Sparsity Tables for Polyhedra Scars and Exact Scars

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October 23, 2025

Polyhedra scars (fixed L). For each polyhedron we list the scar multiplets (including pseudoscars where observed) and their sparsity, defined as sparsity $= \frac{\text{number of non-zero components}}{\dim \mathcal{H}}$. Data are taken from the polyhedra study (icosahedron, dodecahedron, Catalan solids, and the Archimedean solid with a scar).

Table 1: Icosahedron ($V{=}12$, $\dim \mathcal{H}{=}2^{12}{=}4096$). Includes pseudoscars.

Label	Energy E	Degeneracy	Non-zero comps.	Sparsity (%)
Scar S_1	-6	5	280	6.836
Pseudoscar S_1'	-4	1	720	17.578
Pseudoscar S_2'	0	1	600	14.648

Table 2: Dodecahedron ($V{=}20$, $\dim \mathcal{H}{=}2^{20}{=}1{,}048{,}576$).

Label	Energy E	Degeneracy	Non-zero comps.	Sparsity (%)
Scar S_1				

Table 3: Catalan solids (fixed L of each graph).

Solid	Label	Energy E	Degeneracy	Non-zero comps.	Sparsity (%)
Rhombic dodecahedron ($\dim \mathcal{H}{=}16384$)	S_1	-12	3	432	2.637
Rhombic dodecahedron	S_2	-6	18	432	2.637
Rhombic dodecahedron	S_3	6	18	432	2.637
Rhombic dodecahedron	S_4	12	3	432	2.637
Triakis octahedron ($\dim \mathcal{H}{=}16384$)	S_1	-4	1	192	1.172
Triakis octahedron	S_2	0	4	368	2.246
Tetrakis hexahedron ($\dim \mathcal{H}{=}16384$)	S_1	-4	12	1172	7.153
Tetrakis hexahedron	S_2	0	1	192	1.172

Table 4: Archimedean solid with a scar: Truncated tetrahedron ($\dim \mathcal{H}{=}4096$).

Label	Energy E	Degeneracy	Non-zero comps.	Sparsity (%)
S_1	0	1	48	1.172

Exact scars (top-of-tower vs. L). Below we list the sparsity of the *top scar in the tower* as a function of chain length L for each exactly solvable model.

Table 5: Dimer scar (spin-$\frac{1}{2}$).

L	Non-zero comps.	dim $\mathcal{H}$	Sparsity (%)
14	252	$2^{14} = 16384$	1.538
16	510	$2^{16} = 65536$	0.778
18	1020	$2^{18} = 262144$	0.389
20	2046	$2^{20} = 1048576$	0.195

Table 6: Domain-wall conserving spin-$\frac{1}{2}$ model — tower $|S_n\rangle$.

L	Non-zero comps.	dim $\mathcal{H}$	Sparsity (%)
12	54	4096	1.318
14	210	16384	1.282
16	352	65536	0.537
18	1287	262144	0.491

Table 7: Domain-wall conserving spin-$\frac{1}{2}$ model — tower $|S'_n\rangle$.

L	Non-zero comps.	dim $\mathcal{H}$	Sparsity (%)
12	54	4096	1.318
14	210	16384	1.282
16	352	65536	0.537
18	1287	262144	0.491

Table 8: Hubbard/Hirsch model — η tower.

L	Non-zero comps.	dim $\mathcal{H}$	Sparsity (%)
8	70	65536	0.107
10	252	1048576	0.024
12	924	16777216	0.006

Table 9: Hubbard/Hirsch model — τ tower.

L	Non-zero comps.	dim $\mathcal{H}$	Sparsity (%)
8	120	65536	0.183
10	700	1048576	0.067
12	3780	16777216	0.023

Table 10: Hubbard/Hirsch model — ς tower.

L	Non-zero comps.	dim $\mathcal{H}$	Sparsity (%)
8	320	65536	0.488
10	1400	1048576	0.134
12	6048	16777216	0.036

Table 11: Spin-1 XY model — Tower A.

L	Non-zero comps.	$\dim \mathcal{H}$	Sparsity (%)
8	70	6561	1.067
10	252	59049	0.427
12	924	531441	0.174
14	3432	4782969	0.072

Table 12: Spin-1 XY model — Tower B.

L	Non-zero comps.	$\dim \mathcal{H}$	Sparsity (%)
8	69	6561	1.052
10	250	59049	0.423
12	923	531441	0.174
14	3430	4782969	0.072