Multiple-Choice Test

Chapter 01.01
Introduction to Numerical Methods

1. Solving an engineering problem requires four steps. In order of sequence the four steps are
   (A) formulate, model, solve, implement
   (B) formulate, solve, model, implement
   (C) formulate, model, implement, solve
   (D) model, formulate, implement, solve

2. One of the roots of the equation $x^3 - 3x^2 + x - 3 = 0$ is
   (A) -1
   (B) 1
   (C) $\sqrt{3}$
   (D) 3

3. The solution to the set of equations
   $$25a + b + c = 25$$
   $$64a + 8b + c = 71$$
   $$144a + 12b + c = 155$$
   most nearly is $(a, b, c) =$
   (A) (1,1,1)
   (B) (1,-1,1)
   (C) (1,1,-1)
   (D) does not have a unique solution.
   (E)

4. The exact integral of

$$\int_0^{\frac{\pi}{4}} 2\cos 2x dx$$

is most nearly
   (A) -1.000
   (B) 1.000
   (C) 0.000
   (D) 2.000

(E)

5. The value of $\frac{dy}{dx}(1.0)$, given $y = 2\sin(3x)$ most nearly is

(A) -5.9399
(B) -1.980
(C) 0.31402
(D) 5.9918

6. The form of the exact solution of the ordinary differential equation $2\frac{dy}{dx} + 3y = 5e^{-x}$, $y(0) = 5$ is

(A) $Ae^{-1.5x} + Be^{x}$
(B) $Ae^{-1.5x} + Be^{-x}$
(C) $Ae^{1.5x} + Be^{-x}$
(D) $Ae^{-1.5x} + Bxe^{-x}$

For a complete solution, refer to the links at the end of the book.

Multiple-Choice Test

Chapter 01.02
Measuring Errors

1. True error is defined as
   (A) Present Approximation – Previous Approximation
   (B) True Value – Approximate Value
   (C) abs (True Value – Approximate Value)
   (D) abs (Present Approximation – Previous Approximation)

2. The expression for true error in calculating the derivative of $\sin(2x)$ at $x = \pi/4$ by using the approximate expression

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

   is
   (A) $\dfrac{h - \cos(2h) - 1}{h}$
   (B) $\dfrac{h - \cos(h) - 1}{h}$
   (C) $\dfrac{1 - \cos(2h)}{h}$
   (D) $\dfrac{\sin(2h)}{h}$

3. The relative approximate error at the end of an iteration to find the root of an equation is 0.004%. The least number of significant digits we can trust in the solution is
   (A) 2
   (B) 3
   (C) 4
   (D) 5

4. The number 0.01850×10^3 has ________ significant digits
   (A) 3
   (B) 4
   (C) 5
   (D) 6

5. The following gas stations were cited for irregular dispensation by the Department of Agriculture. Which one cheated you the most?

Station	Actual gasoline dispensed	Gasoline reading at pump
Ser	9.90	10.00
Cit	19.90	20.00
Hus	29.80	30.00
She	29.95	30.00

(A) Ser
(B) Cit
(C) Hus
(D) She

6. The number of significant digits in the number 219900 is
(A) 4
(B) 5
(C) 6
(D) 4 or 5 or 6

For a complete solution, refer to the links at the end of the book.

Multiple-Choice Test

Chapter 01.03
Sources of Error

1. Truncation error is caused by approximating
   (A) irrational numbers
   (B) fractions
   (C) rational numbers
   (D) exact mathematical procedures

2. A computer that represents only 4 significant digits with chopping would calculate 66.666*33.333 as
   (A) 2220
   (B) 2221
   (C) 2221.17778
   (D) 2222

3. A computer that represents only 4 significant digits with rounding would calculate 66.666*33.333 as
   (A) 2220
   (B) 2221
   (C) 2221.17778
   (D) 2222

4. The truncation error in calculating $f'(2)$ for $f(x) = x^2$ by
$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$
   with $h = 0.2$ is
   (A) -0.2
   (B) 0.2
   (C) 4.0
   (D) 4.2

5. The truncation error in finding $\int_{-3}^{9} x^3 dx$ using LRAM (left end point Riemann approximation) with equally portioned points $-3 < 0 < 3 < 6 < 9$ is
   (A) 648
   (B) 756
   (C) 972
   (D) 1620

6. The number 1/10 is registered in a fixed 6 bit-register with all bits used for the fractional part. The difference gets accumulated every $1/10^{th}$ of a second for one day. The magnitude of the accumulated difference is
   (A) 0.082
   (B) 135
   (C) 270
   (D) 5400

For a complete solution, refer to the links at the end of the book.

Multiple-Choice Test

Chapter 01.04
Binary Representation

1. $(25)_{10} = (?)_2$
   (A) 100110
   (B) 10011
   (C) 11001
   (D) 110010

2. $(1101)_2 = (?)_{10}$
   (A) 3
   (B) 13
   (C) 15
   (D) 26

3. $(25.375)_{10} = (?.?)_2$
   (A) 100110.011
   (B) 11001.011
   (C) 10011.0011
   (D) 10011.110

4. Representing $\sqrt{2}$ in a fixed point register with 2 bits for the integer part and 3 bits for the fractional part gives a round-off error of most nearly
   (A) -0.085709
   (B) 0.0392
   (C) 0.1642
   (D) 0.2892

5. An engineer working for the Department of Defense is writing a program that transfers non-negative real numbers to integer format. To avoid overflow problems, the maximum non-negative integer that can be represented in a 5-bit integer word is
   (A) 16
   (B) 31
   (C) 63
   (D) 64

6. For a numerically controlled machine, integers need to be stored in a memory location. The minimum number of bits needed for an integer word to represent all integers between 0 and 1024 is
   (A) 8
   (B) 9
   (C) 10
   (D) 11

For a complete solution, refer to the links at the end of the book.

Multiple-Choice Test

Chapter 01.05
Floating Point Representation

1. A hypothetical computer stores real numbers in floating point format in 8-bit words. The first bit is used for the sign of the number, the second bit for the sign of the exponent, the next two bits for the magnitude of the exponent, and the next four bits for the magnitude of the mantissa. The number $e \cong 2.718$ in the 8-bit format is
   (A) 00010101
   (B) 00011010
   (C) 00010011
   (D) 00101010

2. A hypothetical computer stores real numbers in floating point format in 8-bit words. The first bit is used for the sign of the number, the second bit for the sign of the exponent, the next two bits for the magnitude of the exponent, and the next four bits for the magnitude of the mantissa. The number that $(10100111)_2$ represented in the above given 8-bit format is
   (A) -5.75
   (B) -2.875
   (C) -1.75
   (D) -0.359375

3. A hypothetical computer stores floating point numbers in 8-bit words. The first bit is used for the sign of the number, the second bit for the sign of the exponent, the next two bits for the magnitude of the exponent, and the next four bits for the magnitude of the mantissa. The machine epsilon is most nearly
   (A) 2^{-8}
   (B) 2^{-4}
   (C) 2^{-3}
   (D) 2^{-2}

4. A machine stores floating point numbers in 7-bit word. The first bit is used for the sign of the number, the next three for the biased exponent and the next three for the magnitude of the mantissa. The number $(0010110)_2$ represented in base-10 is
   (A) 0.375
   (B) 0.875
   (C) 1.5
   (D) 3.5

5. A machine stores floating point numbers in 7-bit words. The first bit is stored for the sign of the number, the next three for the biased exponent and the next three for the magnitude of the mantissa. You are asked to represent 33.35 in the above word. The error you will get in this case would be
   - (A) underflow
   - (B) overflow
   - (C) NaN
   - (D) No error will be registered.

6. A hypothetical computer stores floating point numbers in 9-bit words. The first bit is used for the sign of the number, the second bit for the sign of the exponent, the next three bits for the magnitude of the exponent, and the next four bits for the magnitude of the mantissa. Every second, the error between 0.1 and its binary representation in the 9-bit word is accumulated. The accumulated error after one day most nearly is
   - (A) 0.002344
   - (B) 20.25
   - (C) 202.5
   - (D) 8640

For a complete solution, refer to the links at the end of the book.

Multiple-Choice Test

Chapter 01.06
Propagation of Errors

1. If $A = 3.56 \pm 0.05$ and $B = 3.25 \pm 0.04$, the values of $A + B$ are
   (A) $6.81 \le A + B \le 6.90$
   (B) $6.72 \le A + B \le 6.90$
   (C) $6.81 \le A + B \le 6.81$
   (D) $6.71 \le A + B \le 6.91$

2. A number A is correctly rounded to 3.18 from a given number B. Then $|A - B| \le C$, where C is
   (A) 0.005
   (B) 0.01
   (C) 0.18
   (D) 0.09999

3. Two numbers A and B are approximated as C and D, respectively. The relative error in $C \times D$ is given by
   (A) $\left|\left(\frac{A-C}{A}\right)\right| \times \left|\left(\frac{B-D}{B}\right)\right|$
   (B) $\left|\left(\frac{A-C}{A}\right)\right| + \left|\left(\frac{B-D}{B}\right)\right| + \left|\left(\frac{A-C}{A}\right)\right| \times \left|\left(\frac{B-D}{B}\right)\right|$
   (C) $\left|\left(\frac{A-C}{A}\right)\right| + \left|\left(\frac{B-D}{B}\right)\right| - \left|\left(\frac{A-C}{A}\right)\right| \times \left|\left(\frac{B-D}{B}\right)\right|$
   (D) $\left(\frac{A-C}{A}\right) - \left(\frac{B-D}{B}\right)$

4. The formula for normal strain in a longitudinal bar is given by $\in = \frac{F}{AE}$ where

   F = normal force applied
   A = cross-sectional area of the bar
   E = Young's modulus

   If $F = 50 \pm 0.5\,\text{N}$, $A = 0.2 \pm 0.002\ \text{m}^2$, and $E = 210 \times 10^9 \pm 1 \times 10^9\ \text{Pa}$, the maximum error in the measurement of strain is
   (A) 10^{-12}
   (B) 2.95×10^{-11}
   (C) 1.22×10^{-9}
   (D) 1.19×10^{-9}

5. A wooden block is measured to be 60 mm by a ruler and the measurements are considered to be good to 1/4th of a millimeter. Then in the measurement of 60 mm, we have ________ significant digits.
   (A) 0
   (B) 1
   (C) 2
   (D) 3

6. In the calculation of the volume of a cube of nominal size $5''$, the uncertainty in the measurement of each side is 10%. The uncertainty in the measurement of the volume would be
   (A) 5.477%
   (B) 10.00%
   (C) 17.32%
   (D) 30.00%

For a complete solution, refer to the links at the end of the book.

Multiple-Choice Test

Chapter 01.07
Taylors Series Revisited

1. The coefficient of the x^5 term in the Maclaurin polynomial for $\sin(2x)$ is
   (A) 0
   (B) 0.0083333
   (C) 0.016667
   (D) 0.26667

2. Given $f(3) = 6$, $f'(3) = 8$, $f''(3) = 11$, and that all other higher order derivatives of $f(x)$ are zero at $x = 3$, and assuming the function and all its derivatives exist and are continuous between $x = 3$ and $x = 7$, the value of $f(7)$ is
   (A) 38.000
   (B) 79.500
   (C) 126.00
   (D) 331.50

3. Given that $y(x)$ is the solution to $\frac{dy}{dx} = y^3 + 2$, $y(0) = 3$ the value of $y(0.2)$ from a second order Taylor polynomial written around $x = 0$ is
   (A) 4.400
   (B) 8.800
   (C) 24.46
   (D) 29.00

4. The series $\sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!} 4^n$ is a Maclaurin series for the following function
   (A) $\cos(x)$
   (B) $\cos(2x)$
   (C) $\sin(x)$
   (D) $\sin(2x)$

5. The function

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

is called the error function. It is used in the field of probability and cannot be calculated exactly for finite values of x. However, one can expand the integrand as a Taylor polynomial and conduct integration. The approximate value of $erf(2.0)$ using the first three terms of the Taylor series around $t = 0$ is

(A) -0.75225
(B) 0.99532
(C) 1.5330
(D) 2.8586

6. Using the remainder of Maclaurin polynomial of n^{th} order for $f(x)$ defined as

$$R_n(x) = \frac{x^{n+1}}{(n+1)!} f^{(n+1)}(c), \quad n \geq 0, \; 0 \leq c \leq x$$

the least order of the Maclaurin polynomial required to get an absolute true error of at most 10^{-6} in the calculation of $\sin(0.1)$ is (do not use the exact value of $\sin(0.1)$ or $\cos(0.1)$ to find the answer, but the knowledge that $|\sin(x)| \leq 1$ and $|\cos(x)| \leq 1$).

(A) 3
(B) 5
(C) 7
(D) 9

For a complete solution, refer to the links at the end of the book.

Multiple-Choice Test

Chapter 02.01
A Primer on Differentiation

1. The definition of the first derivative of a function $f(x)$ is

(A) $f'(x) = \dfrac{f(x+\Delta x) + f(x)}{\Delta x}$

(B) $f'(x) = \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$

(C) $f'(x) = \lim\limits_{\Delta x \to 0} \dfrac{f(x+\Delta x) + f(x)}{\Delta x}$

(D) $f'(x) = \lim\limits_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$

2. Given $y = 5e^{3x} + \sin x$, $\dfrac{dy}{dx}$ is

(A) $5e^{3x} + \cos x$
(B) $15e^{3x} + \cos x$
(C) $15e^{3x} - \cos x$
(D) $2.666e^{3x} - \cos x$

3. Given $y = \sin 2x$, $\dfrac{dy}{dx}$ at $x = 3$ is most nearly

(A) 0.9600
(B) 0.9945
(C) 1.920
(D) 1.989

4. Given $y = x^3 \ln x$, $\dfrac{dy}{dx}$ is

(A) $3x^2 \ln x$
(B) $3x^2 \ln x + x^2$
(C) x^2
(D) $3x$

5. The velocity of a body as a function of time is given as $v(t) = 5e^{-2t} + 4$, where t is in seconds, and v is in m/s. The acceleration in m/s^2 at $t = 0.6$ s is

(A) -3.012
(B) 5.506
(C) 4.147
(D) -10.00

6. If $x^2 + 2xy = y^2$, then $\dfrac{dy}{dx}$ is

(A) $\dfrac{x+y}{y-x}$
(B) $2x + 2y$
(C) $\dfrac{x+1}{y}$
(D) $-x$

<ins>For a complete solution, refer to the links at the end of the book.</ins>

Multiple-Choice Test

Chapter 02.02
Differentiation of Continuous Functions

1. The definition of the first derivative of a function $f(x)$ is
   (A) $f'(x) = \dfrac{f(x+\Delta x) + f(x)}{\Delta x}$
   (B) $f'(x) = \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$
   (C) $f'(x) = \lim\limits_{\Delta x \to 0} \dfrac{f(x+\Delta x) + f(x)}{\Delta x}$
   (D) $f'(x) = \lim\limits_{\Delta x \to 0} \dfrac{f(x+\Delta x) - f(x)}{\Delta x}$

2. The exact derivative of $f(x) = x^3$ at $x = 5$ is most nearly
   (A) 25.00
   (B) 75.00
   (C) 106.25
   (D) 125.00

3. Using the forwarded divided difference approximation with a step size of 0.2, the derivative of $f(x) = 5e^{2.3x}$ at $x = 1.25$ is
   (A) 163.4
   (B) 203.8
   (C) 211.1
   (D) 258.8

4. A student finds the numerical value of $\dfrac{d}{dx}(e^x) = 20.220$ at $x = 3$ using a step size of 0.2. Which of the following methods did the student use to conduct the differentiation?
   (A) Backward divided difference
   (B) Calculus, that is, exact
   (C) Central divided difference
   (D) Forward divided difference

5. Using the backward divided difference approximation, $\frac{d}{dx}(e^x) = 4.3715$ at $x = 1.5$ for a step size of 0.05. If you keep halving the step size to find $\frac{d}{dx}(e^x)$ at $x = 1.5$ before two significant digits can be considered to be at least correct in your answer, the step size would be (you cannot use the exact value to determine the answer)
   (A) 0.05/2
   (B) 0.05/4
   (C) 0.05/8
   (D) 0.05/16

6. The heat transfer rate q over a surface is given by

$$q = -kA\frac{dT}{dy}$$

where

k = thermal conductivity $\left(\frac{\mathrm{J}}{\mathrm{s \cdot m \cdot K}}\right)$

A = surface area $\left(\mathrm{m}^2\right)$

T = temperature (K)

y = distance normal to the surface (m)

Given

$$k = 0.025\frac{\mathrm{J}}{\mathrm{s \cdot m \cdot K}}$$

$$A = 3\,\mathrm{m}^2$$

the temperature T over the surface varies as

$$T = -1493y^3 + 2200y^2 - 1076y + 500$$

The heat transfer rate q at the surface most nearly is
   (A) -1076 W
   (B) 37.5 W
   (C) 80.7 W
   (D) 500 W

For a complete solution, refer to the links at the end of the book.

Multiple-Choice Test

Chapter 02.03
Differentiation of Discrete Functions

1. The definition of the first derivative of a function $f(x)$ is

   (A) $f'(x) = \dfrac{f(x+\Delta x)+f(x)}{\Delta x}$

   (B) $f'(x) = \dfrac{f(x+\Delta x)-f(x)}{\Delta x}$

   (C) $f'(x) = \lim\limits_{\Delta x \to 0} \dfrac{f(x+\Delta x)+f(x)}{\Delta x}$

   (D) $f'(x) = \lim\limits_{\Delta x \to 0} \dfrac{f(x+\Delta x)-f(x)}{\Delta x}$

2. Using the forward divided difference approximation with a step size of 0.2, the derivative of the function at $x = 2$ is given as

x	1.8	2.0	2.2	2.4	2.6
$f(x)$	6.0496	7.3890	9.0250	11.023	13.464

   (A) 6.697
   (B) 7.389
   (C) 7.438
   (D) 8.180

3. A student finds the numerical value of $f'(x) = 20.220$ at $x = 3$ using a step size of 0.2. Which of the following methods did the student use to conduct the differentiation if $f(x)$ is given in the table below?

x	2.6	2.8	3.0	3.2	3.4	3.6
$f(x)$	$e^{2.6}$	$e^{2.8}$	e^{3}	$e^{3.2}$	$e^{3.4}$	$e^{3.6}$

   (A) Backward divided difference
   (B) Calculus, that is, exact
   (C) Central divided difference
   (D) Forward divided difference

4. The upward velocity of a body is given as a function of time as

t, s	10	15	20	22
v, m/s	22	36	57	10

To find the acceleration at $t = 17$ s, a scientist finds a second order polynomial approximation for the velocity, and then differentiates it to find the acceleration. The estimate of the acceleration in m/s^2 at $t = 17$ s is most nearly

(A) 4.060
(B) 4.200
(C) 8.157
(D) 8.498

5. The velocity of a rocket is given as a function of time as

t, s	0	0.5	1.2	1.5	1.8
v, m/s	0	213	223	275	300

Allowed to use the forward divided difference, backward divided difference or central divided difference approximation of the first derivative, your best estimate for the acceleration $\left(a = \dfrac{dv}{dt}\right)$ of the rocket in m/s^2 at $t = 1.5$ seconds is

(A) 83.33
(B) 128.33
(C) 173.33
(D) 183.33

6. In a circuit with an inductor of inductance L, a resistor with resistance R, and a variable voltage source $E(t)$,

$$E(t) = L\frac{di}{dt} + Ri$$

The current, i, is measured at several values of time as

Time, t (secs)	1.00	1.01	1.03	1.1
Current, i (amperes)	3.10	3.12	3.18	3.24

If $L = 0.98$ henries and $R = 0.142$ ohms, the most accurate expression for $E(1.00)$ is

(A) $0.98\left(\dfrac{3.24 - 3.10}{0.1}\right) + (0.142)(3.10)$
(B) 0.142×3.10
(C) $0.98\left(\dfrac{3.12 - 3.10}{0.01}\right) + (0.142)(3.10)$
(D) $0.98\left(\dfrac{3.12 - 3.10}{0.01}\right)$

<u>For a complete solution, refer to the links at the end of the book.</u>

Multiple-Choice Test

Chapter 03.01
Background Nonlinear Equations

1. The value of x that satisfies $f(x)=0$ is called the
   (A) root of an equation $f(x)=0$
   (B) root of a function $f(x)$
   (C) zero of an equation $f(x)=0$
   (D) none of the above

2. A quadratic equation has ________ root(s).
   (A) one
   (B) two
   (C) three
   (D) four

3. For a certain cubic equation, at least one of the roots is known to be a complex root. How many total complex roots does the cubic equation have?
   (A) one
   (B) two
   (C) three
   (D) cannot be determined

4. An equation such as $\tan x = x$ has _________ root(s).
   (A) zero
   (B) one
   (C) two
   (D) infinite

5. A polynomial of order n has __________ zeros.
   (A) $n-1$
   (B) n
   (C) $n+1$
   (D) $n+2$

6. The velocity of a body is given by $v(t)=5e^{-t}+4$, where t is in seconds and v is in m/s. The velocity of the body is 6 m/s at t = _____ seconds.
   (A) 0.1823
   (B) 0.3979
   (C) 0.9163
   (D) 1.609

For a complete solution, refer to the links at the end of the book.

Multiple-Choice Test

Chapter 03.03
Bisection Method

1. The bisection method of finding roots of nonlinear equations falls under the category of a (an) _________ method.
   (A) open
   (B) bracketing
   (C) random
   (D) graphical

2. If $f(x)$ is a real continuous function in $[a,b]$, and $f(a)f(b)<0$, then for $f(x)=0$, there is (are) ____________ in the domain $[a,b]$.
   (A) one root
   (B) an undeterminable number of roots
   (C) no root
   (D) at least one root

3. Assuming an initial bracket of $[1,5]$, the second (at the end of 2 iterations) iterative value of the root of $te^{-t}-0.3=0$ using the bisection method is
   (A) 0
   (B) 1.5
   (C) 2
   (D) 3

4. To find the root of $f(x)=0$, a scientist is using the bisection method. At the beginning of an iteration, the lower and upper guesses of the root are x_l and x_u. At the end of the iteration, the absolute relative approximate error in the estimated value of the root would be
   (A) $\left|\dfrac{x_u}{x_u+x_\ell}\right|$
   (B) $\left|\dfrac{x_\ell}{x_u+x_\ell}\right|$
   (C) $\left|\dfrac{x_u-x_\ell}{x_u+x_\ell}\right|$
   (D) $\left|\dfrac{x_u+x_\ell}{x_u-x_\ell}\right|$

5. For an equation like $x^2 = 0$, a root exists at $x = 0$. The bisection method cannot be adopted to solve this equation in spite of the root existing at $x = 0$ because the function $f(x) = x^2$
   (A) is a polynomial
   (B) has repeated roots at $x = 0$
   (C) is always non-negative
   (D) has a slope equal to zero at $x = 0$

6. The ideal gas law is given by
   $$pv = RT$$
   where p is the pressure, v is the specific volume, R is the universal gas constant, and T is the absolute temperature. This equation is only accurate for a limited range of pressure and temperature. Vander Waals came up with an equation that was accurate for larger ranges of pressure and temperature given by
   $$\left(p + \frac{a}{v^2}\right)(v - b) = RT$$
   Where a and b are empirical constants dependent on a particular gas. Given the value of $R = 0.08$, $a = 3.592$, $b = 0.04267$, $p = 10$ and $T = 300$ (assume all units are consistent), one is going to find the specific volume, v, for the above values. Without finding the solution from the Vander Waals equation, what would be a good initial guess for v?
   (A) 0
   (B) 1.2
   (C) 2.4
   (D) 3.6

<u>For a complete solution, refer to the links at the end of the book.</u>

Multiple-Choice Test

Chapter 03.04
Newton-Raphson Method

1. The Newton-Raphson method of finding roots of nonlinear equations falls under the category of _____________ methods.
   (A) bracketing
   (B) open
   (C) random
   (D) graphical

2. The Newton-Raphson method formula for finding the square root of a real number R from the equation $x^2 - R = 0$ is,
   (A) $x_{i+1} = \dfrac{x_i}{2}$
   (B) $x_{i+1} = \dfrac{3x_i}{2}$
   (C) $x_{i+1} = \dfrac{1}{2}\left(x_i + \dfrac{R}{x_i}\right)$
   (D) $x_{i+1} = \dfrac{1}{2}\left(3x_i - \dfrac{R}{x_i}\right)$

3. The next iterative value of the root of $x^2 - 4 = 0$ using the Newton-Raphson method, if the initial guess is 3, is
   (A) 1.5
   (B) 2.067
   (C) 2.167
   (D) 3.000

4. The root of the equation $f(x) = 0$ is found by using the Newton-Raphson method. The initial estimate of the root is $x_0 = 3$, $f(3) = 5$. The angle the line tangent to the function $f(x)$ makes at $x = 3$ is $57°$ with respect to the x-axis. The next estimate of the root, x_1 most nearly is
   (A) –3.2470
   (B) −0.2470
   (C) 3.2470
   (D) 6.2470

5. The root of $x^3 = 4$ is found by using the Newton-Raphson method. The successive iterative values of the root are given in the table below.

Iteration Number	Value of Root
0	2.0000
1	1.6667
2	1.5911
3	1.5874
4	1.5874

The iteration number at which I would first trust at least two significant digits in the answer is

(A) 1
(B) 2
(C) 3
(D) 4

6. The ideal gas law is given by

$$pv = RT$$

where p is the pressure, v is the specific volume, R is the universal gas constant, and T is the absolute temperature. This equation is only accurate for a limited range of pressure and temperature. Vander Waals came up with an equation that was accurate for larger ranges of pressure and temperature given by

$$\left(p + \frac{a}{v^2}\right)(v - b) = RT$$

where a and b are empirical constants dependent on a particular gas. Given the value of $R = 0.08$, $a = 3.592$, $b = 0.04267$, $p = 10$ and $T = 300$ (assume all units are consistent), one is going to find the specific volume, v, for the above values. Without finding the solution from the Vander Waals equation, what would be a good initial guess for v?

(A) 0
(B) 1.2
(C) 2.4
(D) 3.6

For a complete solution, refer to the links at the end of the book.

Multiple-Choice Test

Secant Method
Chapter 03.05

1. The secant method of finding roots of nonlinear equations falls under the category of ______________ methods.
   - (A) bracketing
   - (B) graphical
   - (C) open
   - (D) random

2. The secant method formula for finding the square root of a real number R from the equation $x^2 - R = 0$ is
   - (A) $\dfrac{x_i x_{i-1} + R}{x_i + x_{i-1}}$
   - (B) $\dfrac{x_i x_{i-1}}{x_i + x_{i-1}}$
   - (C) $\dfrac{1}{2}\left(x_i + \dfrac{R}{x_i}\right)$
   - (D) $\dfrac{2x_i^2 + x_i x_{i-1} - R}{x_i + x_{i-1}}$

3. The next iterative value of the root of $x^2 - 4 = 0$ using secant method, if the initial guesses are 3 and 4, is
   - (A) 2.2857
   - (B) 2.5000
   - (C) 5.5000
   - (D) 5.7143

4. The root of the equation $f(x) = 0$ is found by using the secant method. Given one of the initial estimates is $x_0 = 3$, $f(3) = 5$, and the angle the secant line makes with the x-axis is $57°$, the next estimate of the root, x_1, is
   - (A) –3.2470
   - (B) –0.24704
   - (C) 3.247
   - (D) 6.2470

5. For finding the root of $\sin x = 0$ by the secant method, the following choice of initial guesses would not be appropriate.

(A) $\frac{\pi}{4}$ and $\frac{\pi}{2}$

(B) $\frac{\pi}{4}$ and $\frac{3\pi}{4}$

(C) $-\frac{\pi}{2}$ and $\frac{\pi}{2}$

(D) $\frac{\pi}{3}$ and $\frac{\pi}{2}$

6. When drugs are given orally to a patient, the drug concentration c in the blood stream at time t is given by a formula

$$c = Kte^{-at}$$

where K is dependent on parameters such as the dose administered while a is dependent on the absorption and elimination rates of the drug. If $K = 2$ and $a = 0.25$, and t is in seconds and c is in mg/ml, the time at which the maximum concentration is reached is given by the solution of the equation

(A) $2te^{-0.25t} = 0$

(B) $2e^{-0.25t} - 2te^{-0.25t} = 0$

(C) $2e^{-0.25t} - 0.5te^{-0.25t} = 0$

(D) $2te^{-0.25t} = 2$

For a complete solution, refer to the links at the end of the book.

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Multiple-Choice Test
Background
Simultaneous Linear Equations

1. Given $[A]=\begin{bmatrix} 6 & 2 & 3 & 9 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 6 \end{bmatrix}$ then [A] is a ______________ matrix.

(A) diagonal
(B) identity
(C) lower triangular
(D) upper triangular

2. A square matrix [A] is lower triangular if

(A) $a_{ij} = 0, j > i$
(B) $a_{ij} = 0, i > j$
(C) $a_{ij} \neq 0, i > j$
(D) $a_{ij} \neq 0, j > i$

3. Given

$$[A]=\begin{bmatrix} 12.3 & -12.3 & 20.3 \\ 11.3 & -10.3 & -11.3 \\ 10.3 & -11.3 & -12.3 \end{bmatrix},\ [B]=\begin{bmatrix} 2 & 4 \\ -5 & 6 \\ 11 & -20 \end{bmatrix}$$

then if
$[C] = [A]\,[B]$, then
$c_{31}=$ ______________________

(A) -58.2
(B) -37.6
(C) 219.4
(D) 259.4

4. The following system of equations has ____________ solution(s).

$x + y = 2$
$6x+6y =12$

(A) infinite
(B) no
(C) two
(D) unique

5. Consider there are only two computer companies in a country. The companies are named *Dude* and *Imac*. Each year, company *Dude* keeps $1/5^{th}$ of its customers, while the rest switch to *Imac*. Each year, *Imac* keeps $1/3^{rd}$ of its customers, while the rest switch to *Dude*. If in *2003*, *Dude* had $1/6^{th}$ of the market and *Imac* had $5/6^{th}$ of the market, what will be share of *Dude* computers when the market becomes stable?

(A) 37/90
(B) 5/11
(C) 6/11
(D) 53/90

6. Three kids - Jim, Corey and David receive an inheritance of \$2,253,453. The money is put in three trusts but is not divided equally to begin with. Corey's trust is three times that of David's because Corey made an *A* in Dr. Kaw's class. Each trust is put in an interest generating investment. The three trusts of Jim, Corey and David pays an interest of 6%, 8%, 11%, respectively. The total interest of all the three trusts combined at the end of the first year is \$190,740.57. The equations to find the trust money of Jim (*J*), Corey (*C*) and David (*D*) in a matrix form is

(A) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & -1 \\ 0.06 & 0.08 & 0.11 \end{bmatrix} \begin{bmatrix} J \\ C \\ D \end{bmatrix} = \begin{bmatrix} 2{,}253{,}453 \\ 0 \\ 190{,}740.57 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -3 \\ 0.06 & 0.08 & 0.11 \end{bmatrix} \begin{bmatrix} J \\ C \\ D \end{bmatrix} = \begin{bmatrix} 2{,}253{,}453 \\ 0 \\ 190{,}740.57 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & -3 \\ 6 & 8 & 11 \end{bmatrix} \begin{bmatrix} J \\ C \\ D \end{bmatrix} = \begin{bmatrix} 2{,}253{,}453 \\ 0 \\ 190{,}740.57 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & -1 \\ 6 & 8 & 11 \end{bmatrix} \begin{bmatrix} J \\ C \\ D \end{bmatrix} = \begin{bmatrix} 2{,}253{,}453 \\ 0 \\ 19{,}074{,}057 \end{bmatrix}$

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Multiple Choice Test

Gaussian Elimination

1. The goal of forward elimination steps in Naïve Gauss elimination method is to reduce the coefficient matrix to a (an) ______________ matrix.

(A) diagonal
(B) identity
(C) lower triangular
(D) upper triangular

2. Division by zero during forward elimination steps in Naïve Gaussian elimination of the set of equations [A][X]=[C] implies the coefficient matrix [A] is

(A) invertible
(B) nonsingular
(C) not determinable to be singular or nonsingular
(D) singular

3. Using a computer with four significant digits with chopping, Naïve Gauss elimination solution to

$$0.0030x_1 + 55.23x_2 = 58.12$$
$$6.239x_1 - 7.123x_2 = 47.23$$

is

(A) $x_1 = 26.66$; $x_2 = 1.051$
(B) $x_1 = 8.769$; $x_2 = 1.051$
(C) $x_1 = 8.800$; $x_2 = 1.000$
(D) $x_1 = 8.771$; $x_2 = 1.052$

4. Using a computer with four significant digits with chopping, Gaussian elimination with partial pivoting solution to

$$0.0030x_1 + 55.23x_2 = 58.12$$
$$6.239x_1 - 7.123x_2 = 47.23$$

is

(A) $x_1 = 26.66$; $x_2 = 1.051$
(B) $x_1 = 8.769$; $x_2 = 1.051$
(C) $x_1 = 8.800$; $x_2 = 1.000$
(D) $x_1 = 8.771$; $x_2 = 1.052$

5. At the end of forward elimination steps of Naïve Gauss Elimination method on the following equations

$$\begin{bmatrix} 4.2857\times10^{7} & -9.2307\times10^{5} & 0 & 0 \\ 4.2857\times10^{7} & -5.4619\times10^{5} & -4.2857\times10^{7} & 5.4619\times10^{5} \\ -6.5 & -0.15384 & 6.5 & 0.15384 \\ 0 & 0 & 4.2857\times10^{7} & -3.6057\times10^{5} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} -7.887\times10^{3} \\ 0 \\ 0.007 \\ 0 \end{bmatrix}$$

the resulting equations in the matrix form are given by

$$\begin{bmatrix} 4.2857\times10^{7} & -9.2307\times10^{5} & 0 & 0 \\ 0 & 3.7688\times10^{5} & -4.2857\times10^{7} & 5.4619\times10^{5} \\ 0 & 0 & -26.9140 & 0.579684 \\ 0 & 0 & 0 & 5.62500\times10^{5} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} -7.887\times10^{3} \\ 7.887\times10^{3} \\ 1.19530\times10^{-2} \\ 1.90336\times10^{4} \end{bmatrix}$$

The determinant of the original coefficient matrix is

(A) 0.00

(B) 4.2857×10^{7}

(C) 5.486×10^{19}

(D) -2.445×10^{20}

6. The following data is given for the velocity of the rocket as a function of time. To find the velocity at t=21 s, you are asked to use a quadratic polynomial, $v(t)=at^2+bt+c$ to approximate the velocity profile.

t	*(s)*	0	14	15	20	30	35
v(t)	*m/s*	0	227.04	362.78	517.35	602.97	901.67

The correct set of equations that will find *a*, *b* and *c* are

(A) $$\begin{bmatrix} 176 & 14 & 1 \\ 225 & 15 & 1 \\ 400 & 20 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 227.04 \\ 362.78 \\ 517.35 \end{bmatrix}$$

(B) $$\begin{bmatrix} 225 & 15 & 1 \\ 400 & 20 & 1 \\ 900 & 30 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 362.78 \\ 517.35 \\ 602.97 \end{bmatrix}$$

(C) $$\begin{bmatrix} 0 & 0 & 1 \\ 225 & 15 & 1 \\ 400 & 20 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 362.78 \\ 517.35 \end{bmatrix}$$

(D) $$\begin{bmatrix} 400 & 20 & 1 \\ 900 & 30 & 1 \\ 1225 & 35 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 517.35 \\ 602.97 \\ 901.67 \end{bmatrix}$$

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Multiple Choice Test
LU Decomposition Method
Simultaneous Linear Equations

1. LU decomposition method is computationally more efficient than Naïve Gauss elimination for solving
   (A) a single set of simultaneous linear equations
   (B) multiple sets of simultaneous linear equations with different coefficient matrices and same right hand side vectors.
   (C) multiple sets of simultaneous linear equations with same coefficient matrix and different right hand side vectors.
   (D) less than ten simultaneous linear equations.

2. The lower triangular matrix [L] in the [L][U] decomposition of matrix given below

$$\begin{bmatrix} 25 & 5 & 4 \\ 10 & 8 & 16 \\ 8 & 12 & 22 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \ell_{21} & 1 & 0 \\ \ell_{31} & \ell_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

is

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0.40000 & 1 & 0 \\ 0.32000 & 1.7333 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 25 & 5 & 4 \\ 0 & 6 & 14.400 \\ 0 & 0 & -4.2400 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & 0 \\ 10 & 1 & 0 \\ 8 & 12 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 & 0 \\ 0.40000 & 1 & 0 \\ 0.32000 & 1.5000 & 1 \end{bmatrix}$

3. The upper triangular matrix [U] in the [L][U] decomposition of matrix given below

$$\begin{bmatrix} 25 & 5 & 4 \\ 0 & 8 & 16 \\ 0 & 12 & 22 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \ell_{21} & 1 & 0 \\ \ell_{31} & \ell_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

is

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0.40000 & 1 & 0 \\ 0.32000 & 1.7333 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 25 & 5 & 4 \\ 0 & 6 & 14.400 \\ 0 & 0 & -4.2400 \end{bmatrix}$

(C) $\begin{bmatrix} 25 & 5 & 4 \\ 0 & 8 & 16 \\ 0 & 0 & -2 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0.2000 & 0.16000 \\ 0 & 1 & 2.4000 \\ 0 & 0 & -4.240 \end{bmatrix}$

4. For a given 2000×2000 matrix [A], assume that it takes about 15 seconds to find the inverse of [A] by the use of the [L][U] decomposition method, that is, finding the [L][U] once, and then doing forward substitution and back substitution 2000 times using the 2000 columns of the identity matrix as the right hand side vector. The approximate time, in seconds, that it will take to find the inverse if found by repeated use of Naive Gauss Elimination method, that is, doing forward elimination and back substitution 2000 times by using the 2000 columns of the identity matrix as the right hand side vector is

(A) 300
(B) 1500
(C) 7500
(D) 30000

5. The algorithm in solving the set of equations [A][X] = [C], where [A] = [L][U] involves solving [L][Z] = [C] by forward substitution. The algorithm to solve [L][Z]=[C] is given by

(A)
```
z_1 = c_1 / l_11
for i from 2 to n do
 sum = 0
   for j from 1 to i do
      sum = sum + l_ij * z_j
   end do
 z_i = (c_i – sum) / l_ii
end do
```

(B)
```
z_1 = c_1 / l_11
for i from 2 to n do
 sum = 0
   for j from 1 to (i-1) do
      sum = sum + l_ij * z_j
   end do
 z_i = (c_i – sum) / l_ii
end do
```

(C)
```
z_1 = c_1 / l_11
for i from 2 to n do
   for j from 1 to (i-1) do
      sum = sum + l_ij * z_j
   end do
z_i = (c_i – sum) / l_ii
end do
```

(D)
```
for i from 2 to n do
  sum = 0
   for j from 1 to (i-1) do
      sum = sum + l_ij * z_j
  end do
 z_i = (c_i – sum) / l_ii
end do
```

6. To solve boundary value problems, finite difference method is used resulting in simultaneous linear equations with tri-diagonal coefficient matrices. These are solved using the specialized [L][U] decomposition method. The set of equations in matrix form with a tri-diagonal coefficient matrix for

$$\frac{d^2y}{dx^2} = 6x - 0.5x^2,\ y(0) = 0,\ y(12) = 0,$$

using finite difference method with a second order accurate central divided difference method and a step size of $h = 4$ is

(A) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.0625 & 0.125 & 0.0625 & 0 \\ 0 & 0.0625 & 0.125 & 0.0625 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 16.0 \\ 16.0 \\ 0 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.0625 & -0.125 & 0.0625 & 0 \\ 0 & 0.0625 & -0.125 & 0.0625 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 16.0 \\ 16.0 \\ 0 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0.0625 & -0.125 & 0.0625 & 0 \\ 0 & 0.0625 & -0.125 & 0.0625 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 16.0 \\ 16.0 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0.0625 & 0.125 & 0.0625 & 0 \\ 0 & 0.0625 & 0.125 & 0.0625 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 16.0 \\ 16.0 \end{bmatrix}$

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Multiple-Choice Test
Gauss-Seidel Method of Solving Simultaneous Linear Equations

1. A square matrix $[A]_{nxn}$ is diagonally dominant if

(A) $|a_{ii}| \geq \sum_{\substack{j=1 \\ i \neq j}}^{n} |a_{ij}|, \quad i = 1, 2, \ldots, n$

(B) $|a_{ii}| \geq \sum_{\substack{j=1 \\ i \neq j}}^{n} |a_{ij}|, \; i = 1, 2, \ldots, n$ and $|a_{ii}| > \sum_{\substack{j=1 \\ i \neq j}}^{n} |a_{ij}|,$ for any $i = 1, 2, \ldots, n$

(C) $|a_{ii}| \geq \sum_{j=1}^{n} |a_{ij}|, \; i = 1, 2, \ldots, n$ and $|a_{ii}| > \sum_{j=1}^{n} |a_{ij}|,$ for any $i = 1, 2, \ldots, n$

(D) $|a_{ii}| \geq \sum_{j=1}^{n} |a_{ij}|, \; i = 1, 2, \ldots, n$

2. Using $[x_1 \quad x_2 \quad x_3] = [1 \quad 3 \quad 5]$ as the initial guess, the value of $[x_1 \quad x_2 \quad x_3]$ after three iterations in Gauss-Seidel method for

$$\begin{bmatrix} 12 & 7 & 3 \\ 1 & 5 & 1 \\ 2 & 7 & -11 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ 6 \end{bmatrix}$$

is

(A) [-2.8333 -1.4333 -1.9727]
(B) [1.4959 -0.90464 -0.84914]
(C) [0.90666 -1.0115 -1.0242]
(D) [1.2148 -0.72060 -0.82451]

3. To ensure that the following system of equations,

$$\begin{aligned} 2x_1 + 7x_2 - 11x_3 &= 6 \\ x_1 + 2x_2 + x_3 &= -5 \\ 7x_1 + 5x_2 + 2x_3 &= 17 \end{aligned}$$

converges using Gauss-Seidel Method, one can rewrite the above equations as follows:

(A) $\begin{bmatrix} 2 & 7 & -11 \\ 1 & 2 & 1 \\ 7 & 5 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -5 \\ 17 \end{bmatrix}$

(B) $\begin{bmatrix} 7 & 5 & 2 \\ 1 & 2 & 1 \\ 2 & 7 & -11 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 17 \\ -5 \\ 6 \end{bmatrix}$

(C) $\begin{bmatrix} 7 & 5 & 2 \\ 1 & 2 & 1 \\ 2 & 7 & -11 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -5 \\ 17 \end{bmatrix}$

(D) The equations cannot be rewritten in a form to ensure convergence.

4. For $\begin{bmatrix} 12 & 7 & 3 \\ 1 & 5 & 1 \\ 2 & 7 & -11 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 22 \\ 7 \\ -2 \end{bmatrix}$ and using $\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$ as the initial guess, the values of $\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$ are found at the end of each iteration as

Iteration #	x_1	x_2	x_3
1	0.41666	1.1166	0.96818
2	0.93989	1.0183	1.0007
3	0.98908	1.0020	0.99930
4	0.99898	1.0003	1.0000

At what first iteration number would you trust at least 1 significant digit in your solution?

(A) 1
(B) 2
(C) 3
(D) 4

5. The algorithm for the Gauss-Seidel Method to solve [A] [X] = [C] is given as follows for using nmax iterations. The initial value of [X] is stored in [X].

(A)
```
Sub Seidel(n, a, x, rhs, nmax)
For k = 1 To nmax
For i = 1 To n
For j = 1 To n
If (i <> j) Then
Sum = Sum + a(i, j) * x(j)
endif
Next j
x(i) = (rhs(i) - Sum) / a(i, i)
Next i
Next k
End Sub
```

(B)
```
Sub Seidel(n, a, x, rhs, nmax)
For k = 1 To nmax
For i = 1 To n
Sum = 0
For j = 1 To n
If (i <> j) Then
Sum = Sum + a(i, j) * x(j)
endif
Next j
x(i) = (rhs(i) - Sum) / a(i, i)
Next i
Next k
End Sub
```

(C)
```
Sub Seidel(n, a, x, rhs, nmax)
For k = 1 To nmax
For i = 1 To n
Sum = 0
For j = 1 To n
Sum = Sum + a(i, j) * x(j)
Next j
x(i) = (rhs(i) - Sum) / a(i, i)
Next i
Next k
End Sub
```

(D)
```
Sub Seidel(n, a, x, rhs, nmax)
For k = 1 To nmax
For i = 1 To n
Sum = 0
For j = 1 To n
If (i <> j) Then
Sum = Sum + a(i, j) * x(j)
endif
Next j
x(i) = rhs(i)  / a(i, i)
Next i
Next k
End Sub
```

6. Thermistors measure temperature, have a nonlinear output and are valued for a limited range. So when a thermistor is manufactured, the manufacturer supplies a resistance vs. temperature curve. An accurate representation of the curve is generally given by

$$\frac{1}{T} = a_0 + a_1 \ln(R) + a_2 \{\ln(R)\}^2 + a_3 \{\ln(R)\}^3$$

where T is temperature in Kelvin, R is resistance in ohms, and a_0, a_1, a_2, a_3 are constants of the calibration curve.

Given the following for a thermistor

R	**T**
ohm	$^\circ C$
1101.0	25.113
911.3	30.131
636.0	40.120
451.1	50.128

the value of temperature in $^\circ C$ for a measured resistance of 900 ohms most nearly is

(A) 30.002
(B) 30.472
(C) 31.272
(D) 31.445

Multiple-Choice Test

Chapter 05.01
Background on Interpolation

1. The number of polynomials that can go through two fixed data points (x_1, y_1) and (x_2, y_2) is
   (A) 0
   (B) 1
   (C) 2
   (D) infinite

2. A unique polynomial of degree __________________ passes through $n+1$ data points.
   (A) $n+1$
   (B) $n+1$ or less
   (C) n
   (D) n or less

3. The following function(s) can be used for interpolation:
   (A) polynomial
   (B) exponential
   (C) trigonometric
   (D) all of the above

4. Polynomials are the most commonly used functions for interpolation because they are easy to
   (A) evaluate
   (B) differentiate
   (C) integrate
   (D) evaluate, differentiate and integrate

5. Given $n+1$ data points $(x_0, y_0), (x_1, y_1), \ldots\ldots, (x_{n-1}, y_{n-1}), (x_n, y_n)$, assume you pass a function $f(x)$ through all the data points. If now the value of the function $f(x)$ is required to be found outside the range of the given x-data, the procedure is called
   (A) extrapolation
   (B) interpolation
   (C) guessing
   (D) regression

6. Given three data points (1,6), (3,28), and (10, 231), it is found that the function $y = 2x^2 + 3x + 1$ passes through the three data points. Your estimate of y at $x = 2$ is most nearly

(A) 6
(B) 15
(C) 17
(D) 28

For a complete solution, refer to the links at the end of the book.

Multiple-Choice Test

Chapter 05.02
Direct Method of Interpolation

1. A unique polynomial of degree ________________ passes through $n+1$ data points.
   (A) $n+1$
   (B) $n+1$ or less
   (C) n
   (D) n or less

2. The following data of the velocity of a body is given as a function of time.

Time (s)	0	15	18	22	24
Velocity (m/s)	22	24	37	25	123

   The velocity in m/s at $16\,\text{s}$ using linear polynomial interpolation is most nearly
   (A) 27.867
   (B) 28.333
   (C) 30.429
   (D) 43.000

3. The following data of the velocity of a body is given as a function of time.

Time (s)	0	15	18	22	24
Velocity (m/s)	22	24	37	25	123

   The velocity in m/s at $16\,\text{s}$ using quadratic polynomial interpolation is most nearly
   (A) 27.867
   (B) 28.333
   (C) 30.429
   (D) 43.000

4. The following data of the velocity of a body is given as a function of time.

Time (s)	0	15	18	22	24
Velocity (m/s)	22	24	37	25	123

   Using quadratic interpolation, the interpolant

$$v(t) = 8.667t^2 - 349.67t + 3523, \quad 18 \le t \le 24$$

   approximates the velocity of the body. From this information, the time in seconds at which the velocity of the body is 35 m/s during the above time interval of $t = 18\,\text{s}$ to $t = 24\,\text{s}$ is
   (A) 18.667
   (B) 20.850
   (C) 22.200
   (D) 22.294

5. The following data of the velocity of a body is given as a function of time.

Time (s)	0	15	18	22	24
Velocity (m/s)	22	24	37	25	123

One of the interpolant approximations for the velocity from the above data is given as

$$v(t) = 8.6667t^2 - 349.67t + 3523, \quad 18 \le t \le 24$$

Using the above interpolant, the distance in meters covered by the body between $t = 19$ s and $t = 22$ s is most nearly

(A) 10.337
(B) 88.500
(C) 93.000
(D) 168.00

6. The following data of the velocity of a body is given as a function of time.

Time (s)	0	15	18	22	24
Velocity (m/s)	22	24	37	25	123

If you were going to use quadratic interpolation to find the value of the velocity at $t = 14.9$ seconds, what three data points of time would you choose for interpolation?

(A) 0, 15, 18
(B) 15, 18, 22
(C) 0, 15, 22
(D) 0, 18, 24

For a complete solution, refer to the links at the end of the book.

Multiple-Choice Test

Chapter 05.03
Newton's Divided Difference Polynomial Method

1. If a polynomial of degree n has $n+1$ zeros, then the polynomial is
   (A) oscillatory
   (B) zero everywhere
   (C) quadratic
   (D) not defined

2. The following x, y data is given.

x	15	18	22
y	24	37	25

   The Newton's divided difference second order polynomial for the above data is given by
   $$f_2(x) = b_0 + b_1(x-15) + b_2(x-15)(x-18)$$
   The value of b_1 is most nearly
   (A) –1.0480
   (B) 0.14333
   (C) 4.3333
   (D) 24.000

3. The polynomial that passes through the following x, y data

x	18	22	24
y	?	25	123

   is given by
   $$8.125x^2 - 324.75x + 3237, \quad 18 \le x \le 24$$
   The corresponding polynomial using Newton's divided difference polynomial is given by
   $$f_2(x) = b_0 + b_1(x-18) + b_2(x-18)(x-22)$$
   The value of b_2 is most nearly
   (A) 0.25000
   (B) 8.1250
   (C) 24.000
   (D) not obtainable with the information given

4. Velocity vs. time data for a body is approximated by a second order Newton's divided difference polynomial as

$$v(t)=b_0+39.622(t-20)+0.5540(t-20)(t-15), \quad 10\le t\le 20$$

The acceleration in m/s^2 at $t=15$ is

(A) 0.5540
(B) 39.622
(C) 36.852
(D) not obtainable with the given information

5. The path that a robot is following on a $x-y$ plane is found by interpolating the following four data points as

x	2	4.5	5.5	7
y	7.5	7.5	6	5

$$y(x)=0.1524x^3-2.257x^2+9.605x-3.900$$

The length of the path from $x=2$ to $x=7$ is

(A) $\sqrt{(7.5-7.5)^2+(4.5-2)^2}+\sqrt{(6-7.5)^2+(5.5-4.5)^2}+\sqrt{(5-6)^2+(7-5.5)^2}$
(B) $\int_2^7 \sqrt{1+(0.1524x^3-2.257x^2+9.605x-3.900)^2}\,dx$
(C) $\int_2^7 \sqrt{1+(0.4572x^2-4.514x+9.605)^2}\,dx$
(D) $\int_2^7 (0.1524x^3-2.257x^2+9.605x-3.900)dx$

6. The following data of the velocity of a body is given as a function of time.

Time (s)	0	15	18	22	24
Velocity (m/s)	22	24	37	25	123

If you were going to use quadratic interpolation to find the value of the velocity at $t=14.9$ seconds, the three data points of time you would choose for interpolation are

(A) 0, 15, 18
(B) 15, 18, 22
(C) 0, 15, 22
(D) 0, 18, 24

For a complete solution, refer to the links at the end of the book.

Multiple-Choice Test

Chapter 05.04
Lagrange Method of Interpolation

1. A unique polynomial of degree _______________ passes through $n+1$ data points.
   (A) $n+1$
   (B) n
   (C) n or less
   (D) $n+1$ or less

2. Given the two points $[a, f(a)]$, $[b, f(b)]$, the linear Lagrange polynomial $f_1(x)$ that passes through these two points is given by
   (A) $f_1(x) = \dfrac{x-b}{a-b} f(a) + \dfrac{x-a}{a-b} f(b)$
   (B) $f_1(x) = \dfrac{x}{b-a} f(a) + \dfrac{x}{b-a} f(b)$
   (C) $f_1(x) = f(a) + \dfrac{f(b)-f(a)}{b-a}(b-a)$
   (D) $f_1(x) = \dfrac{x-b}{a-b} f(a) + \dfrac{x-a}{b-a} f(b)$

3. The Lagrange polynomial that passes through the 3 data points is given by

x	15	18	22
y	24	37	25

$$f_2(x) = L_0(x)(24) + L_1(x)(37) + L_2(x)(25)$$

The value of $L_1(x)$ at $x = 16$ is most nearly
   (A) –0.071430
   (B) 0.50000
   (C) 0.57143
   (D) 4.3333

4. The following data of the velocity of a body is given as a function of time.

Time (s)	10	15	18	22	24
Velocity (m/s)	22	24	37	25	123

A quadratic Lagrange interpolant is found using three data points, $t = 15$, 18 and 22. From this information, at what of the times given in seconds is the velocity of the body 26 m/s during the time interval of $t = 15$ to $t = 22$ seconds.

(A) 20.173
(B) 21.858
(C) 21.667
(D) 22.020

5. The path that a robot is following on a x, y plane is found by interpolating four data points as

x	2	4.5	5.5	7
y	7.5	7.5	6	5

$$y(x) = 0.15238x^3 - 2.2571x^2 + 9.6048x - 3.9000$$

The length of the path from $x = 2$ to $x = 7$ is

(A) $\sqrt{(7.5-7.5)^2 + (4.5-2)^2} + \sqrt{(6-7.5)^2 + (5.5-4.5)^2} + \sqrt{(5-6)^2 + (7-5.5)^2}$

(B) $\int_2^7 \sqrt{1 + (0.15238x^3 - 2.2571x^2 + 9.6048x - 3.9000)^2}\,dx$

(C) $\int_2^7 \sqrt{1 + (0.45714x^2 - 4.5142x + 9.6048)^2}\,dx$

(D) $\int_2^7 (0.15238x^3 - 2.2571x^2 + 9.6048x - 3.9000)dx$

6. The following data of the velocity of a body is given as a function of time.

Time (s)	0	15	18	22	24
Velocity (m/s)	22	24	37	25	123

If you were going to use quadratic interpolation to find the value of the velocity at $t = 14.9$ seconds, what three data points of time would you choose for interpolation?

(A) 0, 15, 18
(B) 15, 18, 22
(C) 0, 15, 22
(D) 0, 18, 24

For a complete solution, refer to the links at the end of the book.

Multiple-Choice Test

Chapter 05.05
Spline Method of Interpolation

1. The following n data points, (x_1, y_1), (x_2, y_2), …….. (x_n, y_n), are given. For conducting quadratic spline interpolation the x-data needs to be
   (A) equally spaced
   (B) placed in ascending or descending order of x-values
   (C) integers
   (D) positive

2. In cubic spline interpolation,
   (A) the first derivatives of the splines are continuous at the interior data points
   (B) the second derivatives of the splines are continuous at the interior data points
   (C) the first and the second derivatives of the splines are continuous at the interior data points
   (D) the third derivatives of the splines are continuous at the interior data points

3. The following incomplete y vs. x data is given.

x	1	2	4	6	7
y	5	11	????	????	32

The data is fit by quadratic spline interpolants given by

$$f(x) = ax - 1, \quad 1 \le x \le 2$$
$$f(x) = -2x^2 + 14x - 9, \quad 2 \le x \le 4$$
$$f(x) = bx^2 + cx + d, \quad 4 \le x \le 6$$
$$f(x) = 25x^2 - 303x + 928, \quad 6 \le x \le 7$$

where a, b, c, and d are constants. The value of c is most nearly
   (A) -303.00
   (B) -144.50
   (C) 0.0000
   (D) 14.000

4. The following incomplete y vs. x data is given.

x	1	2	4	6	7
y	5	11	????	????	32

The data is fit by quadratic spline interpolants given by

$f(x) = ax - 1, \quad 1 \le x \le 2,$

$f(x) = -2x^2 + 14x - 9, \quad 2 \le x \le 4$

$f(x) = bx^2 + cx + d, \quad 4 \le x \le 6$

$f(x) = ex^2 + fx + g, \quad 6 \le x \le 7$

where a, b, c, d, e, f, and g are constants. The value of $\frac{df}{dx}$ at $x = 2.6$ most nearly is

(A) -144.50
(B) -4.0000
(C) 3.6000
(D) 12.200

5. The following incomplete y vs. x data is given.

x	1	2	4	6	7
y	5	11	????	????	32

The data is fit by quadratic spline interpolants given by

$f(x) = ax - 1, \quad 1 \le x \le 2,$

$f(x) = -2x^2 + 14x - 9, \quad 2 \le x \le 4$

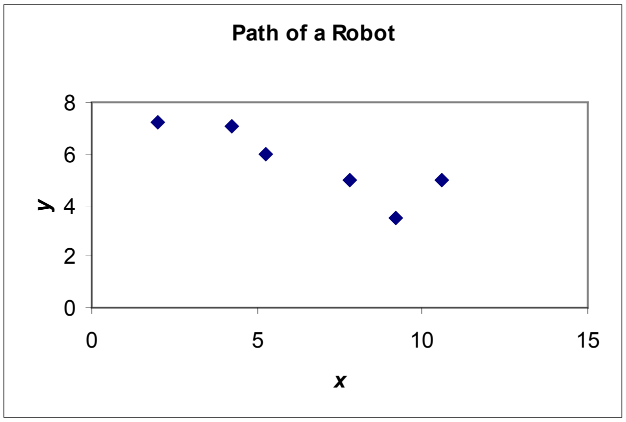
$f(x) = bx^2 + cx + d, \quad 4 \le x \le 6$

$f(x) = 25x^2 - 303x + 928, \quad 6 \le x \le 7$

where a, b, c, and d are constants. What is the value of $\int_{1.5}^{3.5} f(x)dx$?

(A) 23.500
(B) 25.667
(C) 25.750
(D) 28.000

6. A robot needs to follow a path that passes consecutively through six points as shown in the figure. To find the shortest path that is also smooth you would recommend which of the following?
   (A) Pass a fifth order polynomial through the data
   (B) Pass linear splines through the data
   (C) Pass quadratic splines through the data
   (D) Regress the data to a second order polynomial



<u>For a complete solution, refer to the links at the end of the book.</u>

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Multiple-Choice Test
Background
Regression

1. The average and standard deviation of the following numbers are

2	4	10	12	1.6	6.4

(A) 6.0, 4.0857
(B) 6.0, 4.2783
(C) 7.2, 4.0857
(D) 7.2, 4.4757

2. The average of 7 numbers is given 12.6. If 6 of the numbers are 5, 7, 9, 12,17 and 10, the remaining number is

(A) -47.9
(B) -47.4
(C) 15.6
(D) 28.2

3. The average and standard deviation of 7 numbers is given a 8.142 and 5.005, respectively. If 5 numbers are 5, 7, 9, 12 and 17, the other two numbers are

(A) -0.1738, 7.175
(B) 3.396, 12.890
(C) 3.500, 3.500
(D) 4.488, 2.512

quiz_reg_background.doc

4. The sum of the square of the difference between data point and its average for the data is

2	5	10	12	2.5	6.7

(A) 4.023
(B) 13.49
(C) 16.19
(D) 80.93

5. Two medication are tried to heal esophageal ulcers in patients. The time to heal is reported as the time the patient reports 1 or less heartburn episode per week.

Pacalo	Reggon
26	25
23	31
21	32
25	23
32	19
37	26

The medication with less recovery time with standard deviation and mean is

(A) Pacalo, $\bar{x} = 27.33, \sigma = 6.022\,2$
(B) Reggon, $\bar{x} = 26.00, \sigma = 4.900$
(C) Pacalo, $\bar{x} = 27.33, \sigma = 5.497\,2$
(D) Pacalo, $\bar{x} = 27.33, \sigma = 5.497\,2$

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6. A very large number of data points are chosen on a function $y = 3e^{2x}$ from $x = 0.2$ to 2.1. The average value of these values most nearly is

(A) 51.5

(B) 78.2

(C) 97.8

(D) 102

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Multiple-Choice Test
Take this multiple-choice test on linear regression online

Linear Regression
Regression

1. Given $(x_1, y_1), (x_2, y_2), \ldots\ldots\ldots\ldots(x_n, y_n)$, best fitting data to $y = f(x)$ by least squares requires minimization of

   (A) $\sum_{i=1}^{n}[y_i - f(x_i)]$

   (B) $\sum_{i=1}^{n}|y_i - f(x_i)|$

   (C) $\sum_{i=1}^{n}[y_i - f(x_i)]^2$

   (D) $\sum_{i=1}^{n}[y_i - \bar{y}]^2, \bar{y} = \frac{\sum_{i=1}^{n} y_i}{n}$

2. The following data

x	1	20	30	40
y	1	400	800	1300

is regressed with least squares regression to $y = a_o + a_1 x$. The value of a_1 most nearly is

   A) 27.480
   B) 28.956
   C) 32.625
   D) 40.000

3. The following data

x	1	20	30	40
y	1	400	800	1300

is regressed with least squares regression to $y = a_1 x$. The value of a_1 most nearly is

   A) 27.480
   B) 28.956
   C) 32.625
   D) 40.000

4. An instructor gives the same y vs x data as given below to four students.

x	1	10	20	30	40
y	1	100	400	600	1200

They each come up with four different answers for the straight line regression model. Only one is correct. The correct model is

A) $y = 60x - 1200$

B) $y = 30x - 200$

C) $y = -139.43 + 29.684x$

D) $y = 1 + 22.782x$

5. A torsion spring of a mousetrap is twisted through an angle of 180^0. The torque vs angle data is given below.

T	N-m	0.110	0.189	0.230	0.250
θ	rad	0.10	0.50	1.1	1.5

The amount of strain energy stored in the mousetrap spring in Joules is

A) 0.2987
B) 0.4174
C) 0.8420
D) 1562

6. A scientist finds that regressing the y vs x data given below to straight-line $y = a_0 + a_1 x$ results in the coefficient of determination for the straight-line model, r^2 to be zero.

x	1	3	11	17
y	2	6	22	?

The missing value for y at $x = 17$ most nearly is

A) -2.444
B) 2.000
C) 6.889
D) 34.00

Take this multiple-choice test on linear regression online

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Multiple-Choice Test
Nonlinear Regression
Regression

1. When using the linearized data model to find the constants of the regression model $y = ae^{bx}$ to best fit $(x_1, y_1), (x_2, y_2), \ldots\ldots..(x_n, y_n)$, the sum is the square of the residuals that is minimized is

(A) $\sum_{i=1}^{n}\left(y_i - ae^{bx_i}\right)^2$

(B) $\sum_{i=1}^{n}\left(\ln(y_i) - \ln a - bx_i\right)^2$

(C) $\sum_{i=1}^{n}\left(y_i - \ln a - bx_i\right)^2$

(D) $\sum_{i=1}^{n}\left(\ln(y_i) - \ln a - b\ln(x_i)\right)^2$

2. It is suspected from theoretical considerations that the rate of flow from a firehouse is proportional to some power of the nozzle pressure. Assume pressure data is more accurate. You are linearizing the data.

Flow rate, F (gallons/min)	96	129	135	145	168	235
Pressure, p (psi)	11	17	20	25	40	55

The exponent of the power of the nozzle pressure in the regression model, $F=ap^b$ most nearly is

(A) 0.497
(B) 0.556
(C) 0.578
(D) 0.678

3. The linearized data model for the stress-strain curve $\sigma = K_1 \varepsilon e^{-K_2 \varepsilon}$ for concrete in compression, where σ is the stress and ε is the strain is

(A) $\ln \sigma = \ln k_1 + \ln \varepsilon - k_2 \varepsilon$

(B) $\ln \dfrac{\sigma}{\varepsilon} = \ln k_1 - k_2 \varepsilon$

(C) $\ln \dfrac{\sigma}{\varepsilon} = \ln k_1 + k_2 \varepsilon$

(D) $\ln \sigma = \ln(k_1 \varepsilon) - k_2 \varepsilon$

4. In nonlinear regression, finding the constants of the model requires solution of simultaneous nonlinear equations. However in the exponential model, $y = ae^{bx}$ that is best fit to $(x_1, y_1), (x_2, y_2), \ldots\ldots (x_n, y_n)$, the value of b can be found as a solution of a sample nonlinear equation. That equation is given by

(A) $\sum_{i=1}^{n} y_i x_i e^{bx_i} - \sum_{i=1}^{n} y_i e^{bx_i} \sum_{i=1}^{n} x_i = 0$

(B) $\sum_{i=1}^{n} y_i x_i e^{bx_i} - \dfrac{\sum_{i=1}^{n} y_i e^{bx_i}}{\sum_{i=1}^{n} e^{2bx_i}} \sum_{i=1}^{n} x_i e^{2bx_i} = 0$

(C) $\sum_{i=1}^{n} y_i x_i e^{bx_i} - \dfrac{\sum_{i=1}^{n} y_i e^{bx_i}}{\sum_{i=1}^{n} e^{2bx_i}} \sum_{i=1}^{n} e^{bx_i} = 0$

(D) $\sum_{i=1}^{n} y_i e^{bx_i} - \dfrac{\sum_{i=1}^{n} y_i e^{bx_i}}{\sum_{i=1}^{n} e^{2bx_i}} \sum_{i=1}^{n} x_i e^{2bx_i} = 0$

5. There is a functional relationship between the mass density p of air and altitude h above the sea level

Altitude above sea level, h (km)	0.32	0.64	1.28	1.60
Mass Density, ρ (kg/m^3)	1.15	1.10	1.05	0.95

In the regression model $\rho = k_1 e^{.k_2 h}$, the constant k_2 is found as $k_2 = 0.1315$. Assuming the mass density of air at the top of the atmosphere is $1/1000^{th}$ of the mass density of air at sea level. The altitude in *km* of the top of the atmosphere most nearly is

(A) 46.2
(B) 46.6
(C) 49.7
(D) 52.5

6. A steel cylinder at 80^{o}F of length 12" is placed in a liquid nitrogen bath $(-315^o F)$. If thermal expansion coefficient of steel behaves as a second order polynomial of temperature and the polynomial is found by regressing the data below,

Temperature (^{o}F)	Thermal expansion coefficient (μ in/in/^{o}F)
-320	2.76
-240	3.83
-160	4.72
-80	5.43
0	6.00
80	6.47

the reduction in the length of cylinder most nearly is

(A) 0.0219"
(B) 0.0231"
(C) 0.0235"
(D) 0.0307"

Multiple-Choice Test

Chapter 07.01
Background

1. Physically, integrating $\int_a^b f(x)dx$ means finding the

   (A) area under the curve from a to b
   (B) area to the left of point a
   (C) area to the right of point b
   (D) area above the curve from a to b

2. The mean value of a function $f(x)$ from a to b is given by

   (A) $\dfrac{f(a)+f(b)}{2}$

   (B) $\dfrac{f(a)+2f\left(\dfrac{a+b}{2}\right)+f(b)}{4}$

   (C) $\int_a^b f(x)dx$

   (D) $\dfrac{\int_a^b f(x)dx}{b-a}$

3. The exact value of $\int_{0.2}^{2.2} xe^x dx$ is most nearly

   (A) 7.8036
   (B) 11.807
   (C) 14.034
   (D) 19.611

4. $\int_{0.2}^{2} f(x)dx$ for

$$\begin{aligned} f(x) &= x, \qquad 0 \le x \le 1.2 \\ &= x^2, \quad 1.2 < x \le 2.4 \end{aligned}$$

is most nearly

   (A) 1.9800
   (B) 2.6640
   (C) 2.7907
   (D) 4.7520

5. The area of a circle of radius a can be found by the following integral

(A) $\int_0^a \left(a^2 - x^2\right) dx$

(B) $\int_0^{2\pi} \sqrt{a^2 - x^2}\, dx$

(C) $4\int_0^a \sqrt{a^2 - x^2}\, dx$

(D) $\int_0^a \sqrt{a^2 - x^2}\, dx$

6. Velocity distribution of a fluid flow through a pipe varies along the radius and is given by $v(r)$. The flow rate through the pipe of radius a is given by

(A) $\pi v(a) a^2$

(B) $\pi \dfrac{v(0) + v(a)}{2} a^2$

(C) $\int_0^a v(r)\, dr$

(D) $2\pi \int_0^a v(r) r\, dr$

<ins>For a complete solution, refer to the links at the end of the book.</ins>

Multiple Choice Test

Chapter 07.02
Trapezoidal Rule

1. The two-segment trapezoidal rule of integration is exact for integrating at most ________ order polynomials.
   (A) first
   (B) second
   (C) third
   (D) fourth

2. The value of $\int_{0.2}^{2.2} xe^x dx$ by using the one-segment trapezoidal rule is most nearly
   (A) 11.672
   (B) 11.807
   (C) 20.099
   (D) 24.119

3. The value of $\int_{0.2}^{2.2} xe^x dx$ by using the three-segment trapezoidal rule is most nearly
   (A) 11.672
   (B) 11.807
   (C) 12.811
   (D) 14.633

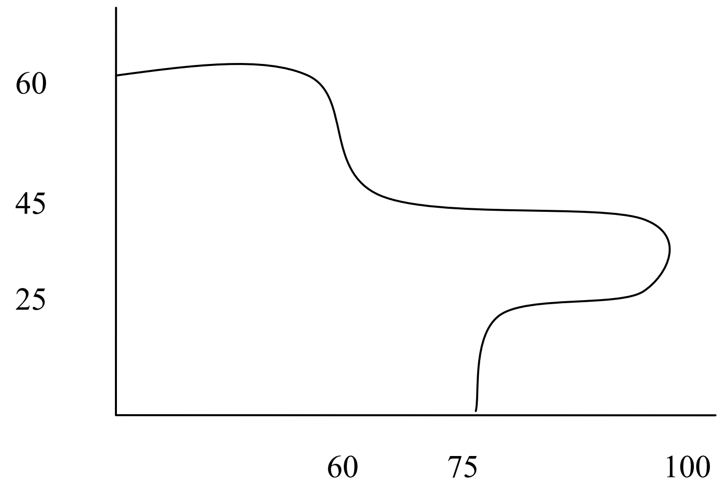
4. The velocity of a body is given by

$$\begin{aligned} v(t) &= 2t, \quad & 1 \le t \le 5 \\ &= 5t^2 + 3, \quad & 5 < t \le 14 \end{aligned}$$

   where t is given in seconds, and v is given in m/s. Use the two-segment trapezoidal rule to find the distance in meters covered by the body from $t = 2$ to $t = 9$ seconds.
   (A) 935.00
   (B) 1039.7
   (C) 1260.9
   (D) 5048.9

5. The shaded area shows a plot of land available for sale. The units of measurement are in meters. Your best estimate of the area of the land in m^2 is most nearly

(A) 2500
(B) 4775
(C) 5250
(D) 6000



6. The following data of the velocity of a body is given as a function of time.

Time (s)	0	15	18	22	24
Velocity (m/s)	22	24	37	25	123

The distance in meters covered by the body from $t = 12\text{ s}$ to $t = 18\text{ s}$ calculated using the trapezoidal rule with unequal segments is

(A) 162.90
(B) 166.00
(C) 181.70
(D) 436.50

<u>For a complete solution, refer to the links at the end of the book.</u>

Multiple-Choice Test

Chapter 07.03
Simpson's 1/3 Rule

1. The highest order of polynomial integrand for which Simpson's 1/3 rule of integration is exact is
   (A) first
   (B) second
   (C) third
   (D) fourth

2. The value of $\int_{0.2}^{2.2} e^x dx$ by using 2-segment Simpson's 1/3 rule most nearly is
   (A) 7.8036
   (B) 7.8423
   (C) 8.4433
   (D) 10.246

3. The value of $\int_{0.2}^{2.2} e^x dx$ by using 4-segment Simpson's 1/3 rule most nearly is
   (A) 7.8036
   (B) 7.8062
   (C) 7.8423
   (D) 7.9655

4. The velocity of a body is given by

$$\begin{aligned} v(t) &= 2t, & 1 \le t \le 5 \\ &= 5t^2 + 3, & 5 < t \le 14 \end{aligned}$$

   where t is given in seconds, and v is given in m/s. Using two-segment Simpson's 1/3 rule, the distance in meters covered by the body from $t = 2$ to $t = 9$ seconds most nearly is
   (A) 949.33
   (B) 1039.7
   (C) 1200.5
   (D) 1442.0

5. The value of $\int_{3}^{19} f(x)dx$ by using 2-segment Simpson's 1/3 rule is estimated as 702.039. The estimate of the same integral using 4-segment Simpson's 1/3 rule most nearly is

(A) $702.039 + \frac{8}{3}\left[2f(7) - f(11) + 2f(15)\right]$

(B) $\frac{702.039}{2} + \frac{8}{3}\left[2f(7) - f(11) + 2f(15)\right]$

(C) $702.039 + \frac{8}{3}\left[2f(7) + 2f(15)\right]$

(D) $\frac{702.039}{2} + \frac{8}{3}\left[2f(7) + 2f(15)\right]$

6. The following data of the velocity of a body is given as a function of time.

Time (s)	4	7	10	15
Velocity (m/s)	22	24	37	46

The best estimate of the distance in meters covered by the body from $t = 4$ to $t = 15$ using combined Simpson's 1/3 rule and the trapezoidal rule would be

(A) 354.70
(B) 362.50
(C) 368.00
(D) 378.80

For a complete solution, refer to the links at the end of the book.

Multiple-Choice Test

Chapter 07.04
Romberg Rule

1. If I_n is the value of integral $\int_a^b f(x)dx$ using n-segment trapezoidal rule, a better estimate of the integral can be found using Richardson's extrapolation as

   (A) $I_{2n} + \dfrac{I_{2n} - I_n}{15}$

   (B) $I_{2n} + \dfrac{I_{2n} - I_n}{3}$

   (C) I_{2n}

   (D) $I_{2n} + \dfrac{I_{2n} - I_n}{I_{2n}}$

2. The estimate of an integral of $\int_3^{19} f(x)dx$ is given as 1860.9 using 1-segment trapezoidal rule. Given $f(7) = 20.27$, $f(11) = 45.125$, and $f(14) = 82.23$, the value of the integral using 2-segment trapezoidal rule would most nearly be

   (A) 787.32
   (B) 1072.0
   (C) 1144.9
   (D) 1291.5

3. The value of an integral $\int_a^b f(x)dx$ given using 1, 2, and 4 segments trapezoidal rule is given as 5.3460, 2.7708, and 1.7536, respectively. The best estimate of the integral you can find using Romberg integration is most nearly

   (A) 1.3355
   (B) 1.3813
   (C) 1.4145
   (D) 1.9124

4. Without using the formula for one-segment trapezoidal rule for estimating $\int_a^b f(x)dx$ the true error, E_t can be found directly as well as exactly by using the formula

$$E_t = -\frac{(b-a)^3}{12} f''(\xi),\ a \le \xi \le b$$

for

(A) $f(x) = e^x$
(B) $f(x) = x^3 + 3x$
(C) $f(x) = 5x^2 + 3$
(D) $f(x) = 5x^2 + e^x$

5. For $\int_a^b f(x)dx$, the true error, E_t in one-segment trapezoidal rule is given by

$$E_t = -\frac{(b-a)^3}{12} f''(\xi),\ a \le \xi \le b$$

The value of ξ for the integral $\int_{2.5}^{7.2} 3e^{0.2x}dx$ is most nearly

(A) 2.7998
(B) 4.8500
(C) 4.9601
(D) 5.0327

6. Given the velocity vs. time data for a body

t (s)	2	4	6	8	10	25
v (m/s)	0.166	0.55115	1.8299	6.0755	20.172	8137.5

The best estimate for distance covered in meters between $t = 2$ s and $t = 10$ s by using Romberg rule based on trapezoidal rule results would be

(A) 33.456
(B) 36.877
(C) 37.251
(D) 81.350

For a complete solution, refer to the links at the end of the book.

Multiple-Choice Test

Chapter 07.05
Gauss Quadrature Rule

1. $\int_{5}^{10} f(x)dx$ is exactly

   (A) $\int_{-1}^{1} f(2.5x+7.5)dx$

   (B) $2.5\int_{-1}^{1} f(2.5x+7.5)dx$

   (C) $5\int_{-1}^{1} f(5x+5)dx$

   (D) $5\int_{-1}^{1} (2.5x+7.5)f(x)dx$

2. For a definite integral of any third order polynomial, the two-point Gauss quadrature rule will give the same results as the

   (A) 1-segment trapezoidal rule
   (B) 2-segment trapezoidal rule
   (C) 3-segment trapezoidal rule
   (D) Simpson's 1/3 rule

3. The value of $\int_{0.2}^{2.2} xe^{x}dx$ by using the two-point Gauss quadrature rule is most nearly

   (A) 11.672
   (B) 11.807
   (C) 12.811
   (D) 14.633

4. A scientist uses the one-point Gauss quadrature rule based on getting exact results of integration for functions $f(x)=1$ and x. The one-point Gauss quadrature rule approximation for $\int_a^b f(x)dx$ is

(A) $\frac{b-a}{2}[f(a)+f(b)]$

(B) $(b-a)f\left(\frac{a+b}{2}\right)$

(C) $\frac{b-a}{2}\left[f\left(\frac{b-a}{2}\left\{-\frac{1}{\sqrt{3}}\right\}+\frac{b+a}{2}\right)+f\left(\frac{b-a}{2}\left\{\frac{1}{\sqrt{3}}\right\}+\frac{b+a}{2}\right)\right]$

(D) $(b-a)f(a)$

5. A scientist develops an approximate formula for integration as

$$\int_a^b f(x)dx \approx c_1 f(x_1), \text{ where } a \le x_1 \le b$$

The values of c_1 and x_1 are found by assuming that the formula is exact for functions of the form $a_0 x + a_1 x^2$. The resulting formula would therefore be exact for integrating

(A) $f(x)=2$

(B) $f(x)=2+3x+5x^2$

(C) $f(x)=5x^2$

(D) $f(x)=2+3x$

6. You are asked to estimate the water flow rate in a pipe of radius 2 m at a remote area location with a harsh environment. You already know that velocity varies along the radial location, but you do not know how it varies. The flow rate Q is given by

$$Q=\int_0^2 2\pi r V dr$$

To save money, you are allowed to put only two velocity probes (these probes send the data to the central office in New York, NY via satellite) in the pipe. Radial location, r is measured from the center of the pipe, that is $r=0$ is the center of the pipe and $r=2\text{m}$ is the pipe radius. The radial locations you would suggest for the two velocity probes for the most accurate calculation of the flow rate are

(A) 0, 2

(B) 1, 2

(C) 0, 1

(D) 0.42, 1.58

For a complete solution, refer to the links at the end of the book.

Multiple-Choice Test

Chapter 08.01
Background

1. The differential equation $2\frac{dy}{dx}+x^2y=2x+3,\ y(0)=5$ is
   (A) linear
   (B) nonlinear
   (C) linear with fixed constants
   (D) undeterminable to be linear or nonlinear

2. A differential equation is considered to be ordinary if it has
   (A) one dependent variable
   (B) more than one dependent variable
   (C) one independent variable
   (D) more than one independent variable

3. Given

$$2\frac{dy}{dx}+3y=\sin 2x,\ y(0)=6$$

$y(2)$ most nearly is
   (A) 0.17643
   (B) 0.29872
   (C) 0.32046
   (D) 0.58024

4. The form of the exact solution to

$$2\frac{dy}{dx}+3y=e^{-x},\ y(0)=5$$

is
   (A) $Ae^{-1.5x}+Be^{-x}$
   (B) $Ae^{-1.5x}+Bxe^{-x}$
   (C) $Ae^{1.5x}+Be^{-x}$
   (D) $Ae^{1.5x}+Bxe^{-x}$

5. The following nonlinear differential equation can be solved exactly by separation of variables.

$$\frac{d\theta}{dt} = -10^{-6}\left(\theta^2 - 81\right),\ \theta(0) = 1000$$

The value of $\theta(100)$ most nearly is

(A) -99.99
(B) 909.10
(C) 1000.32
(D) 1111.10

6. A solid spherical ball taken out of a furnace at 1200 K is allowed to cool in air. Given the following,

radius of the ball $= 2$ cm
density of the ball $= 7800\ \text{kg/m}^3$
specific heat of the ball $= 420\ \text{J/kg}\cdot\text{K}$
emmittance $= 0.85$
Stefan-Boltzman constant $= 5.67\times 10^{-8}\ \text{J/s}\cdot\text{m}^2\cdot\text{K}^4$
ambient temperature $= 300\ \text{K}$
convection coefficient to air $= 350\ \text{J/s}\cdot\text{m}^2\cdot\text{K}$

the differential equation governing the temperature θ of the ball as a function of time t is given by

(A) $\frac{d\theta}{dt} = -2.2067\times 10^{-12}\left(\theta^4 - 81\times 10^8\right)$

(B) $\frac{d\theta}{dt} = -1.6026\times 10^{-2}\left(\theta - 300\right)$

(C) $\frac{d\theta}{dt} = 2.2067\times 10^{-12}\left(\theta^4 - 81\times 10^8\right) + 1.6026\times 10^{-12}\left(\theta - 300\right)$

(D) $\frac{d\theta}{dt} = -2.2067\times 10^{-12}\left(\theta^4 - 81\times 10^8\right) - 1.6026\times 10^{-2}\left(\theta - 300\right)$

For a complete solution, refer to the links at the end of the book.

Multiple-Choice Test

Chapter 08.02
Euler's Method

1. To solve the ordinary differential equation

$$3\frac{dy}{dx}+5y^2=\sin x,\ y(0)=5$$

by Euler's method, you need to rewrite the equation as

(A) $\frac{dy}{dx}=\sin x-5y^2,\ y(0)=5$

(B) $\frac{dy}{dx}=\frac{1}{3}\left(\sin x-5y^2\right),\ y(0)=5$

(C) $\frac{dy}{dx}=\frac{1}{3}\left(-\cos x-\frac{5y^3}{3}\right),\ y(0)=5$

(D) $\frac{dy}{dx}=\frac{1}{3}\sin x,\ y(0)=5$

2. Given

$$3\frac{dy}{dx}+5y^2=\sin x,\ y(0.3)=5$$

and using a step size of $h=0.3$, the value of $y(0.9)$ using Euler's method is most nearly

(A) -35.318
(B) -36.458
(C) -658.91
(D) -669.05

3. Given

$$3\frac{dy}{dx}+\sqrt{y}=e^{0.1x},\ y(0.3)=5$$

and using a step size of $h=0.3$, the best estimate of $\frac{dy}{dx}(0.9)$ using Euler's method is most nearly

(A) -0.37319
(B) -0.36288
(C) -0.35381
(D) -0.34341

4. The velocity ($\mathrm{m/s}$) of a body is given as a function of time (seconds) by

$$v(t) = 200\ln(1+t) - t,\ t \geq 0$$

Using Euler's method with a step size of 5 seconds, the distance in meters traveled by the body from $t = 2$ to $t = 12$ seconds is most nearly

(A) 3133.1
(B) 3939.7
(C) 5638.0
(D) 39397

5. Euler's method can be derived by using the first two terms of the Taylor series of writing the value of y_{i+1}, that is the value of y at x_{i+1}, in terms of y_i and all the derivatives of y at x_i. If $h = x_{i+1} - x_i$, the explicit expression for y_{i+1} if the first three terms of the Taylor series are chosen for the ordinary differential equation

$$2\frac{dy}{dx} + 3y = e^{-5x}, y(0) = 7$$

would be

(A) $y_{i+1} = y_i + \frac{1}{2}\left(e^{-5x_i} - 3y_i\right)h$

(B) $y_{i+1} = y_i + \frac{1}{2}\left(e^{-5x_i} - 3y_i\right)h - \frac{1}{2}\left(\frac{5}{2}e^{-5x_i}\right)h^2$

(C) $y_{i+1} = y_i + \frac{1}{2}\left(e^{-5x_i} - 3y_i\right)h + \frac{1}{2}\left(-\frac{13}{4}e^{-5x_i} + \frac{9}{4}y_i\right)h^2$

(D) $y_{i+1} = y_i + \frac{1}{2}\left(e^{-5x_i} - 3y_i\right)h - \frac{3}{2}y_i h^2$

6. A homicide victim is found at 6:00 PM in an office building that is maintained at $72\ ^\circ\mathrm{F}$. When the victim was found, his body temperature was at $85\ ^\circ\mathrm{F}$. Three hours later at 9:00 PM, his body temperature was recorded at $78\ ^\circ\mathrm{F}$. Assume the temperature of the body at the time of death is the normal human body temperature of $98.6\ ^\circ\mathrm{F}$.
The governing equation for the temperature θ of the body is
$$\frac{d\theta}{dt} = -k(\theta - \theta_a)$$
where,
θ = temperature of the body, $^\circ\mathrm{F}$
θ_a = ambient temperature, $^\circ\mathrm{F}$
t = time, hours
k = constant based on thermal properties of the body and air.
The estimated time of death most nearly is
(A) 2:11 PM
(B) 3:13 PM
(C) 4:34 PM
(D) 5:12 PM

For a complete solution, refer to the links at the end of the book.

Multiple-Choice Test

Chapter 08.03
Runge-Kutta 2nd Order Method

1. To solve the ordinary differential equation

$$3\frac{dy}{dx} + xy^2 = \sin x,\ y(0) = 5$$

by the Runge-Kutta 2nd order method, you need to rewrite the equation as

(A) $\frac{dy}{dx} = \sin x - xy^2,\ y(0) = 5$

(B) $\frac{dy}{dx} = \frac{1}{3}\left(\sin x - xy^2\right),\ y(0) = 5$

(C) $\frac{dy}{dx} = \frac{1}{3}\left(-\cos x - \frac{xy^3}{3}\right),\ y(0) = 5$

(D) $\frac{dy}{dx} = \frac{1}{3}\sin x,\ y(0) = 5$

2. Given

$$3\frac{dy}{dx} + 5y^2 = \sin x,\ y(0.3) = 5$$

and using a step size of $h = 0.3$, the value of $y(0.9)$ using the Runge-Kutta 2nd order Heun method is most nearly

(A) –4297.4
(B) –4936.7
(C) -0.21336×10^{14}
(D) -0.24489×10^{14}

3. Given

$$3\frac{dy}{dx} + 5\sqrt{y} = e^{0.1x},\ y(0.3) = 5$$

and using a step size of $h = 0.3$, the best estimate of $\frac{dy}{dx}(0.9)$ using the Runge-Kutta 2nd order midpoint method most nearly is

(A) –2.2473
(B) –2.2543
(C) –2.6188
(D) –3.2045

4. The velocity (m/s) of a body is given as a function of time (seconds) by

$$v(t) = 200\ln(1+t) - t,\ t \geq 0$$

Using the Runge-Kutta 2nd order Ralston method with a step size of 5 seconds, the distance in meters traveled by the body from $t = 2$ to $t = 12$ seconds is estimated most nearly as

(A) 3904.9
(B) 3939.7
(C) 6556.3
(D) 39397

5. The Runge-Kutta 2nd order method can be derived by using the first three terms of the Taylor series of writing the value of y_{i+1} (that is the value of y at x_{i+1}) in terms of y_i (that is the value of y at x_i) and all the derivatives of y at x_i. If $h = x_{i+1} - x_i$, the explicit expression for y_{i+1} if the first three terms of the Taylor series are chosen for solving the ordinary differential equation

$$\frac{dy}{dx} + 5y = 3e^{-2x}, y(0) = 7$$

would be

(A) $y_{i+1} = y_i + \left(3e^{-2x_i} - 5y_i\right)h + 5\dfrac{h^2}{2}$

(B) $y_{i+1} = y_i + \left(3e^{-2x_i} - 5y_i\right)h + \left(-21e^{-2x_i} + 25y_i\right)\dfrac{h^2}{2}$

(C) $y_{i+1} = y_i + \left(3e^{-2x_i} - 5y_i\right)h + \left(-6e^{-2x_i}\right)\dfrac{h^2}{2}$

(D) $y_{i+1} = y_i + \left(3e^{-2x_i} - 5y_i\right)h + \left(-6e^{-2x_i} + 5\right)\dfrac{h^2}{2}$

6. A spherical ball is taken out of a furnace at 1200 K and is allowed to cool in air. You are given the following

radius of ball = 2 cm

specific heat of ball $= 420\ \text{J/kg} \cdot \text{K}$

density of ball $= 7800\ \text{kg/m}^3$

convection coefficient $= 350\ \text{J/s} \cdot \text{m}^2 \cdot \text{K}$

ambient temperature $= 300\ \text{K}$

The ordinary differential equation that is given for the temperature θ of the ball is

$$\frac{d\theta}{dt} = -2.20673 \times 10^{-13}\left(\theta^4 - 81 \times 10^8\right)$$

if only radiation is accounted for. The ordinary differential equation if convection is accounted for in addition to radiation is

(A) $\dfrac{d\theta}{dt} = -2.20673 \times 10^{-13}\left(\theta^4 - 81 \times 10^8\right) - 1.6026 \times 10^{-2}(\theta - 300)$

(B) $\dfrac{d\theta}{dt} = -2.20673 \times 10^{-13}\left(\theta^4 - 81 \times 10^8\right) - 4.3982 \times 10^{-2}(\theta - 300)$

(C) $\dfrac{d\theta}{dt} = -1.6026 \times 10^{-2}(\theta - 300)$

(D) $\dfrac{d\theta}{dt} = -4.3982 \times 10^{-2}(\theta - 300)$

<u>For a complete solution, refer to the links at the end of the book.</u>

Multiple-Choice Test

Chapter 08.04
Runge-Kutta 4th Order Method

1. To solve the ordinary differential equation

$$3\frac{dy}{dx} + xy^2 = \sin x, y(0) = 5,$$

by Runge-Kutta 4$^{\text{th}}$ order method, you need to rewrite the equation as

(A) $\frac{dy}{dx} = \sin x - xy^2, y(0) = 5$

(B) $\frac{dy}{dx} = \frac{1}{3}\left(\sin x - xy^2\right), y(0) = 5$

(C) $\frac{dy}{dx} = \frac{1}{3}\left(-\cos x - \frac{xy^3}{3}\right), y(0) = 5$

(D) $\frac{dy}{dx} = \frac{1}{3}\sin x, y(0) = 5$

2. Given $3\frac{dy}{dx} + 5y^2 = \sin x, y(0.3) = 5$ and using a step size of $h = 0.3$, the value of $y(0.9)$ using Runge-Kutta 4$^{\text{th}}$ order method is most nearly

(A) -0.25011×10^{40}
(B) -4297.4
(C) -1261.5
(D) 0.88498

3. Given $3\frac{dy}{dx} + y^2 = e^x, y(0.3) = 5$, and using a step size of $h = 0.3$, the best estimate of $\frac{dy}{dx}(0.9)$ Runge-Kutta 4$^{\text{th}}$ order method is most nearly

(A) -1.6604
(B) -1.1785
(C) -0.45831
(D) 2.7270

4. The velocity ($\mathrm{m/s}$) of a parachutist is given as a function of time (seconds) by

$$v(t) = 55.8\tanh(0.17t),\ \ t \geq 0$$

Using Runge-Kutta 4th order method with a step size of 5 seconds, the distance in meters traveled by the body from $t = 2$ to $t = 12$ seconds is estimated most nearly as

(A) 341.43
(B) 428.97
(C) 429.05
(D) 703.50

5. Runge-Kutta method can be derived from using first three terms of Taylor series of writing the value of y_{i+1}, that is the value of y at x_{i+1}, in terms of y_i and all the derivatives of y at x_i. If $h = x_{i+1} - x_i$, the explicit expression for y_{i+1} if the first five terms of the Taylor series are chosen for the ordinary differential equation

$$\frac{dy}{dx} + 5y = 3e^{-2x}, y(0) = 7,$$

would be

(A) $y_{i+1} = y_i + \left(3e^{-2x_i} - 5y_i\right)h + \frac{5h^2}{2}$

(B) $y_{i+1} = y_i + \left(3e^{-2x_i} - 5y_i\right)h + \left(-21e^{-2x_i} + 25y_i\right)\frac{h^2}{2}$
$+\left(-483e^{-2x_i} + 625y_i\right)\frac{h^3}{6} + \left(-300909e^{-2x_i} + 390625y_i\right)\frac{h^4}{24}$

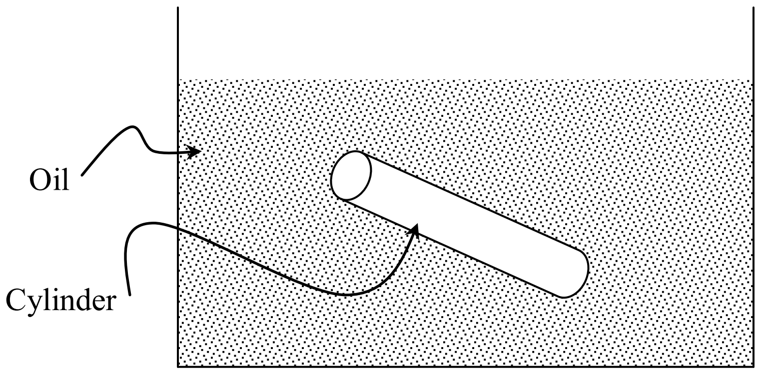
(C) $y_{i+1} = y_i + \left(3e^{-2x_i} - 5y_i\right)h + \left(-6e^{-2x_i}\right)\frac{h^2}{2} + \left(12e^{-2x_i}\right)\frac{h^3}{6}$
$+\left(-24e^{-2x_i}\right)\frac{h^4}{24}$

(D) $y_{i+1} = y_i + \left(3e^{-2x_i} - 5y_i\right)h + \left(-6e^{-2x_i} + 5\right)\frac{h^2}{2} + \left(12e^{-2x_i}\right)\frac{h^3}{6}$
$+\left(-24e^{-2x_i}\right)\frac{h^4}{24}$

6. A hot solid cylinder is immersed in a cool oil bath as part of a quenching process. This process makes the temperature of the cylinder, θ_c, and the bath, θ_b, change with time. If the initial temperature of the bar and the oil bath is given as 600° C and 27°C, respectively, and

Length of cylinder = 30 cm
Radius of cylinder = 3 cm
Density of cylinder = 2700 kg/m^3
Specific heat of cylinder = 895 J/kg · K
Convection heat transfer coefficient = 100 $\text{W/m}^2 \cdot \text{K}$
Specific heat of oil = 1910 J/kg · K
Mass of oil = 2 kg

the coupled ordinary differential equation giving the heat transfer are given by



(A) $$362.4\frac{d\theta_c}{dt}+\theta_c=\theta_b$$
$$675.5\frac{d\theta_b}{dt}+\theta_b=\theta$$

(B) $$362.4\frac{d\theta_c}{dt}-\theta_c=\theta_b$$
$$675.5\frac{d\theta_b}{dt}-\theta_b=\theta_c$$

(C) $$675.5\frac{d\theta_c}{dt}+\theta_c=\theta_b$$
$$362.4\frac{d\theta_b}{dt}+\theta_b=\theta_c$$

(D) $$675.5\frac{d\theta_c}{dt}-\theta_c=\theta_b$$

For a complete solution, refer to the links at the end of the book.

Multiple-Choice Test

Chapter 08.06
Shooting Method

1. The exact solution to the boundary value problem

$$\frac{d^2y}{dx^2} = 6x - 0.5x^2,\ y(0) = 0,\ y(12) = 0$$

for $y(4)$ is

(A) -234.66
(B) 0.00
(C) 16.000
(D) 106.66

2. Given

$$\frac{d^2y}{dx^2} = 6x - 0.5x^2,\ y(0) = 0,\ y(12) = 0,$$

the exact value of $\frac{dy}{dx}(0)$ is

(A) -72.0
(B) 0.00
(C) 36.0
(D) 72.0

3. Given

$$\frac{d^2y}{dx^2} = 6x - 0.5x^2,\ y(0) = 0,\ y(12) = 0,$$

If one was using shooting method with Euler's method with a step size of $h = 4$, and an assumed value of $\frac{dy}{dx}(0) = 20$, then the estimated value of $y(12)$ in the first iteration most nearly is

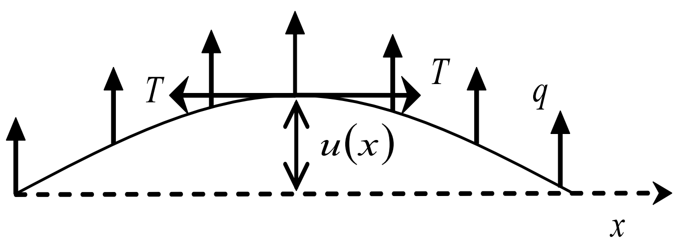
(A) 60.00
(B) 496.0
(C) 1088
(D) 1102

4. The transverse deflection, u of a cable of length, L, fixed at both ends, is given as a solution to

$$\frac{d^2u}{dx^2}=\frac{Tu}{R}+\frac{qx(x-L)}{2R}$$

where

T = tension in cable
R = flexural stiffness
q = distributed transverse load



Given are $L = 50''$, $\mathrm{T} = 2000\ \mathrm{lbs}$, $\mathrm{q} = 75\ \mathrm{lbs/in}$ $\mathrm{R} = 75\times 10^6\ \mathrm{lbs}\cdot\mathrm{in}^2$. The shooting method is used with Euler's method assuming a step size of $h = 12.5''$. Initial slope guesses at $x = 0$ of $\frac{du}{dx} = 0.003$ and $\frac{du}{dx} = 0.004$ are used in order, and then refined for the next iteration using linear interpolation after the value of $u(L)$ is found. The deflection in inches at the center of the cable found during the second iteration is most nearly

(A) 0.075000
(B) 0.10000
(C) -0.061291
(D) 0.00048828

5. The radial displacement, u is a pressurized hollow thick cylinder (inner radius=5″, outer radius=8″) is given at different radial locations.

Radius	Radial Displacement
(in)	(in)
5.0	0.0038731
5.6	0.0036165
6.2	0.0034222
6.8	0.0032743
7.4	0.0031618
8.0	0.0030769

The maximum normal stress, in psi, on the cylinder is given by

$$\sigma_{\max} = 3.2967\times 10^{6}\left(\frac{u(5)}{5} + 0.3\frac{du}{dr}(5)\right)$$

The maximum stress, in psi, with second order accuracy for the approximation of the first derivative most nearly is

(A) 2079.3
(B) 2104.5
(C) 2130.7
(D) 2182.0

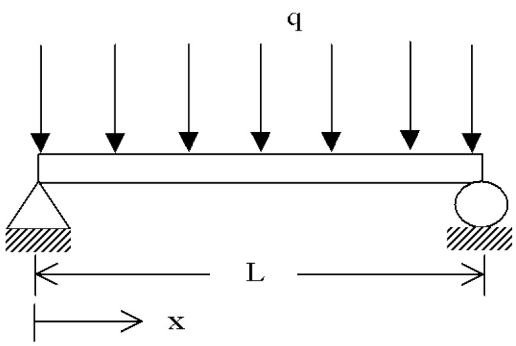
6. For a simply supported beam (at $x=0$ and $x=L$) with a uniform load q, the vertical deflection $v(x)$ is described by the boundary value ordinary differential equation as

$$\frac{d^2v}{dx^2}=\frac{qx(x-L)}{2EI},\ 0\le x\le L$$

where

E = Young's modulus of elasticity of beam,
I = second moment of area.

This ordinary differential equations is based on assuming that $\frac{dv}{dx}$ is small. If $\frac{dv}{dx}$ is not small, then the ordinary differential equation is



(A) $$\frac{\frac{d^2v}{dx^2}}{\sqrt{1+\left(\frac{dv}{dx}\right)^2}}=\frac{qx(x-L)}{2EI}$$

(B) $$\frac{\frac{d^2v}{dx^2}}{\left(1+\left(\frac{dv}{dx}\right)^2\right)^{3/2}}=\frac{qx(x-L)}{2EI}$$

(C) $$\frac{\frac{d^2v}{dx^2}}{\sqrt{1+\left(\frac{dv}{dx}\right)}}=\frac{qx(x-L)}{2EI}$$

(D) $$\frac{\frac{d^2v}{dx^2}}{1+\frac{dv}{dx}}=\frac{qx(x-L)}{2EI}$$

For a complete solution, refer to the links at the end of the book.

Multiple-Choice Test

Chapter 08.07
Finite Difference Method

1. The exact solution to the boundary value problem

$$\frac{d^2y}{dx^2} = 6x - 0.5x^2, \; y(0) = 0, \; y(12) = 0$$

for $y(4)$ is

(A) -234.67
(B) 0.00
(C) 16.000
(D) 37.333

2. Given

$$\frac{d^2y}{dx^2} = 6x - 0.5x^2, \; y(0) = 0, \; y(12) = 0$$

the value of $\frac{d^2y}{dx^2}$ at $y(4)$ using the finite difference method and a step size of $h = 4$ can be approximated by

(A) $\frac{y(8) - y(0)}{8}$
(B) $\frac{y(8) - 2y(4) + y(0)}{16}$
(C) $\frac{y(12) - 2y(8) + y(4)}{16}$
(D) $\frac{y(4) - y(0)}{4}$

3. Given

$$\frac{d^2y}{dx^2} = 6x - 0.5x^2, \; y(0) = 0, \; y(12) = 0,$$

the value of $y(4)$ using the finite difference method with a second order accurate central divided difference method and a step size of $h = 4$ is

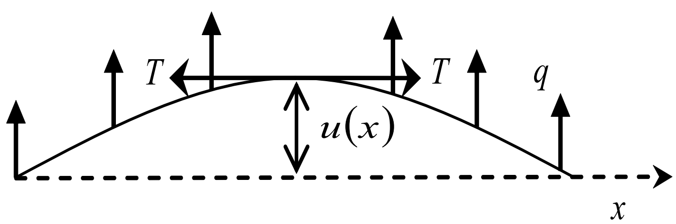
(A) 0.000
(B) 37.333
(C) -234.67
(D) -256.00

4. The transverse deflection u of a cable of length L that is fixed at both ends, is given as a solution to

$$\frac{d^2u}{dx^2} = \frac{Tu}{R} + \frac{qx(x-L)}{2R}$$

where

T = tension in cable
R = flexural stiffness
q = distributed transverse load



Given $L = 50"$, $T = 2000\text{ lbs}$, $q = 75\,\frac{\text{lbs}}{\text{in}}$, and $R = 75\times10^6\text{ lbs}\cdot\text{in}^2$

Using finite difference method modeling with second order central divided difference accuracy and a step size of $h = 12.5"$, the value of the deflection at the center of the cable most nearly is

(A) 0.072737″
(B) 0.080832″
(C) 0.081380″
(D) 0.084843″

5. The radial displacement u of a pressurized hollow thick cylinder (inner radius =5″, outer radius = 8″) is given at different radial locations.

Radius (in)	Radial Displacement (in)
5.0	0.0038731
5.6	0.0036165
6.2	0.0034222
6.8	0.0032743
7.4	0.0031618
8.0	0.0030769

The maximum normal stress, in psi, on the cylinder is given by

$$\sigma_{\max} = 3.2967 \times 10^6 \left(\frac{u(5)}{5} + 0.3 \frac{du}{dr}(5) \right)$$

The maximum stress, in psi, with second order accuracy is

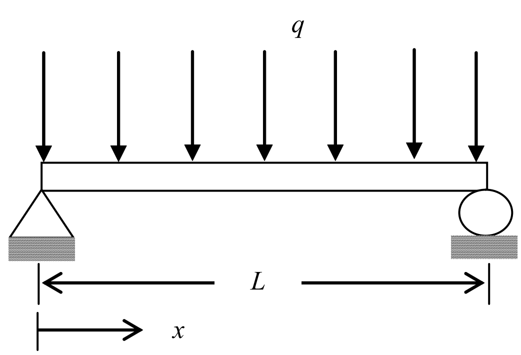
(A) 2079.6
(B) 2104.5
(C) 2130.7
(D) 2182.0

6. For a simply supported beam (at $x = 0$ and $x = L$) with a uniform load q, the vertical deflection $v(x)$ is described by the boundary value ordinary differential equation as

$$\frac{d^2v}{dx^2} = \frac{qx(x-L)}{2EI}, \; 0 \le x \le L$$

where
E = Young's modulus of the beam
I = second moment of area

This ordinary differential equation is based on assuming that $\frac{dv}{dx}$ is small. If $\frac{dv}{dx}$ is not small, then the ordinary differential equation is given by



(A) $$\frac{\frac{d^2v}{dx^2}}{\sqrt{1+\left(\frac{dv}{dx}\right)^2}} = \frac{qx(x-L)}{2EI}$$

(B) $$\frac{\frac{d^2v}{dx^2}}{\left(1+\left(\frac{dv}{dx}\right)^2\right)^{3/2}} = \frac{qx(x-L)}{2EI}$$

(C) $$\frac{\frac{d^2v}{dx^2}}{\sqrt{1+\left(\frac{dv}{dx}\right)}} = \frac{qx(x-L)}{2EI}$$

(D) $$\frac{\frac{d^2v}{dx^2}}{1+\frac{dv}{dx}} = \frac{qx(x-L)}{2EI}$$

<u>For a complete solution, refer to the links at the end of the book.</u>

Multiple-Choice Test

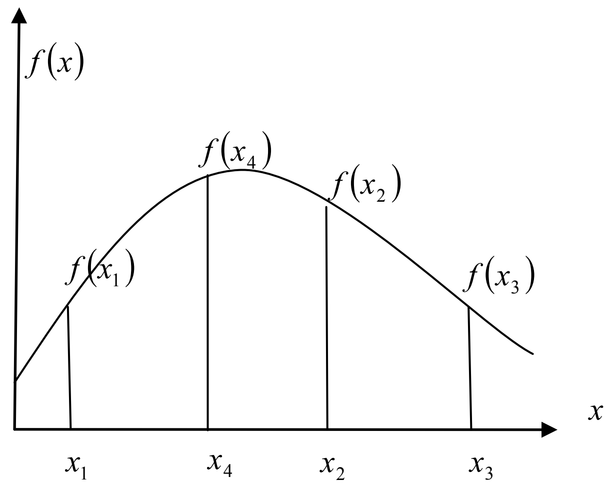
Chapter 09.01
Golden Section Search Method

1. Which of the following statements is incorrect regarding the Equal Interval Search and Golden Section Search methods?
(A) Both methods require an initial boundary region to start the search
(B) The number of iterations in both methods are affected by the size of ε
(C) Everything else being equal, the Golden Section Search method should find an optimal solution faster.
(D) Everything else being equal, the Equal Interval Search method should find an optimal solution faster.

2. Which of the following parameters is not required to use the Golden Section Search method for optimization?
(A) The lower bound for the search region
(B) The upper bound for the search region
(C) The golden ratio
(D) The function to be optimized

3. When applying the Golden Section Search method to a function $f(x)$ to find its maximum, the $f(x_1) > f(x_2)$ condition holds true for the intermediate points x_1 and x_2. Which of the following statements is incorrect?
(A) The new search region is determined by $[x_2, x_u]$
(B) The intermediate point x_1 stays as one of the intermediate points
(C) The upper bound x_u stays the same
(D) The new search region is determined by $[x_l, x_1]$

4. In the graph below, the lower and upper boundary of the search is given by x_1 and x_3 respectively. If x_4 and x_2 are the initial intermediary points, which of the following statement is false?

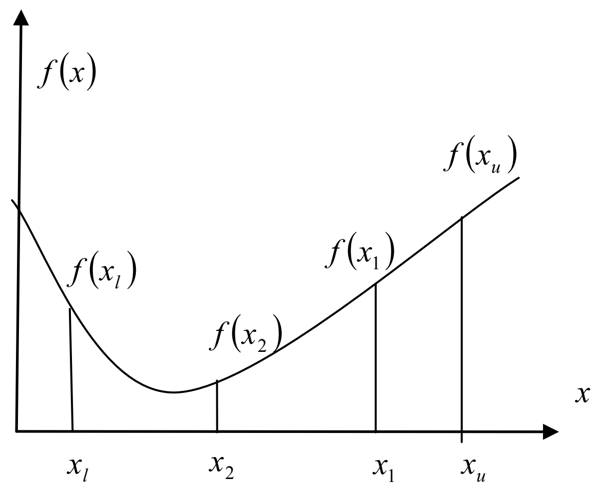


(A) The distance between x_2 and x_1 is equal to the distance between x_4 and x_3
(B) The distance between x_4 and x_2 is approximately 0.618 times the distance between x_2 and x_1
(C) The distance between x_4 and x_1 is approximately 0.618 times the distance between x_4 and x_3
(D) The distance between x_4 and x_1 is equal to the distance between x_2 and x_3

5. Using the Golden Section Search method, find two numbers whose sum is 90 and their product is as large as possible. Use the interval [0,90].

(A) 30 and 60
(B) 45 and 45
(C) 38 and 52
(D) 20 and 70

6. Consider the problem of finding the minimum of the function shown below. Given the intermediate points in the drawing, what would be the search region in the next iteration?



(A) $[x_2, x_u]$
(B) $[x_1, x_u]$
(C) $[x_l, x_1]$
(D) $[x_l, x_2]$

Multiple-Choice Test

Chapter 09.02
Newton's Method

1. Which of the following is NOT required for using Newton's method for optimization?
   (A) The lower bound for search region.
   (B) Twice differentiable optimization function.
   (C) The function to be optimized.
   (D) A good initial estimate that is reasonably close to the optimal.

2. Which of the following statements is INCORRECT?
   (A) If the second derivative at x_i is negative, then x_i is a maximum.
   (B) If the first derivative at x_i is zero, then x_i is an optimum.
   (C) If x_i is a minimum, then the second derivative at x_i is positive
   (D) The value of the function can be positive or negative as any optima.

3. For what value of x, is the function $x^2 - 2x - 6$ minimized?
   (A) 0
   (B) 1
   (C) 5
   (D) 3

4. We need to enclose a field with a fence. We have 500 feet of fencing material with a building on one side of the field where we will not need any fencing. Determine the maximum area of the field that can be enclosed by the fence.
   (A) $x = 125, y = 250$
   (B) $x = 150, y = 200$
   (C) $x = 125, y = 100$
   (D) $x = 200, y = 150$

5. A rectangular box with a square base and no top has a volume of 500 cubic inches. Find the length, l of the edge of the square base and height, h for the box that requires the least amount of material to build. Conduct two iterations using an initial guess of $l = 5in$

(A) Base edge length is 10.00 and height is 5.00
(B) Base edge length is 9.17 and height is 6.00
(C) Base edge length is 9.00 and height is 6.17
(D) Base edge length is 10.00 and height is 10.00

6. A rectangular box with a square base with no top has a surface area of 108 ft^2. Find the dimensions that will maximize the volume. Conduct two iterations using an initial guess of $l = 3\ ft$

(A) Base edge length is 4.15 and height is 4.85
(B) Base edge length is 6.15 and height is 2.85
(C) Base edge length is 6.00 and height is 3.00
(D) Base edge length is 3.85 and height is 6.15

Multiple-Choice Test

Chapter 09.03
Multidimensional Direct Search Method

1. Which of the following statement is FALSE?

   (A) Multidimensional direct search methods are similar to one-dimensional direct search methods.
   (B) Enumerating all possible solutions in a search space and selecting the optimal solutions is an effective method for problems with very high dimensional solution spaces.
   (C) Multidimensional direct search methods do not require a twice differentiable function as an optimization function
   (D) Genetic Algorithms belong to the family of multidimensional direct search methods.

2. Which of the following statements is FALSE?
   (A) Multidimensional direct search methods require an upper and lower bound for their search region.
   (B) Coordinate cycling method relies on single dimensional search methods to determine an optimal solution along each coordinate direction iteratively.
   (C) If the optimization function is twice differentiable, multidimensional direct search methods cannot be used to find an optimal solution.
   (D) Multidimensional direct search methods are not guaranteed to find the global optimum.

3. The first cycle of Example 1 in Chapter 09.03 results in an optimal solution of $f(2.6459, 0.8668) = 4.8823$ for the gutter design problem. The next iteration starts with a search along dimension l (length) looking for the optimal solution of the function $f(l, 0.8668)$ as shown in Table 3 and reproduced below where $\theta = 0.8668$ and $f(x_i) = (6 - 2l + l\cos(0.8668))l\sin(0.8668))$. What is the optimal solution for the length of the gutter side at the end of iteration 10?

Iteration	x_l	x_u	x_1	x_2	$f(x_1)$	$f(x_2)$	ε
1	0.0000	3.0000	1.8541	1.1459	4.9354	3.8871	3.0000
2	1.1459	3.0000	2.2918	1.8541	5.0660	4.9354	1.8541
3	1.8541	3.0000	2.5623	2.2918	4.9491	5.0660	1.1459

4	1.8541	2.5623	2.2918	2.1246	5.0660	5.0627	0.7082
5	2.1246	2.5623	2.3951	2.2918	5.0391	5.0660	0.4377
6	2.1246	2.3951	2.2918	2.2279	5.0660	5.0715	0.2705
7	2.1246	2.2918	2.2279	2.1885	5.0715	5.0708	0.1672
8	2.1885	2.2918	2.2523	2.2279	5.0704	5.0715	0.1033
9	2.1885	2.2523	2.2279	2.2129	5.0715	5.0716	0.0639
10	2.1885	2.2279	2.2129	2.2035	5.0716	5.0714	0.0395

(A) 2.1885
(B) 2.2279
(C) 5.0715
(D) 2.2082

4. What is the maximum size for the area of gutter at the optimal point determined in multiple-choice question 3? (Hint: You do not need to do any calculations to answer this question)
(A) 5.0716
(B) 5.0714
(C) 5.0715
(D) 2.2082

5. To find the minimum of the function $f(x, y) = 5x^2 - 6xy + 5y^2 - 2$ hold $y = 0$ and use 2 and -2 as your upper and lower bounds for your one-dimensional search along the x coordinate using golden search method. What would be the optimal solution for x after the first iteration?
(A) 3.1146
(B) 0.4721
(C) 0
(D) 0.0015

6. Considering the scenario in Question 5, what would be the optimal solution for after the first iteration? (Can you explain the difference?)
(A) 0
(B) 0.7639
(C) 0.4721
(D) 7.5728

Multiple-Choice Test

Chapter 09.04
Multidimensional Gradient Method

1. Which of the following statements is incorrect?

   (A) Direct search methods are useful when the optimization function is not differentiable
   (B) The gradient of $f(x, y)$ is the a vector pointing in the direction of the steepest slope at that point.
   (C) The Hessian is the Jacobian Matrix of second-order partial derivatives of a function.
   (D) The second derivative of the optimization function is used to determine if we have reached an optimal point.

2. An initial estimate of an optimal solution is given to be used in conjunction with the steepest ascent method to determine the maximum of the function. Which of the following statements is correct?
   (A) The function to be optimized must be differentiable.
   (B) If the initial estimate is different than the optimal solution, then the magnitude of the gradient is nonzero.
   (C) As more iterations are performed, the function values of the solutions at the end of each subsequent iteration must be increasing.
   (D) All 3 statements are correct.

3. What are the gradient and the determinant of the Hessian of the function $f(x, y) = x^2 y^2$ at its global optimum?
   (A) $\nabla f = 0\mathrm{i} + 0\mathrm{j}$ and $|\mathrm{H}| > 0$
   (B) $\nabla f = 0\mathrm{i} + 0\mathrm{j}$ and $|\mathrm{H}| = 0$
   (C) $\nabla f = 1\mathrm{i} + 1\mathrm{j}$ and $|\mathrm{H}| < 0$
   (D) $\nabla f = 1\mathrm{i} + 1\mathrm{j}$ and $|\mathrm{H}| = 0$

4. Determine the gradient of the function $x^2 - 2y^2 - 4y + 6$ at point (0, 0)?
   (A) $\nabla f = 2\mathrm{i} - 4\mathrm{j}$
   (B) $\nabla f = 0\mathrm{i} - 4\mathrm{j}$
   (C) $\nabla f = 0\mathrm{i} + 0\mathrm{j}$

(D) $\nabla f = -4\mathrm{i} - 4\mathrm{j}$

5. Determine the determinant of hessian of the function $x^2 - 2y^2 - 4y + 6$ at point (0, 0)?
   (A) 2
   (B) -4
   (C) 0
   (D) -8

6. Determine the minimum of the function $f(x, y) = x^2 + y^2$? Use the point (2, 1) as the initial estimate of the optimal solution. Conduct one iteration.
   (A) $(2,1)$
   (B) $(-6,-3)$
   (C) $(0,0)$
   (D) $(1,-1)$

Multiple-Choice Test

Chapter 10.01
Introduction to Partial Differential Equations

1. A partial differential equation requires
   (A) exactly one independent variable
   (B) two or more independent variables
   (C) more than one dependent variable
   (D) equal number of dependent and independent variables

2. Using substitution, which of the following equations are solutions to the partial differential equation?

$$\frac{\partial^2 u}{\partial x^2} = 9\frac{\partial^2 u}{\partial y^2}$$

   (A) $\cos(3x - y)$
   (B) $x^2 + y^2$
   (C) $\sin(3x - 3y)$
   (D) $e^{-3\pi x}\sin(\pi y)$

3. The partial differential equation

$$5\frac{\partial^2 z}{\partial x^2} + 6\frac{\partial^2 z}{\partial y^2} = xy$$

   is classified as
   (A) elliptic
   (B) parabolic
   (C) hyperbolic
   (D) none of the above

4. The partial differential equation

$$xy\frac{\partial z}{\partial x} = 5\frac{\partial^2 z}{\partial y^2}$$

   is classified as
   (A) elliptic
   (B) parabolic
   (C) hyperbolic
   (D) none of the above

5. The partial differential equation

$$\frac{\partial^2 z}{\partial x^2} - 5\frac{\partial^2 z}{\partial y^2} = 0$$

is classified as

(A) elliptic
(B) parabolic
(C) hyperbolic
(D) none of the above

6. The following is true for the following partial differential equation used in nonlinear mechanics known as the Korteweg-de Vries equation.

$$\frac{\partial w}{\partial t} + \frac{\partial^3 w}{\partial x^3} - 6w\frac{\partial w}{\partial x} = 0$$

(A) linear; 3rd order
(B) nonlinear; 3rd order
(C) linear; 1st order
(D) nonlinear; 1st order

Multiple-Choice Test

Chapter 10.02
Parabolic Partial Differential Equations

1. In a general second order linear partial differential equation with two independent variables

$$A\frac{\partial^2 u}{\partial x^2}+B\frac{\partial^2 u}{\partial x\partial y}+C\frac{\partial^2 u}{\partial y^2}+D=0$$

where A, B, C are functions of x and y, and D is a function of x, y, $\frac{\partial u}{\partial x},\frac{\partial u}{\partial y}$,

then the partial differential equation is parabolic if

(A) $B^2-4AC<0$
(B) $B^2-4AC>0$
(C) $B^2-4AC=0$
(D) $B^2-4AC\neq 0$

2. The region in which the following partial differential equation

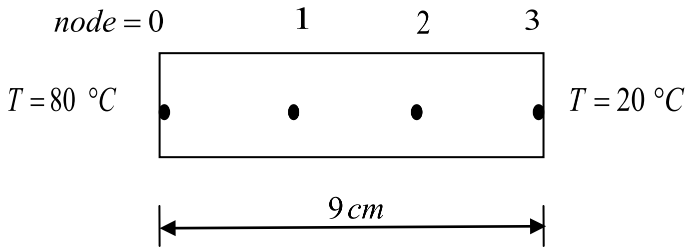
$$x^3\frac{\partial^2 u}{\partial x^2}+27\frac{\partial^2 u}{\partial y^2}+3\frac{\partial^2 u}{\partial x\partial y}+5u=0$$

acts as parabolic equation is

(A) $x>\left(\frac{1}{12}\right)^{1/3}$
(B) $x<\left(\frac{1}{12}\right)^{1/3}$
(C) for all values of x
(D) $x=\left(\frac{1}{12}\right)^{1/3}$

3. The partial differential equation of the temperature in a long thin rod is given by

$$\frac{\partial T}{\partial t}=\alpha\frac{\partial^2 T}{\partial x^2}$$

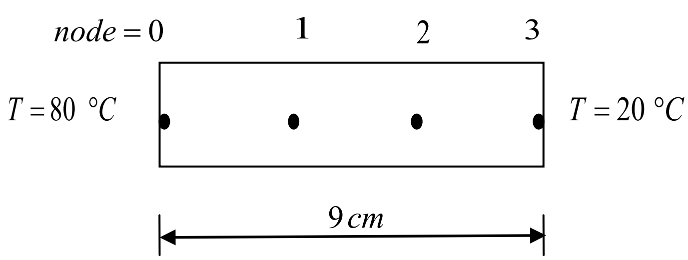


If $\alpha = 0.8\,cm^2/s$, the initial temperature of rod is $40°C$, and the rod is divided into three equal segments, the temperature at node 1 (using $\Delta t = 0.1s$) by using an explicit solution at $t = 0.2\sec$ is

(A) $40.7134\,^0C$
(B) $40.6882\,^0C$
(C) $40.7033\,^0C$
(D) $40.6956\,^0C$

4. The partial differential equation of the temperature in a long thin rod is given by

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

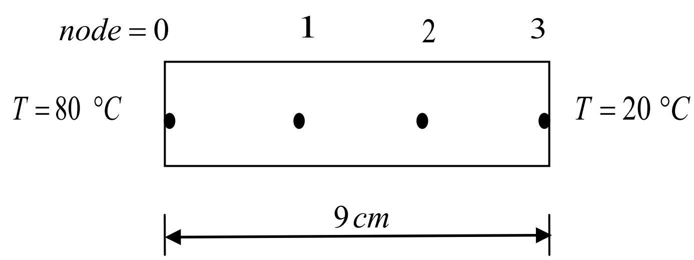


If $\alpha = 0.8\,cm^2/s$, the initial temperature of rod is $40°C$, and the rod is divided into three equal segments, the temperature at node 1 (using $\Delta t = 0.1s$) by using an implicit solution for $t = 0.2\sec$ is

(A) $40.7134\,^0C$
(B) $40.6882\,^0C$
(C) $40.7033\,^0C$
(D) $40.6956\,^0C$

5. The partial differential equation of the temperature in a long thin rod is given by

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$

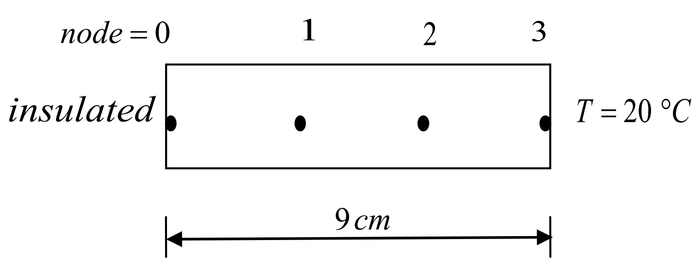


If $\alpha = 0.8\, cm^2/s$, the initial temperature of rod is $40°C$, and the rod is divided into three equal segments, the temperature at node 1 (using $\Delta t = 0.1 s$) by using a <u>Crank-Nicolson</u> solution for $t = 0.2\sec$ is

(A) $40.7134\,^0C$
(B) $40.6882\,^0C$
(C) $40.7033\,^0C$
(D) $40.6956\,^0C$

6. The partial differential equation of the temperature in a long thin rod is given by

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$



If $\alpha = 0.8\, cm^2/s$, the initial temperature of rod is $40°C$, and the rod is divided into three equal segments, the temperature at node 1 (using $\Delta t = 0.1 s$) by using an <u>explicit</u> solution at $t = 0.2\sec$ is

(For node 0, $k\dfrac{\partial T}{\partial x} = h(T_a - T_0)$), where $k = 9W/(m°C)$, $h = 20W/m^2$, $T_a = 25°C$, and $T_0 =$ (the temperature of rod at node 0)

(A) $41.6478\,^0C$
(B) $38.4356\,^0C$
(C) $39.9983\,^0C$
(D) $37.5798\,^0C$

Multiple-Choice Test

Chapter 10.03
Elliptic Partial Differential Equations

1. In a general second order linear partial differential equation with two independent variables,

$$A\frac{\partial^2 u}{\partial x^2}+B\frac{\partial^2 u}{\partial x\partial y}+C\frac{\partial^2 u}{\partial y^2}+D=0$$

where A, B, C are functions of x and y, and D is a function of x, y, $\frac{\partial u}{\partial x},\frac{\partial u}{\partial y}$,

then the PDE is elliptic if

(A) $B^2-4AC<0$
(B) $B^2-4AC>0$
(C) $B^2-4AC=0$
(D) $B^2-4AC\neq 0$

2. The region in which the following equation

$$x^3\frac{\partial^2 u}{\partial x^2}+27\frac{\partial^2 u}{\partial y^2}+3\frac{\partial^2 u}{\partial x\partial y}+5u=0$$

acts as an elliptic equation is

(A) $x>\left(\frac{1}{12}\right)^{1/3}$
(B) $x<\left(\frac{1}{12}\right)^{1/3}$
(C) for all values of x
(D) $x=\left(\frac{1}{12}\right)^{1/3}$

3. The finite difference approximation of $\frac{\partial^2 u}{\partial x^2}$ in the elliptic equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

at (x, y) can be approximated as

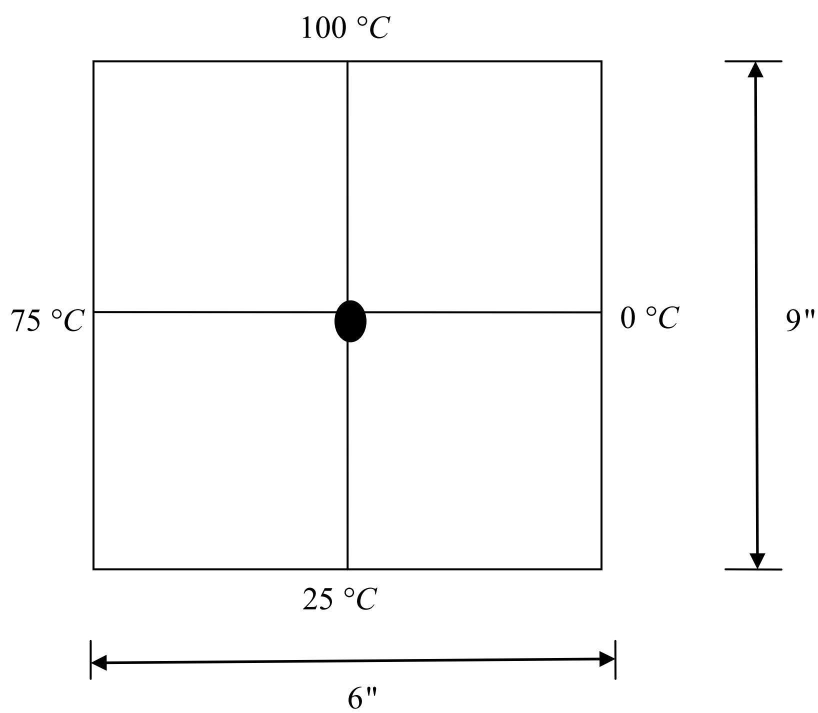
(A) $\frac{\partial^2 u}{\partial x^2} \cong \frac{u(x+\Delta x, y) - 2u(x, y) + u(x-\Delta x, y)}{(\Delta x)^2}$

(B) $\frac{\partial^2 u}{\partial x^2} \cong \frac{u(x+\Delta x, y) - u(x, y) + u(x-\Delta x, y)}{(\Delta x)^2}$

(C) $\frac{\partial^2 u}{\partial x^2} \cong \frac{u(x, y+\Delta y) - 2u(x, y) + u(x, y-\Delta y)}{(\Delta x)^2}$

(D) $\frac{\partial^2 u}{\partial x^2} \cong \frac{u(x+\Delta x, y) - u(x-\Delta x, y)}{2\Delta x}$

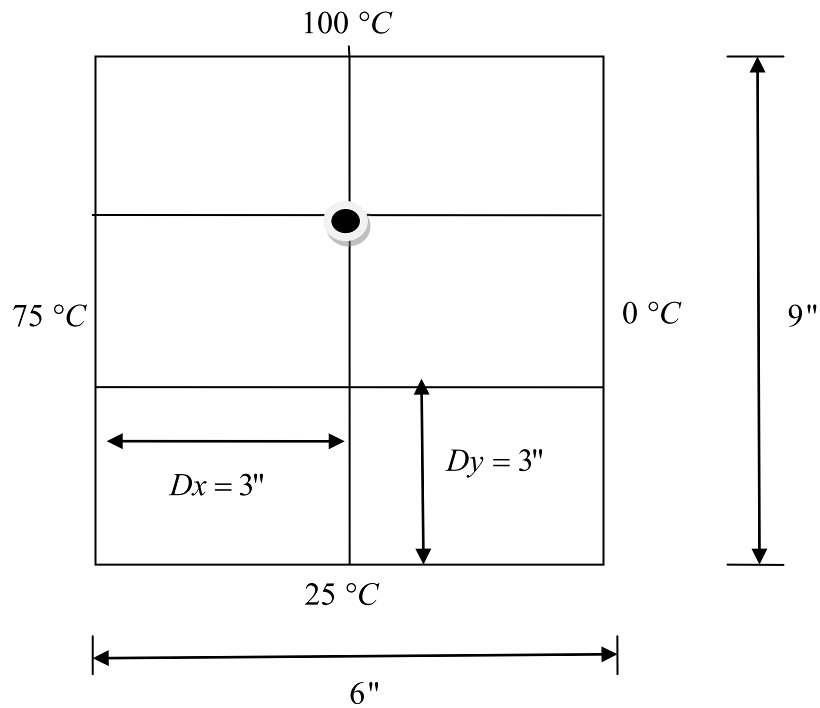
4. Find the temperature at the interior node given in the following figure using the direct method



(A) 45.19 °C
(B) 48.64°C
(C) 50.00 °C

(D) $56.79\ {}^{\circ}C$

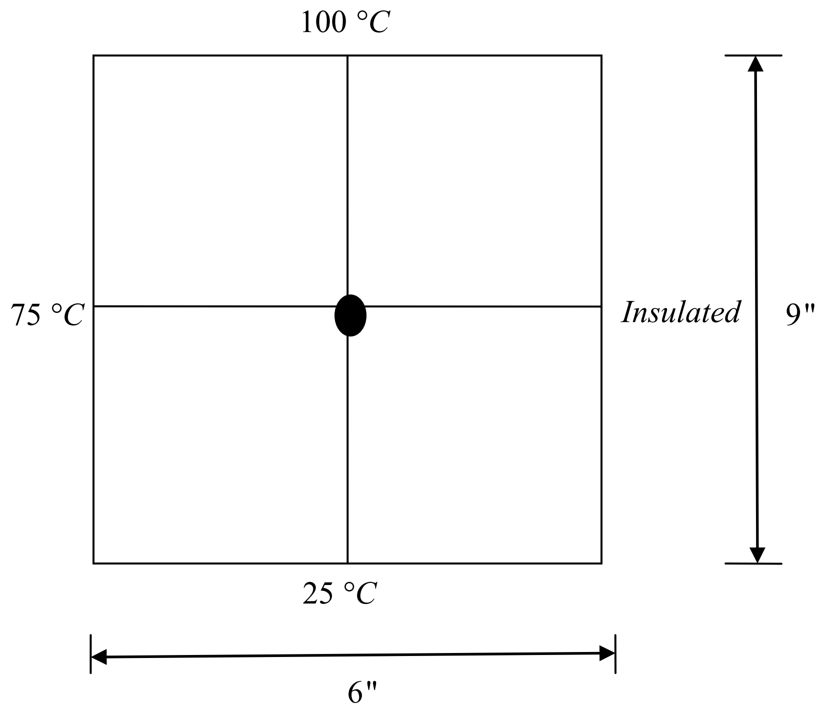
5. Find the temperature at the interior node given in the following figure



Using the Lieberman method and relaxation factor of 1.2, the temperature at $x = 3, y = 6$ estimated after 2 iterations is (use the temperature of interior nodes as $50\ {}^{\circ}C$ for the initial guess)

(A) $52.36\ {}^{\circ}C$
(B) $53.57\ {}^{\circ}C$
(C) $56.20\ {}^{\circ}C$
(D) $58.64\ {}^{\circ}C$

6. Find the steady-state temperature at the interior node as given in the following figure



(A) $53.57°C$
(B) $66.40°C$
(C) $68.20°C$
(D) $69.59°C$

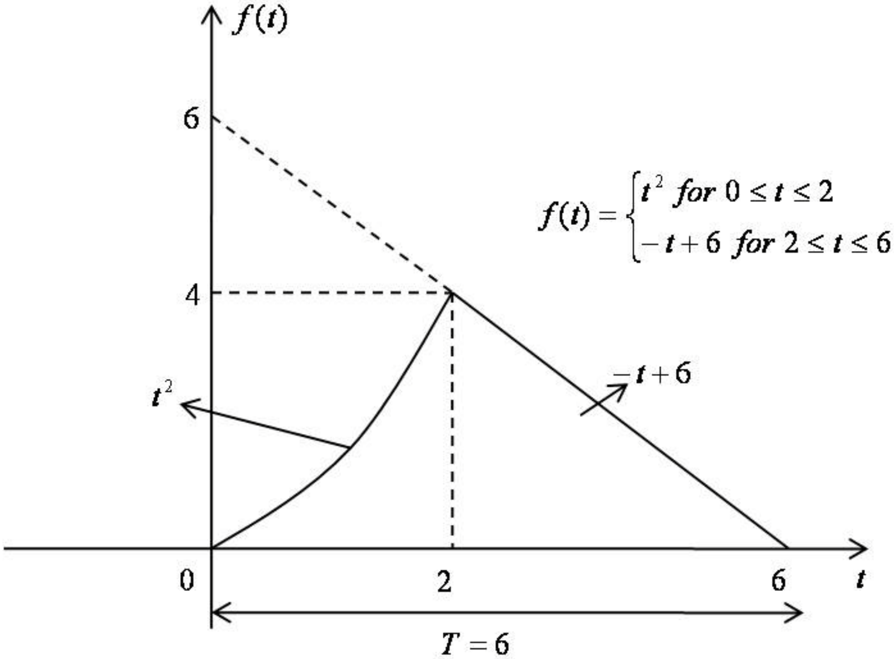
Multiple Choice Test

Chapter 11.02
Continuous Fourier Series

1. Which of the following is an "even" function of t?
   (A) t^2
   (B) $t^2 - 4t$
   (C) $\sin(2t) + 3t$
   (D) $t^3 + 6$

2. A "periodic function" is given by a function which
   (A) has a period $T = 2\pi$
   (B) satisfies $f(t+T) = f(t)$
   (C) satisfies $f(t+T) = -f(t)$
   (D) has a period $T = \pi$

3. Given the following periodic function, $f(t)$.



The coefficient a_0 of the continuous Fourier series associated with the above given function $f(t)$ can be computed as

(A) $\frac{8}{9}$
(B) $\frac{16}{9}$
(C) $\frac{24}{9}$
(D) $\frac{32}{9}$

4. For the given periodic function $f(t)=\begin{cases} 2t \text{ for } 0 \le t \le 2 \\ 4 \text{ for } 2 \le t \le 6\ (=T) \end{cases}$. The coefficient b_1 of the continuous Fourier series associated with the given function $f(t)$ can be computed as

(A) -75.6800
(B) -7.5680
(C) -6.8968
(D) -0.7468

5. For the given periodic function $f(t)=\begin{cases} 2t \text{ for } 0 \le t \le 2 \\ 4 \text{ for } 2 \le t \le 6 \end{cases}$ with a period $T=6$. The Fourier coefficient a_1 can be computed as

(A) -9.2642
(B) -8.1275
(C) -0.9119
(D) -0.5116

6. For the given periodic function $f(t)=\begin{cases} 2t \text{ for } 0 \le t \le 2 \\ 4 \text{ for } 2 \le t \le 6 \end{cases}$ with a period $T=6$ as shown in Problem 5. The complex form of the Fourier series can be expressed as $f(t)=\sum_{k=-\infty}^{\infty}\widetilde{C}_k e^{ikw_0 t}$. The complex coefficient $\widetilde{C}_1$ can be expressed as

(A) $0.4560+0.3734i$
(B) $0.4560-0.3734i$
(C) $-0.4560+0.3734i$
(D) $0.3734-0.4560i$

Multiple Choice-Test

Chapter 11.03
Fourier Transform Pair: Frequency and Time Domain

1. Given two complex numbers: $C_1 = 2 - 3i$, and $C_2 = 1 + 4i$. The product $P = C_1 \times C_2$ can be computed as
   (A) $2 + 5i$
   (B) $-10 + 5i$
   (C) $-14 + 5i$
   (D) $14 + 5i$

2. Given the complex number $C_1 = 3 + 4i$. In polar coordinates, the above complex number can be expressed as $C_1 = Ae^{i\theta}$, where A and θ is called the amplitude and phase angle of C_1, respectively. The amplitude A can be computed as
   (A) 3
   (B) 4
   (C) 5
   (D) 7

3. Given the complex number $C_1 = 3 + 4i$. In polar coordinates, the above complex number can be expressed as $C_1 = Ae^{i\theta}$, where A and θ is called the amplitude and phase angle of C_1, respectively. The phase angle θ in radians can be computed as
   (A) 0.6435
   (B) 0.9273
   (C) 2.864
   (D) 5.454

4. For the complex number $C = -3 + 4i$, the phase angle θ in radians can be computed as
   (A) 0.6435
   (B) 0.9273
   (C) 1.206
   (D) 2.2143

5. Given the function $f_{np}(t) = \delta(t-a) = \begin{cases} 1, \, if \;\; t = a \\ 0, \, elsewhere \end{cases}$. The Fourier transform $\hat{F}(iw_0)$ which will transform the function from time domain to frequency domain can be computed as

(A) $\delta(a+t)$
(B) $e^{-i(2\pi f)a}$
(C) 1
(D) $\delta(t-a)$

6. Given the function $\hat{F}(iw_0) = 1$. The inverse Fourier transform $f_{np}(t)$ which will transform the function from frequency domain to time domain can be computed as

(A) e^{it}
(B) e^{-it}
(C) $\delta(t-0)$
(D) $e^{-i(2\pi f)t}$

Multiple Choice Test

Chapter 11.04
Discrete Fourier Transform

1. Given that $W = e^{-i\left(\frac{2\pi}{N}\right)}$, where $N = 3$. Then $F = W^N$ can be computed as $F =$
   (A) 0
   (B) 1
   (C) -1
   (D) e

2. Given that $W = e^{-i\left(\frac{2\pi}{N}\right)}$, where $N = 3$. $F = W^{\frac{N}{2}}$ can be computed as $F =$
   (A) 0
   (B) 1
   (C) -1
   (D) e

3. Given that $N = 2$, $\{f\} = \begin{Bmatrix} 4-6i \\ -2+4i \end{Bmatrix}$. The values for vector $\{\widetilde{C}^R\}$ shown in

$$\widetilde{C}_n^R = \sum_{k=0}^{N-1} \left\{ f^R(k)\cos(\theta) + f^I(k)\sin(\theta) \right\}$$

can be computed as:

   (A) $\begin{Bmatrix} -2 \\ -6 \end{Bmatrix}$
   (B) $\begin{Bmatrix} -2 \\ 6 \end{Bmatrix}$
   (C) $\begin{Bmatrix} 2 \\ -6 \end{Bmatrix}$
   (D) $\begin{Bmatrix} 2 \\ 6 \end{Bmatrix}$

4. Given that $N = 2$, $\{f\} = \begin{Bmatrix} 4-6i \\ -2+4i \end{Bmatrix}$. The values for $\{\widetilde{C}^I\}$ shown in Equation (22D)

$$\widetilde{C}_n^I = \sum_{k=0}^{N-1} \{f^I(k)\cos(\theta) - f^R(k)\sin(\theta)\}$$

can be computed as

(A) $\begin{Bmatrix} -2 \\ -10 \end{Bmatrix}$

(B) $\begin{Bmatrix} -1 \\ -10 \end{Bmatrix}$

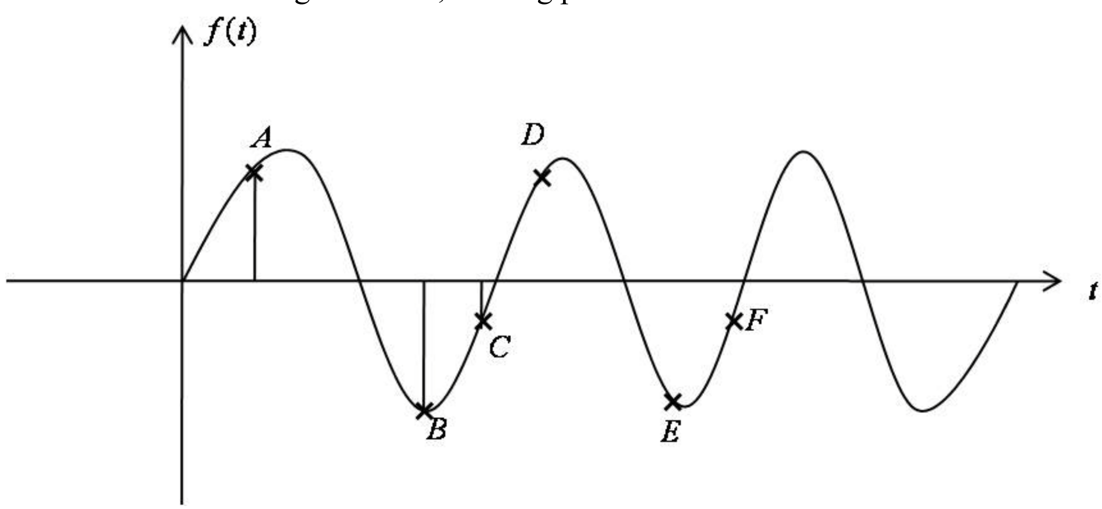
(C) $\begin{Bmatrix} -2 \\ -5 \end{Bmatrix}$

(D) $\begin{Bmatrix} -1 \\ -5 \end{Bmatrix}$

5. If the forcing function $F(t)$ is given as $F(t) = \sum_{n=0}^{7} 10 \times \sin(2\pi n t)$. Then, to avoid aliasing phenomenon, the minimum number of sample data points $N_{\min}$ should be

(A) 8
(B) 16
(C) 24
(D) 32

6. Based on the figure below, aliasing phenomena will not occur because there were



(A) 2 sample data points per cycle.
(B) 4 sample data points per cycle.
(C) 4 sample data points per 2 cycles.
(D) 6 sample data points per 2 cycles.

Multiple Choice Test

Chapter 11.05
Informal Development of Fast Fourier Transform

1. Using the definition $E = e^{-i\frac{2\pi}{N}}$, and the Euler identity $e^{\pm i\theta} = \cos(\theta) \pm i\sin(\theta)$, the value of $E^{\frac{N}{6}}$ can be computed as
   (A) $0.866 - 0.5i$
   (B) $-0.866 + 0.5i$
   (C) $-0.5 - 0.866i$
   (D) $0.5 - 0.866i$

2. Using the definition $E = e^{-i\frac{2\pi}{N}}$, and the Euler identity $e^{\pm i\theta} = \cos(\theta) \pm i\sin(\theta)$, the value of E^{6N} can be computed as
   (A) $1 + i$
   (B) $1 - i$
   (C) 1
   (D) -1

3. Given $N = 2$, and $\{f\} = \begin{Bmatrix} f(0) \\ f(1) \end{Bmatrix} = \begin{Bmatrix} 14 + 6i \\ -2 + 4i \end{Bmatrix}$. The first part of

$\widetilde{C}_n = \widetilde{C}(n) = \sum_{k=0}^{N-1} f(k) E^{nk}$ can be expressed as

$$\widetilde{C}(0) = \sum_{k=0}^{1} f(k) E^{nk} = f(0)E^{(0)(0)} + f(1)E^{(0)(1)}$$

$$\widetilde{C}(1) = f(0)E^{(1)(0)} + f(1)E^{(1)(1)}$$

The values for $\begin{Bmatrix} \widetilde{C}(0) \\ \widetilde{C}(1) \end{Bmatrix}$ can be computed as

(A) $\begin{Bmatrix} 12 + 10i \\ 16 + 2i \end{Bmatrix}$

(B) $\begin{Bmatrix} 10 + 12i \\ 2 + 16i \end{Bmatrix}$

(C) $\begin{Bmatrix} -12 + 10i \\ -16 + 2i \end{Bmatrix}$

(D) $\begin{Bmatrix} 10 - 12i \\ 2 - 16i \end{Bmatrix}$

4. For $N = 2^4 = 16$, level $L = 2$ and referring to Figure 1, the only terms of vector $f_2(-)$ which only need to be computed are:
   (A) $f_2(4-7, 12-15)$
   (B) $f_2(0-3, 8-11)$
   (C) $f_2(0-7)$
   (D) $f_2(8-15)$

5. For $N = 2^4 = 16$, level $L = 3$ and referring to the Figure 1, the only companion nodes associated with $f_3(0)$ and $f_3(1)$ are
   (A) $f_3(4)$ and $f_3(5)$
   (B) $f_3(6)$ and $f_3(7)$
   (C) $f_3(14)$ and $f_3(15)$
   (D) $f_3(2)$ and $f_3(3)$

6. Given $N = 4$, and

$$f_0\begin{pmatrix}0\\1\\2\\3\end{pmatrix} = \begin{Bmatrix}-4+i\\1-2i\\-2+3i\\3-4i\end{Bmatrix}.$$

Corresponding to level $L = 1$, one can compute $f_1(2)$ as
   (A) $-2-2i$
   (B) $4-6i$
   (C) $4+6i$
   (D) $-4-4i$

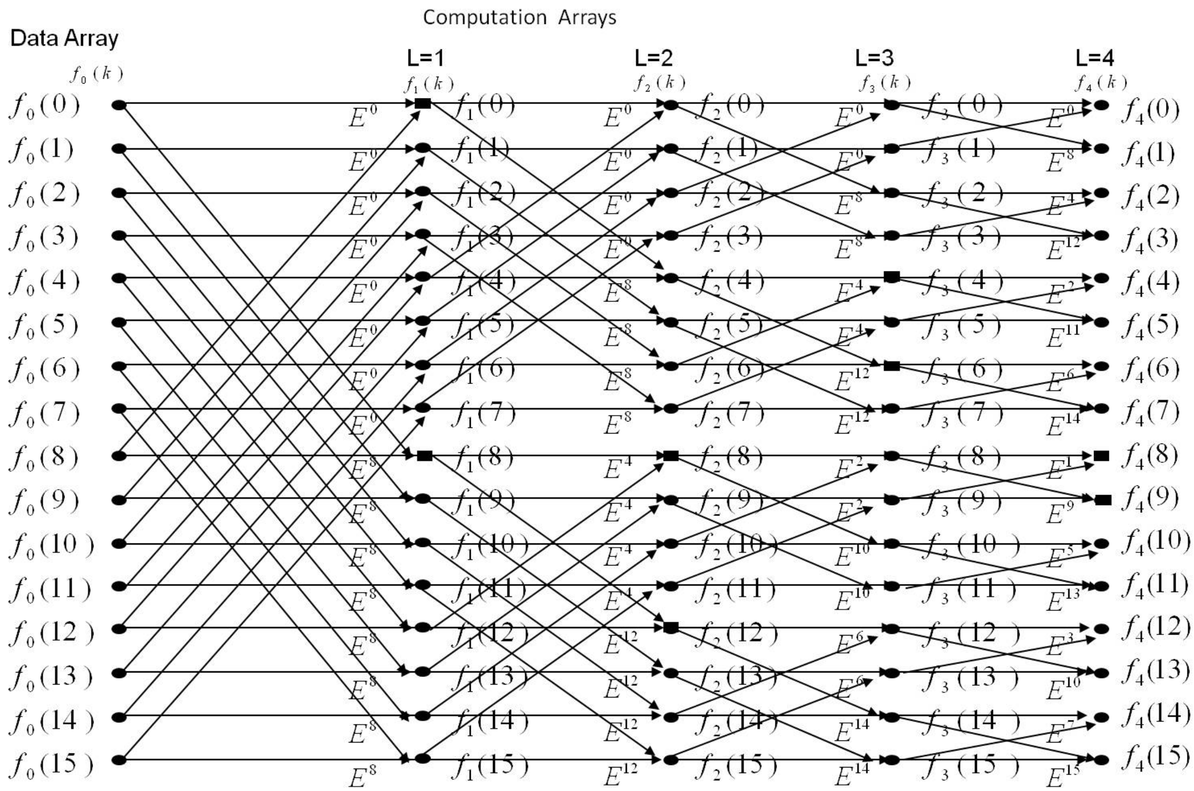


Figure 1: Figure referring to Question 4&5