>`.
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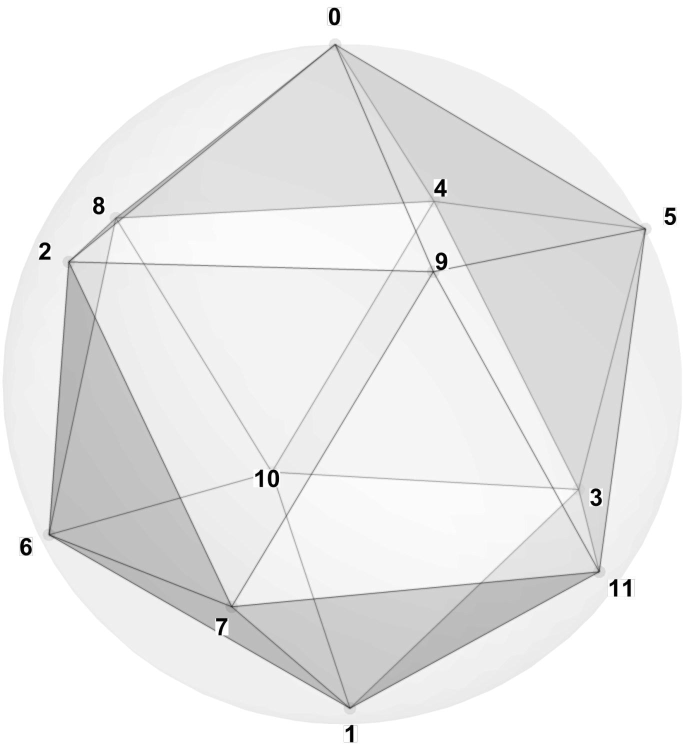
Scar set $S_{\texttt{<k>}}$:

Multiplet label	Energy E	Degeneracy	Annihilated by	Non-zero components (vs. 2^V)
<m1>	$\pm$ <int>	<deg>	H_{Ising} / H_{TF} / both	<# non-zero> / $2^{\texttt{<V>}}$

Local properties (RDMs).



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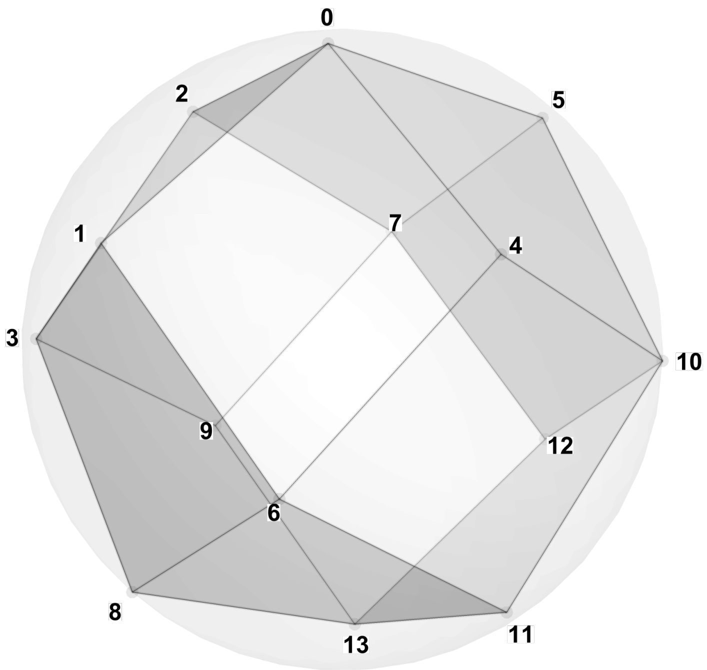
- **Compactness criterion:** `<how subsets chosen>`.
- **Diagnostics:** `<observables, entropies, purity, etc.>`.

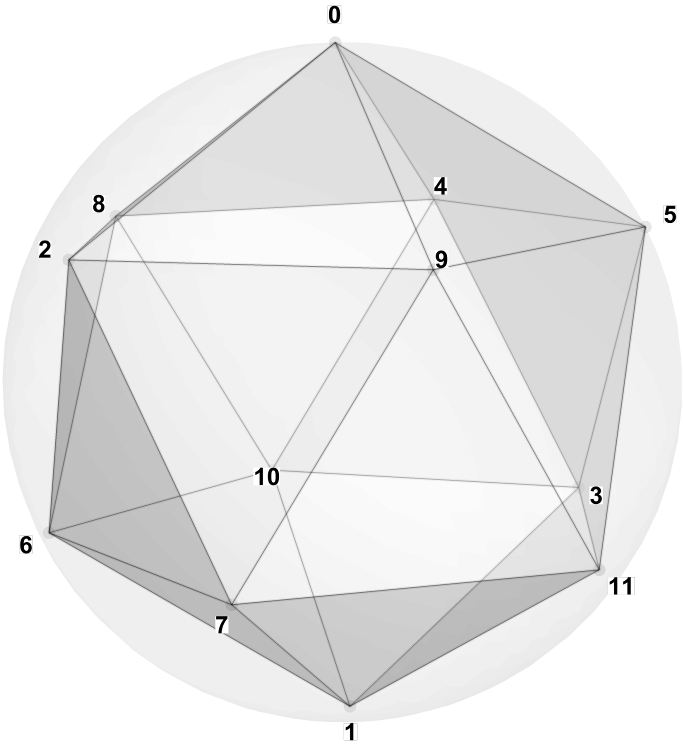
Rhombic Dodecahedron

Overview and data.



- **Duality / paired solid:** `<DUALITY or "self-dual">`.
- **Vertices (V), Faces (F), Edges (E):** $V =$ `<V>`, $F =$ `<F>`, $E =$ `<E>`.
- **Solid point group:** `<POINT-GROUP>`.
  Vertex stabilizer subgroup: `<STABILIZER>`.
- **Hilbert space:** $\dim \mathcal{H} = 2^V$ (spin-$\frac{1}{2}$ on each vertex).
- **Eigenvalue range:** `<Specify conventions>`.

Scar structure: sets and multiplets.

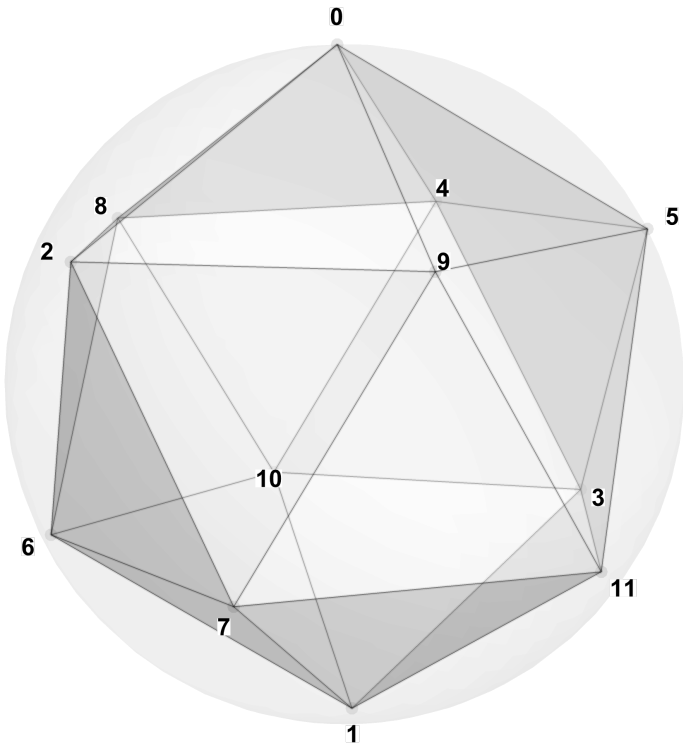


- **Number of scar sets:** `<1 or 2>`.
- For each scar set S_k, fill one table per set:

Scar set $S_{\texttt{<k>}}$:

Multiplet label	**Energy** E	**Degeneracy**	**Annihilated by**	**Non-zero components (vs.** 2^V**)**
`<m1>`	$\pm$ `<int>`	`<deg>`	H_{Ising} / H_{TF} / both	`<# non-zero>` / $2^{\texttt{<V>}}$

Local properties (RDMs).



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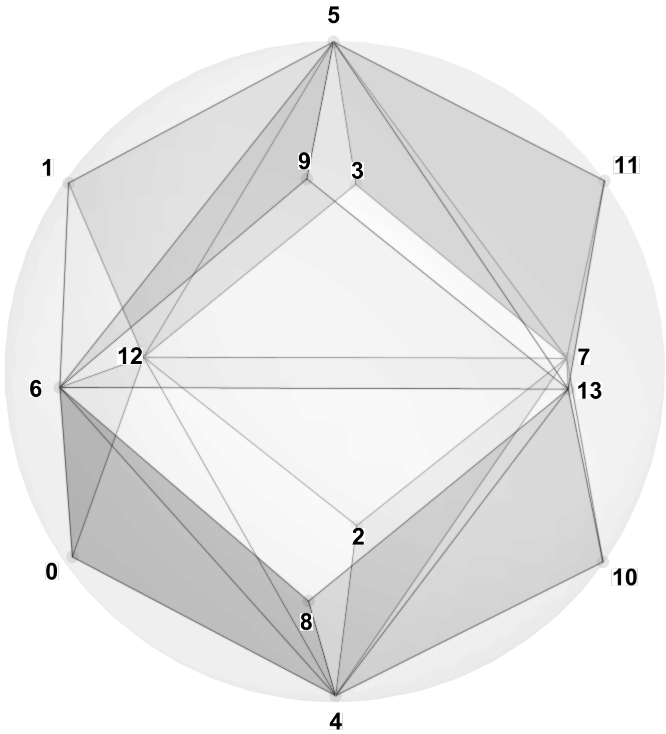
- **Compactness criterion:** `<how subsets chosen>`.
- **Diagnostics:** `<observables, entropies, purity, etc.>`.

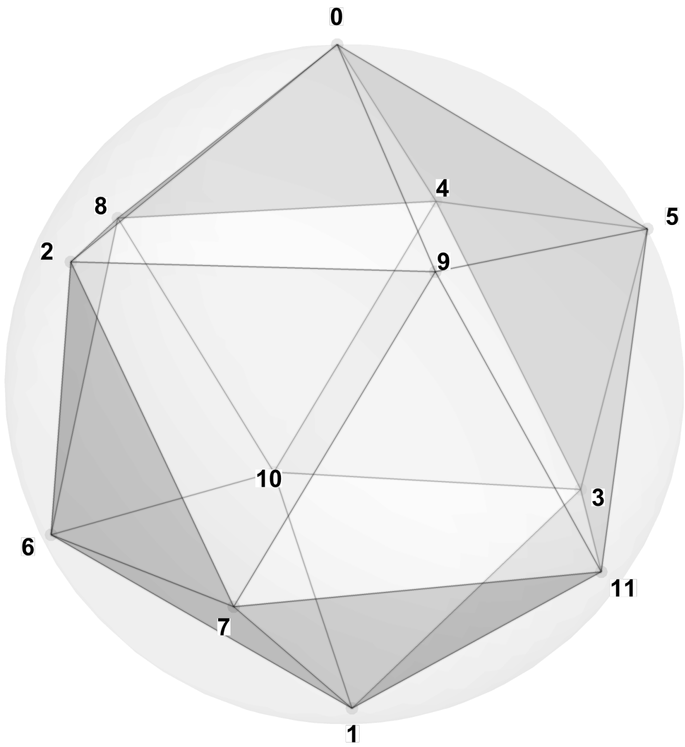
Triakis Octahedron

Overview and data.



- **Duality / paired solid:** `<DUALITY or "self-dual">`.
- **Vertices (V), Faces (F), Edges (E):** $V =$ `<V>`, $F =$ `<F>`, $E =$ `<E>`.
- **Solid point group:** `<POINT-GROUP>`.
  Vertex stabilizer subgroup: `<STABILIZER>`.
- **Hilbert space:** $\dim \mathcal{H} = 2^V$ (spin-$\frac{1}{2}$ on each vertex).
- **Eigenvalue range:** `<Specify conventions>`.

Scar structure: sets and multiplets.

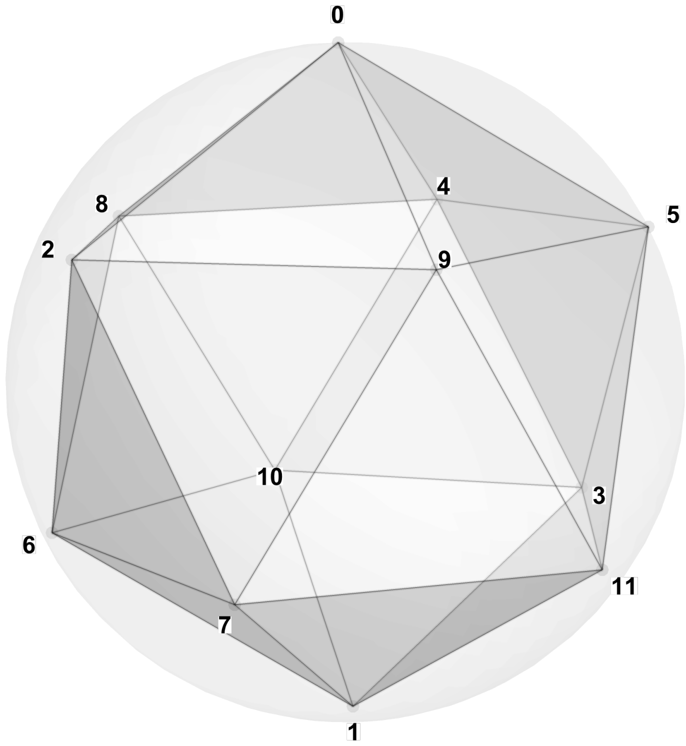


- **Number of scar sets:** `<1 or 2>`.
- For each scar set S_k, fill one table per set:

Scar set $S_{\texttt{<k>}}$:

Multiplet label	**Energy** E	**Degeneracy**	**Annihilated by**	**Non-zero components (vs.** 2^V**)**
`<m1>`	$\pm$ `<int>`	`<deg>`	H_{Ising} / H_{TF} / both	`<# non-zero>` / $2^{\texttt{<V>}}$

Local properties (RDMs).



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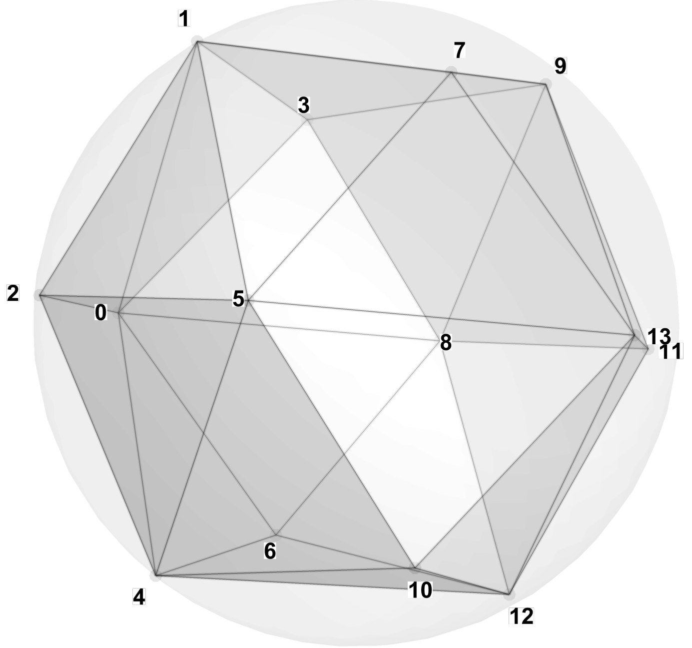
- **Compactness criterion:** `<how subsets chosen>`.
- **Diagnostics:** `<observables, entropies, purity, etc.>`.

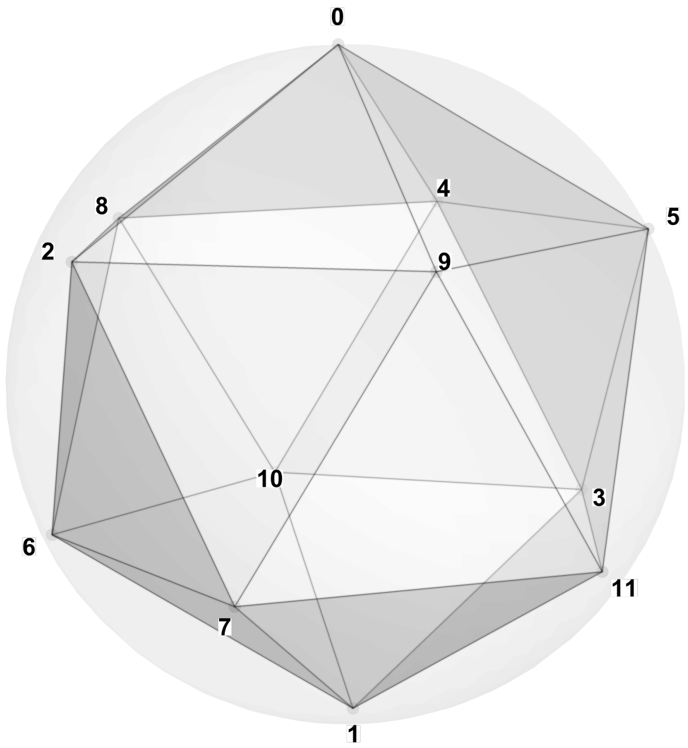
Tetrakis Hexahedron

Overview and data.



- **Duality / paired solid:** <DUALITY or "self-dual">.
- **Vertices (V), Faces (F), Edges (E):** $V =$ <V>, $F =$ <F>, $E =$ <E>.
- **Solid point group:** <POINT-GROUP>.
  Vertex stabilizer subgroup: <STABILIZER>.
- **Hilbert space:** $\dim \mathcal{H} = 2^V$ (spin-$\frac{1}{2}$ on each vertex).
- **Eigenvalue range:** <Specify conventions>.

Scar structure: sets and multiplets.



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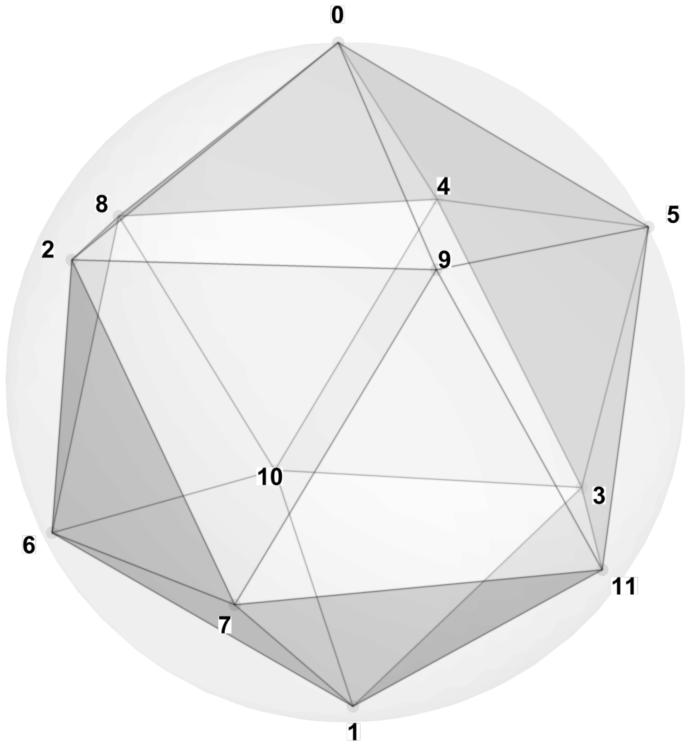
Scar set $S_{\texttt{<k>}}$:

Multiplet label	Energy E	Degeneracy	Annihilated by	Non-zero components (vs. 2^V)
`<m1>`	$\pm$ `<int>`	`<deg>`	H_{Ising} / H_{TF} / both	`<# non-zero>` / $2^{\texttt{<V>}}$

Local properties (RDMs).



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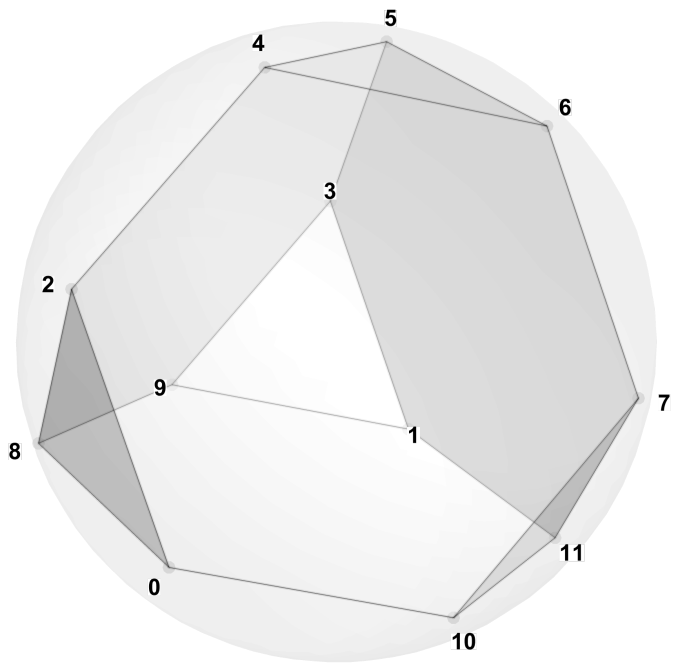
- **Compactness criterion:** `<how subsets chosen>`.
- **Diagnostics:** `<observables, entropies, purity, etc.>`.

Archimedean Solids

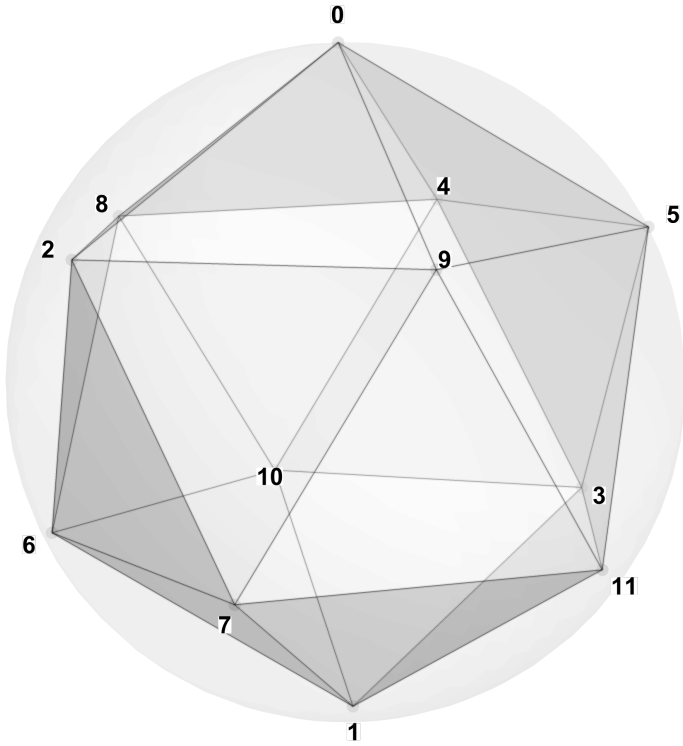
Truncated Tetrahedron

Overview and data.



- **Duality / paired solid:** <DUALITY or "self-dual">.
- **Vertices (V), Faces (F), Edges (E):** $V =$ <V>, $F =$ <F>, $E =$ <E>.
- **Solid point group:** <POINT-GROUP>.
  Vertex stabilizer subgroup: <STABILIZER>.
- **Hilbert space:** $\dim \mathcal{H} = 2^V$ (spin-$\frac{1}{2}$ on each vertex).
- **Eigenvalue range:** <Specify conventions>.

Scar structure: sets and multiplets.



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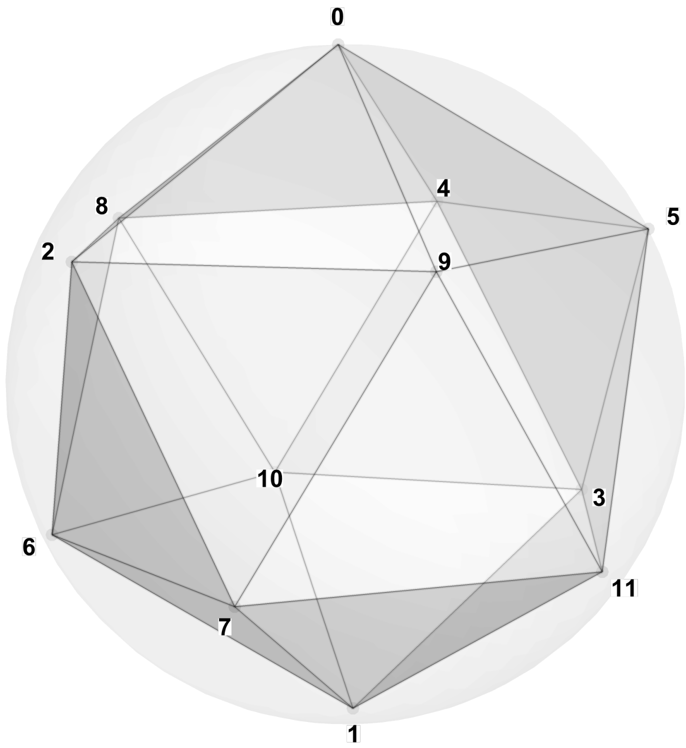
Scar set $S_{\texttt{<k>}}$:

Multiplet label	Energy E	Degeneracy	Annihilated by	Non-zero components (vs. 2^V)
`<m1>`	$\pm$ `<int>`	`<deg>`	H_{Ising} / H_{TF} / both	`<# non-zero>` / $2^{\texttt{<V>}}$

Local properties (RDMs).



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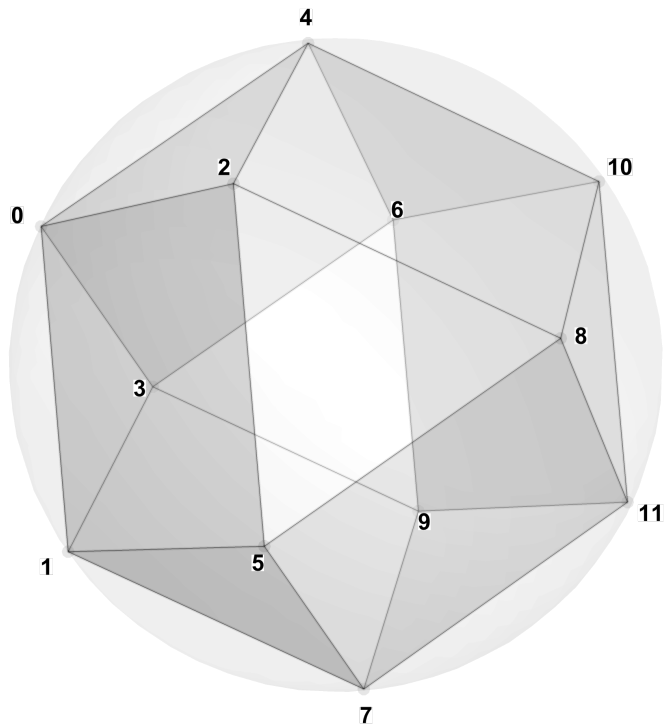
- **Compactness criterion:** `<how subsets chosen>`.
- **Diagnostics:** `<observables, entropies, purity, etc.>`.

Cuboctahedron

Overview and data.



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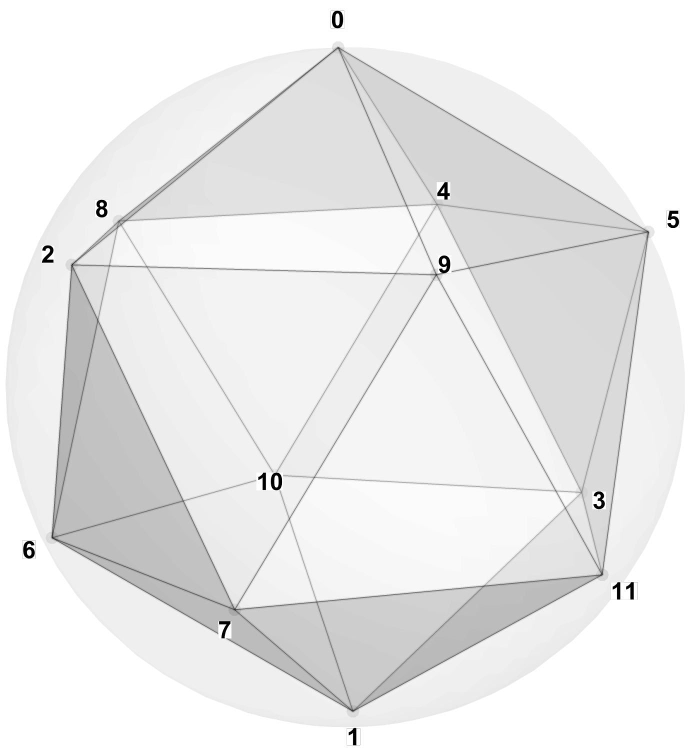
- **Solid point group:** `<POINT-GROUP>`.
  Vertex stabilizer subgroup: `<STABILIZER>`.
- **Hilbert space:** $\dim \mathcal{H} = 2^V$ (spin-$\frac{1}{2}$ on each vertex).
- **Eigenvalue range:** `<Specify conventions>`.

Scar structure: sets and multiplets.



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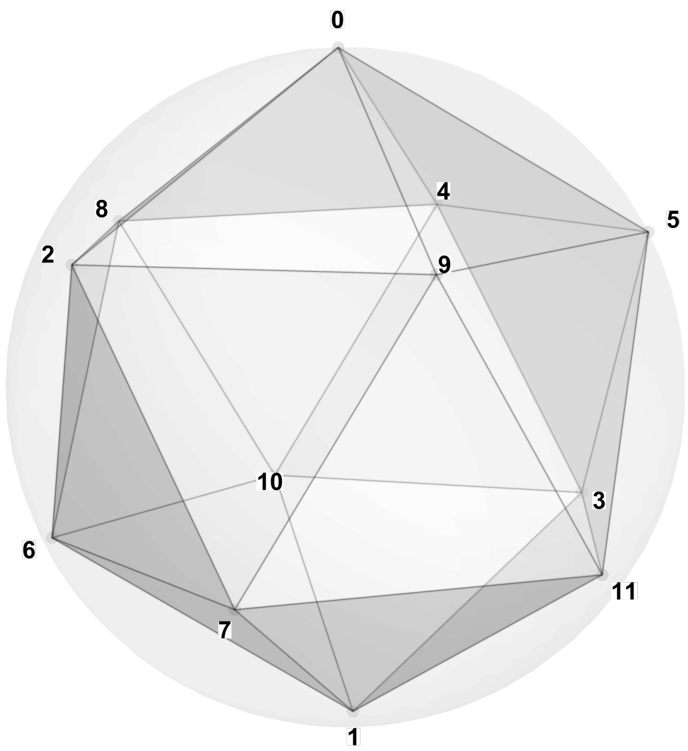
`<m1>`	$\pm$ `<int>`	`<deg>`	H_{Ising} / H_{TF} / both	`<# non-zero>` / $2^{\texttt{<V>}}$

Local properties (RDMs).



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