MATH3430: DIFFERENTIAL EQUATIONS

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TEXTBOOK: MORRIS TENENBAUM & HARRY POLLARD

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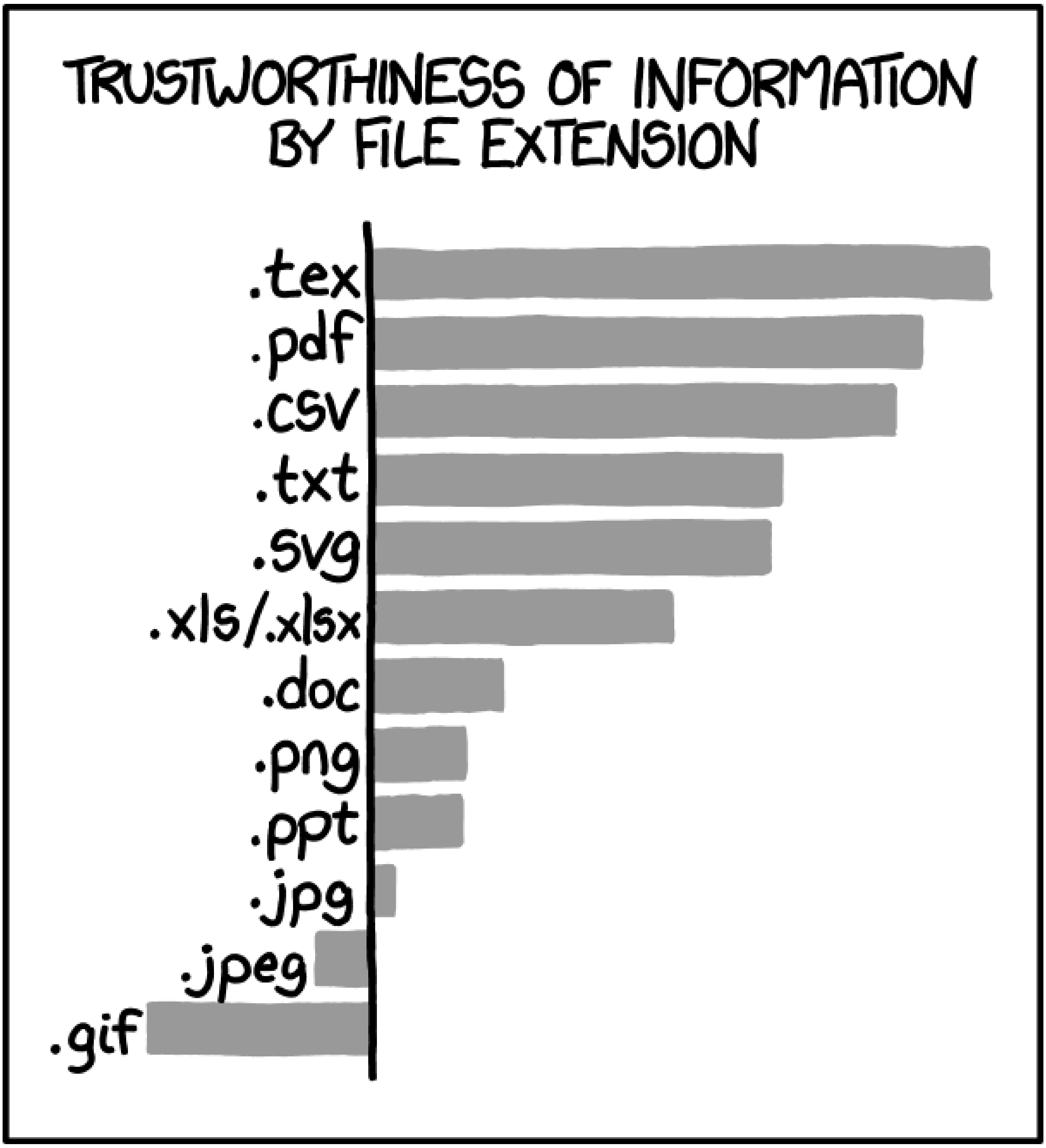
EDITION 1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Preface

To the interested reader,

This document is a compilation of lecture notes taken during the Spring 2023 semester for MATH3430: Ordinary Differential Equations at the University of Colorado Boulder. The course used *Ordinary Differential Equations*[1] by Morris Tenenbaum and Harry Pollard as its primary text. As such, many theorems, definitions, and content may be quoted from the book. This course was taught by Sheagan John, Ph. D.

The author would like to provide the following resources for students currently taking a differential equations course:

1. MIT OpenCourseWare Differential Equations Lectures From Spring 2006.
2. 3Blue1Brown's *Overview of Differential Equations*.

While much effort has been put in to remove typos and mathematical errors, it is very likely that some errors, both small and large, are present. Please keep in mind that the author wrote this resource during his second semester of his undergraduate studies. If an error needs to be resolved, please contact Adithya Bhaskara at adithya.bhaskara@colorado.edu.

Finally, the author would like to dedicate this set of lecture notes to *Aidan Janney*, *Erika Sjöblom*, and *Tate McDonald*, three of the author's closest friends who plan to take Differential Equations in the upcoming semester; Fall 2023, at the time of writing.

Best Regards,
Adithya Bhaskara

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[1] Tenenbaum, M., & Pollard, H. (1985). *Ordinary Differential Equations.* Dover Publications.

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Basic Notions

1.1 Lecture 1: January 20, 2023

1.1.1 Definition of an Ordinary Differential Equation

Consider the following definitions.

Definition 1.1.1: Ordinary Differential Equations

An ordinary differential equation is an equation of the form

$$F(x, y(x), y'(x), \dots, y^{(n)}(x)) = 0$$

where x is an independent variable and y is nth order differentiable.

We remark that every ordinary differential equation is valid as an expression only when we specify the values of x for which it is defined.

Definition 1.1.2: Order of a Differential Equation

The order of an ordinary differential equation is the highest nontrivial derivative present in the equation.

Consider the following ordinary differential equations. From now on, we will omit the term "ordinary," as partial differential equations are not considered in this text.

1. $F(x, y, y') = y' + y = 0$ is a first order differential equation.
2. $F(x, y, y') = \cos(xy') + y^2 y' + x^2 = 0$ is a first order differential equation.
3. $F(x, y, y', y'') = -\frac{1}{1-x^2} + y'' = 0$ is a second order differential equation.

4. $F(x, y, y', y'', y''', y'''') = e^{xy} y'''' - x^2 y'' - \sin x = 0$ is a fourth order differential equation.

We will now consider explicit and implicit solutions to differential equations.

Definition 1.1.3: Explicit Solutions to Ordinary Differential Equations

Let $F(\cdots) = 0$ be a differential equation defined on the interval I. Then, an explicit solution to F is a function

$$y : \mathbb{R} \to \mathbb{C}$$

for which $y(x)$ is well-defined on some set X such that $I \cap X \neq \emptyset$ and y satisfies the differential equation for all $x \in I$

Definition 1.1.4: Implicit Solutions to Ordinary Differential Equations

Let $F(\cdots) = 0$ be a differential equation defined on the interval I. Then, an implicit solution to F is a relation $F(x, y) = 0$ if and only if it defines y as an implicit function of x on I, and if $y(x)$ satisfies the differential equation for all $x \in I$.

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Higher Order Differential Equations

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Systems of Differential Equations