MATH1300: CALCULUS I

ADITHYA BHASKARA

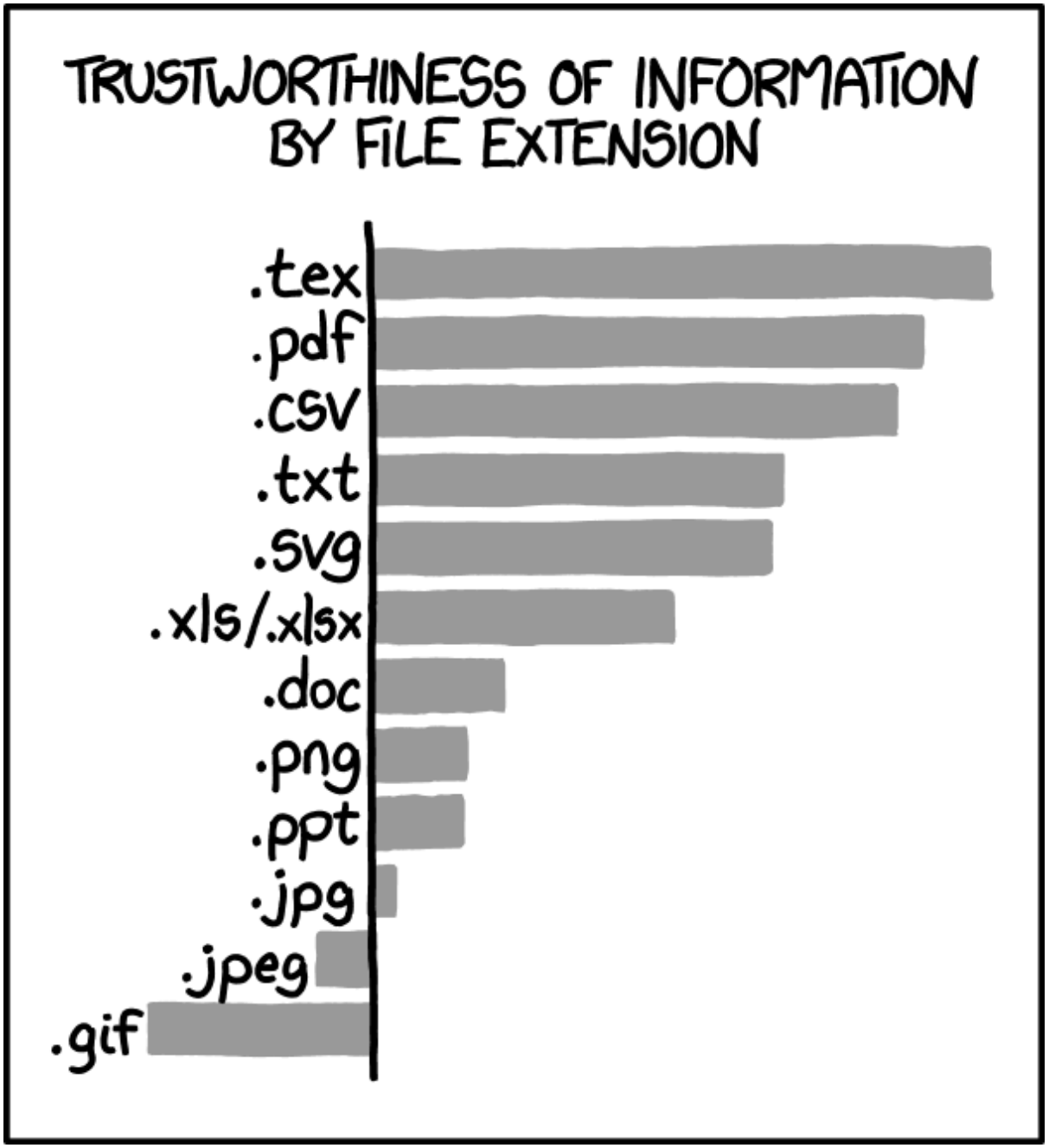
COURSE COORDINATORS: HARRISON STALVEY & CHRISTOPHER EBLEN

TEXTBOOK: JAMES STEWART

UNIVERSITY OF COLORADO BOULDER

EDITION 1

i


TRUSTWORTHINESS OF INFORMATION BY FILE EXTENSION
.tex
.pdf
.csv
.txt
.svg
.xls/.xlsx
.doc
.png
.ppt
.jpg
.jpeg
.gif

Contents

Preface **iii**

1 Vectors and Matrices **1**

2 Systems of Linear Equations **2**

3 Determinants and Eigenvalues **3**

4 Finite Dimensional Vector Spaces **4**

5 Linear Transformations **5**

6 Orthogonality **6**

Appendices

List of Theorems and Definitions **8**

Preface

To the interested reader,

This document is a compilation of lecture notes taken during the Spring 2023 semester for MATH1300: Calculus I at the University of Colorado Boulder. The course used *Calculus – Concepts and Contexts*[1] by James Stewart as its primary text. Additionally, in creating these notes, the author used *Calculus* by Ron Larson and Bruce Edwards. As such, many theorems, definitions, and examples may be quoted or derived from the aforementioned books.

The author would like to provide the following resources for students currently taking a Calculus I course:

1. Paul's *Online Math Notes* for Calculus I at Lamar University.
2. Professor Leonard's YouTube Calculus I Lectures.
3. 3Blue1Brown's *Essence of Calculus*.

Theorems, definitions, and examples may be quoted or derived from the aforementioned resources as well.

While much effort has been put in to remove typos and mathematical errors, it is very likely that some errors, both small and large, are present. If an error needs to be resolved, please contact Adithya Bhaskara at adithya.bhaskara@colorado.edu.

Best Regards,
Adithya Bhaskara

REVISED: January 13, 2023

[1] Andrilli, S., & Hecker, D. (2016). *Elementary Linear Algebra* (5th ed.). Academic Press.

1

Vectors and Matrices

2

Systems of Linear Equations

3

Determinants and Eigenvalues

4

Finite Dimensional Vector Spaces

5
Linear Transformations

6
Orthogonality

Appendices

Chapter 1, Page 1
Definition **??**, Page **??** *The Zero Vector*
Definition **??**, Page **??** *Vector Equality*
Definition **??**, Page **??** *Vector Magnitude*
Definition **??**, Page **??** *Scalar Multiplication*
Theorem **??**, Page **??** *Scalar Multiplication and Magnitude*
Definition **??**, Page **??** *Vector Direction*
Definition **??**, Page **??** *Unit Vectors*
Theorem **??**, Page **??** *Nonzero Vector Implies Nonzero Magnitude*
Theorem **??**, Page **??** *Unit Vectors Represent Direction*
Definition **??**, Page **??** *Vector Addition*
Theorem **??**, Page **??** *Properties of Vector Addition and Scalar Multiplication*
Theorem **??**, Page **??** *Scalar Multiplication Producing the Zero Vector*
Definition **??**, Page **??** *Matrices*
Definition **??**, Page **??** *Square Matrices*
Definition **??**, Page **??** *Diagonal Matrices*
Definition **??**, Page **??** *The Identity Matrix*
Definition **??**, Page **??** *The Zero Matrix*
Definition **??**, Page **??** *Upper Triangular Matrices*
Definition **??**, Page **??** *Lower Triangular Matrices*
Definition **??**, Page **??** *The Set of All $m \times n$ Matrices*
Definition **??**, Page **??** *Matrix Addition*
Definition **??**, Page **??** *Scalar Multiplication With Matrices*
Definition **??**, Page **??** *Matrix Transpose*
Definition **??**, Page **??** *Symmetric and Skew-Symmetric Matrices*
Theorem **??**, Page **??** *Properties of the Transpose*
Definition **??**, Page **??** *Trace*
Theorem **??**, Page **??** *Sum and Difference of Matrices and Their Transpose*
Theorem **??**, Page **??** *The Relation Between Square, Symmetric, and Skew-Symmetric*
Definition **??**, Page **??** *The Dot Product*
Theorem **??**, Page **??** *Properties of the Dot Product*
Theorem **??**, Page **??** *The Angle Between Two Vectors*
Theorem **??**, Page **??** *A Useful Lemma for the Cauchy-Schwarz Inequality*
Theorem **??**, Page **??** *The Cauchy-Schwarz Inequality*
Theorem **??**, Page **??** *The Triangle Inequality*
Definition **??**, Page **??** *The Projection Onto a Vector*
Theorem **??**, Page **??** *Sum of Parallel and Perpendicular Projections*
Theorem **??**, Page **??** *Projections Depend on Lines*
Definition **??**, Page **??** *Matrix Multiplication*
Theorem **??**, Page **??** *Properties of Matrix Multiplication*
Definition **??**, Page **??** *Raising a Matrix to a Power*
Chapter 2, Page 2
Definition **??**, Page **??** *Linear Equations*
Definition **??**, Page **??** *Systems of Linear Equations*
Theorem **??**, Page **??** *Characterizing Solutions of Linear Systems*
Theorem **??**, Page **??** *Row Operations*
Definition **??**, Page **??** *Row Echelon Form*
Definition **??**, Page **??** *Reduced Row Echelon Form*
Theorem **??**, Page **??** *Number of Solutions to a Linear System*
Definition **??**, Page **??** *Homogeneous Systems*
Theorem **??**, Page **??** *Solutions to Homogeneous Systems*
Definition **??**, Page **??** *Equivalence of Linear Systems*
Definition **??**, Page **??** *Row Equivalence*
Definition **??**, Page **??** *Equivalence Relations*
Theorem **??**, Page **??** *System Equivalence and Row Equivalence are Equivalence Relations*
Theorem **??**, Page **??** *Row Equivalence Implies System Equivalence*
Theorem **??**, Page **??** *Uniqueness of Reduced Row Echelon Form*
Definition **??**, Page **??** *Rank*
Theorem **??**, Page **??** *Number of Solutions to Homogeneous Systems*
Definition **??**, Page **??** *Linear Combinations*
Definition **??**, Page **??** *Row Space*
Theorem **??**, Page **??** *Transitivity of Linear Combinations*
Theorem **??**, Page **??** *Row Equivalence Implies Equal Row Space*
Definition **??**, Page **??** *Linear Maps*
Definition **??**, Page **??** *Multiplicative Inverse of a Matrix*
Theorem **??**, Page **??** *Inverse Commutativity*
Definition **??**, Page **??** *Singularity*
Theorem **??**, Page **??** *Uniqueness of the Inverse*
Definition **??**, Page **??** *Negative Integral Powers of a Nonsingular Matrices*
Theorem **??**, Page **??** *Properties of Nonsingular Matrices*
Theorem **??**, Page **??** *Matrix Exponent Laws*
Theorem **??**, Page **??** *2×2 Inverse*
Theorem **??**, Page **??** *Uniqueness of Solutions to Linear Systems*
Chapter 3, Page 3
Theorem **??**, Page **??** *The Determinant Determines the Area in $\mathbb{R}^2$*
Theorem **??**, Page **??** *The Determinant Determines the Volume in $\mathbb{R}^3$*
Definition **??**, Page **??** *The (i, j) Submatrix*
Definition **??**, Page **??** *The (i, j) Minor*
Definition **??**, Page **??** *The (i, j) Cofactor*
Definition **??**, Page **??** *The Determinant*
Theorem **??**, Page **??** *The Determinant of an Upper Triangular Matrix*
Theorem **??**, Page **??** *Determinants and Row Operations*
Theorem **??**, Page **??** *Inverses and Determinants*
Theorem **??**, Page **??** *Properties of Determinants*

Definition **??**, Page **??** *Similarity*
Definition **??**, Page **??** *Diagonalizability*
Definition **??**, Page **??** *Eigenvalues and Eigenvectors*
Definition **??**, Page **??** *Eigenspace*
Theorem **??**, Page **??** *Finding Eigenvectors and Eigenvalues*
Theorem **??**, Page **??** *The Process of Diagonalization*
Definition **??**, Page **??** *Linear Independence*
Theorem **??**, Page **??** *Diagonalizability*
Theorem **??**, Page **??** *Diagonalizability and Rank*
Theorem **??**, Page **??** *Raising Matrices to Powers*
Definition **??**, Page **??** *The Complex Numbers*
Definition **??**, Page **??** *The Set $\mathbb{C}^n$*
Definition **??**, Page **??** *Definitions of Operations With Complex Numbers*
Theorem **??**, Page **??** *Properties of Complex Numbers*
Theorem **??**, Page **??** *The Fundamental Theorem of Algebra*
Definition **??**, Page **??** *Vector Addition in $\mathbb{C}^n$*
Definition **??**, Page **??** *Scalar Multiplication in $\mathbb{C}^n$*
Definition **??**, Page **??** *The Dot Product in $\mathbb{C}^n$*
Definition **??**, Page **??** *The Magnitude in $\mathbb{C}^n$*
Definition **??**, Page **??** *The Adjoint*
Theorem **??**, Page **??** *The Transpose and The Dot Product*
Theorem **??**, Page **??** *The Adjoint and The Dot Product*
Definition **??**, Page **??** *Hermitian and Skew-Hermitian Matrices*
Chapter 4, Page 4
Definition **??**, Page **??** *Vector Spaces*
Theorem **??**, Page **??** *Derived Property of Vector Space 1*
Theorem **??**, Page **??** *Derived Property of Vector Space 2*
Theorem **??**, Page **??** *Derived Property of Vector Space 3*
Theorem **??**, Page **??** *Derived Property of Vector Space 4*
Definition **??**, Page **??** *Subspaces*
Theorem **??**, Page **??** *Showing a Vector Space is a Subspace*
Theorem **??**, Page **??** *Eigenspaces are Subspaces*
Definition **??**, Page **??** *Finite Linear Combinations*
Theorem **??**, Page **??** *Subspaces are Closed Under Linear Combinations*
Definition **??**, Page **??** *Span*
Theorem **??**, Page **??** *A Complete Characterization of the Span*
Theorem **??**, Page **??** *Two Subsets of a Vector Space and Their Spans*
Theorem **??**, Page **??** *Span as an Intersection of Subspaces*
Theorem **??**, Page **??** *Span and Row Space*
Theorem **??**, Page **??** *Is a Vector in the Span?*
Definition **??**, Page **??** *Linear Independence and Dependence*
Definition **??**, Page **??** *Generalizing Linear Independence and Dependence to Infinite Sets*
Theorem **??**, Page **??** *Showing Linear Independence 1*
Theorem **??**, Page **??** *Showing Linear Independence 2*
Theorem **??**, Page **??** *Linear Dependence of Sets With One or Two Elements*
Theorem **??**, Page **??** *Linear Dependence of Finite Subsets of a Vector Space Containing $\vec{0}$*
Theorem **??**, Page **??** *Linear Independence of Nonempty Sets*
Theorem **??**, Page **??** *A Test for Linear Independence in $\mathbb{R}^n$*
Theorem **??**, Page **??** *A Test for Linear Dependence in $\mathbb{R}^n$*
Definition **??**, Page **??** *Basis*
Theorem **??**, Page **??** *A Useful Lemma for Bases*
Theorem **??**, Page **??** *Bases Have Equivalent Cardinality*
Definition **??**, Page **??** *Dimension*
Theorem **??**, Page **??** *General Statements About Span and Dimension 1*
Theorem **??**, Page **??** *General Statements About Span and Dimension 2*
Theorem **??**, Page **??** *General Statements About Linear Independence and Dimension 1*
Theorem **??**, Page **??** *General Statements About Linear Independence and Dimension 2*
Theorem **??**, Page **??** *Dimensions of Subspaces*
Theorem **??**, Page **??** *Diagonalizability, Revisited: Part I*
Theorem **??**, Page **??** *Finding a Basis for span(S) by Contraction*
Theorem **??**, Page **??** *Finding a Basis by Expansion*
Definition **??**, Page **??** *Ordered Bases*
Definition **??**, Page **??** *Coordinates With Respect to a Basis*
Theorem **??**, Page **??** *Coordinatization*
Theorem **??**, Page **??** *Properties of Coordinatization*
Definition **??**, Page **??** *Transition Matrices*
Theorem **??**, Page **??** *Finding a Transition Matrix*
Theorem **??**, Page **??** *Properties of Transition Matrices*
Theorem **??**, Page **??** *Diagonalizability, Revisited: Part II*
Chapter 5, Page 5
Definition **??**, Page **??** *Linear Transformations*
Theorem **??**, Page **??** *Properties of Linear Transformations*
Theorem **??**, Page **??** *Compositions of Linear Transformations*
Definition **??**, Page **??** *Linear Operators*
Definition **??**, Page **??** *The Identity Linear Operator*
Definition **??**, Page **??** *The Zero Linear Operator*
Theorem **??**, Page **??** *Linear Transformations and Subspaces*
Theorem **??**, Page **??** *Linear Transformations and Bases*
Theorem **??**, Page **??** *Matrices and Linear Transformations*
Theorem **??**, Page **??** *Matrices for Linear Transformation, Considering Different Bases*
Theorem **??**, Page **??** *Similar Matrices and Linear Operators*
Theorem **??**, Page **??** *The Matrix of a Composition of Linear Transformations*
Definition **??**, Page **??** *Kernel*
Definition **??**, Page **??** *Range*
Theorem **??**, Page **??** *Kernel and Range are Subspaces*
Theorem **??**, Page **??** *Finding the Kernel of a Linear Transformation*
Theorem **??**, Page **??** *Finding the Range of a Linear Transformation*
Definition **??**, Page **??** *Nullity of a Linear Transformation*
Definition **??**, Page **??** *Rank of a Linear Transformation*
Theorem **??**, Page **??** *The Dimension Theorem (The Rank-Nullity Theorem), in $\mathbb{R}^n$*
Theorem **??**, Page **??** *Determining Injectivity and Surjectivity*
Theorem **??**, Page **??** *Determining Injectivity and Surjectivity With Equivalent Dimensions*
Theorem **??**, Page **??** *Injectivity Implies Linear Independence, Surjectivity Implies Spanning*
Definition **??**, Page **??** *Isomorphisms*
Definition **??**, Page **??** *Invertible Linear Transformations*
Theorem **??**, Page **??** *Isomorphism If And Only If Invertible*
Theorem **??**, Page **??** *Finding an Inverse Matrix, if it Exists*
Theorem **??**, Page **??** *Isomorphisms Preserve Linear Independence and Span*
Definition **??**, Page **??** *Isomorphic Vector Spaces*
Theorem **??**, Page **??** *$\cong$ is an Equivalence Relation*
Theorem **??**, Page **??** *The Dimension Theorem (The Rank-*

Nullity Theorem)
THEOREM **??**, PAGE **??** *Isomorphism Implies Equivalent Dimension*
THEOREM **??**, PAGE **??** *All n-Dimensional Vector Spaces are Isomorphic to* $\mathbb{F}^n$
DEFINITION **??**, PAGE **??** *Eigenvalues and Eigenvectors*
DEFINITION **??**, PAGE **??** *Eigenspace*
THEOREM **??**, PAGE **??** *Finding Eigenvectors and Eigenvalues*
DEFINITION **??**, PAGE **??** *Diagonalizability of a Linear Operator*
THEOREM **??**, PAGE **??** *Diagonalizability of a Linear Operator*
THEOREM **??**, PAGE **??** *Eigenvectors With Distinct Eigenvalues are Linearly Independent*
DEFINITION **??**, PAGE **??** *Algebraic Multiplicity*
DEFINITION **??**, PAGE **??** *Geometric Multiplicity*
THEOREM **??**, PAGE **??** *Algebraic and Geometric Multiplicities and Diagonalizability*
THEOREM **??**, PAGE **??** *Union and Intersection of Bases for Eigenspaces*
THEOREM **??**, PAGE **??** *The Process of Diagonalization for Linear Operators*
Chapter 6, PAGE 6
DEFINITION **??**, PAGE **??** *Inner Products and Inner Product Spaces*
THEOREM **??**, PAGE **??** *Properties of Inner Products*
DEFINITION **??**, PAGE **??** *Norms*
THEOREM **??**, PAGE **??** *Properties of Norms*
THEOREM **??**, PAGE **??** *The Cauchy-Schwarz Inequality*
THEOREM **??**, PAGE **??** *The Triangle Inequality*
DEFINITION **??**, PAGE **??** *Distance*
DEFINITION **??**, PAGE **??** *Angle*
DEFINITION **??**, PAGE **??** *Orthogonality*
THEOREM **??**, PAGE **??** *Orthonormal Implies Linearly Independent*
DEFINITION **??**, PAGE **??** *Orthogonal Bases*
DEFINITION **??**, PAGE **??** *Orthonormal Bases*
THEOREM **??**, PAGE **??** *The Gram-Schmidt Process*
THEOREM **??**, PAGE **??** *Every Inner Product Space Has an Orthonormal Basis*
THEOREM **??**, PAGE **??** *Coordinatization With Respect to an Orthogonal Basis*
THEOREM **??**, PAGE **??** *Coordinatization With Respect to an Orthonormal Basis*
DEFINITION **??**, PAGE **??** *Orthogonal Complements*
THEOREM **??**, PAGE **??** *A Useful Lemma for Finding Orthogonal Complements*
THEOREM **??**, PAGE **??** *Subsets and Subspaces*
THEOREM **??**, PAGE **??** *Finite Dimensional Inner Product Spaces and Subspaces*
THEOREM **??**, PAGE **??** *The Orthogonal Complement of the Ambient Space*
THEOREM **??**, PAGE **??** *The Orthogonal Complement of the Zero Vector*
DEFINITION **??**, PAGE **??** *Projections Onto a Subspace With Orthogonal Bases*
THEOREM **??**, PAGE **??** *Projections Onto a Subspace With Orthonormal Bases*
THEOREM **??**, PAGE **??** *Projection Theorem*
Appendix **??**, PAGE **??**
DEFINITION **??**, PAGE **??** *Proofs*
Appendix **??**, PAGE **??**
DEFINITION **??**, PAGE **??** *Functions, Domains, and Codomains*
DEFINITION **??**, PAGE **??** *Images and Pre-Images*
DEFINITION **??**, PAGE **??** *Range*
DEFINITION **??**, PAGE **??** *Injective Functions*
DEFINITION **??**, PAGE **??** *Surjective Functions*
DEFINITION **??**, PAGE **??** *Bijective Functions*
DEFINITION **??**, PAGE **??** *Compositions*
THEOREM **??**, PAGE **??** *Compositions, Injectivity, and Surjectivity*
DEFINITION **??**, PAGE **??** *Inverse Functions*
THEOREM **??**, PAGE **??** *Existence of Inverse Functions*
THEOREM **??**, PAGE **??** *Uniqueness of Inverse Functions*
THEOREM **??**, PAGE **??** *The Pigeonhole Principle*
THEOREM **??**, PAGE **??** *The Extended Pigeonhole Principle*