Total No. of printed pages = 6

MATHEMATICS –I

Full Marks – 70

Pass Marks – 21

Time – Three hours

The figures in the margin indicate full marks for the questions.

GROUP–A

Answer any *seven* questions.   7×5=35

1. (a) If 4+√3 is a root of a quadratic equation, then find the equation.   3

   (b) If one root of $x^2-px+q = 0$ is twice the other show that $2p^2 = 9q$.   2

2. (a) If w is the imaginary cube root of unity evaluate   3

$$\begin{vmatrix} 1 & w & w^2 \\ w & w^2 & 1 \\ w^2 & 1 & w \end{vmatrix}$$

[Turn over

(b) Show that $\sqrt{i} + \sqrt{-i} = \sqrt{2}$.   2

3. (a) Show that $^{n}P_{r} = {}^{n-1}P_{r} + r \cdot {}^{n-1}P_{r-1}$ 3

   (b) How many odd numbers of 5 distinct significant digits can be formed with 0, 1, 2, 3, 4 ?   2

4. (a) Find 'a' if $\log_{a} 324 = 4$.   2

   (b) Prove that $7\log\dfrac{10}{9} - 2\log\dfrac{25}{24} + 3\log\dfrac{81}{80} = \log 2$

                                                                        3

5. (a) If the sum and product of three numbers in a G.P are 49 and 512 respectively, find the numbers.   3

   (b) If a, b, c be the pth, qth and rth terms of an A.P. prove that—   2

       $a(q-r) + b(r-p) + c(p-q) = 0$

6. (a) Solve by Crammer's rule   3

       $2x - 3y + z = -1$

       $3x + y - 2z = 1$

       $4x - y + z = 9.$

(b) Without expansion prove that $\qquad$ 2

$$\begin{vmatrix} a & b & c \\ x & y & z \\ p & q & r \end{vmatrix} = \begin{vmatrix} y & b & q \\ x & a & p \\ z & c & r \end{vmatrix}$$

7. (a) Find the co-efficient of $\dfrac{1}{x}$ in the expansion

   of $\left(\dfrac{3}{x^2} - \dfrac{x^3}{2} \right)^8$ $\qquad$ 3

   (b) Show that $\log (1+2+3) = \log 1 + \log 2 + \log 3$.

   2

8. (a) Show that $\qquad$ 3

$$\frac{\dfrac{1}{\lfloor 2} + \dfrac{1}{\lfloor 4} + \dfrac{1}{\lfloor 6} + \ldots\ldots}{\dfrac{1}{\lfloor 1} + \dfrac{1}{\lfloor 3} + \dfrac{1}{\lfloor 5} + \ldots\ldots} = \frac{c-1}{c+1}$$

   (b) If $A^2 \infty BC$, $B^2 \infty CA$, $C^2 \infty AB$ show that the product of the three constants of variation is unity. $\qquad$ 2

9. (a) If w is the imaginary cube root of unity, prove that $(1-w+w^2)^2 + (1+w-w^2)^2 = -4$.

2

(b) Prove that $\dfrac{1.3}{\underline{|1}} + \dfrac{2.4}{\underline{|2}} + \dfrac{3.5}{\underline{|3}} + \cdots \infty = 4c$

3

GROUP–B

Answer any *four* questions.   4×5=20

10. (a) If $\tan\theta = \dfrac{x}{y}$ find the value of $x\sin 2\theta + y\cos 2\theta$.

3

(b) Prove that $\tan^{-1}\dfrac{1}{2} + \tan^{-1}\dfrac{1}{3} = \dfrac{\pi}{4}$

2

11. (a) Find the value of Cosec $(-660°)$ and cot $(-1575°)$

3

(b) Show that $\cos(60°-A)\cos(30°-B) - \sin(60°-A)\sin(30°-B) = \sin(A+B)$

2

12. (a) Prove that in any triangle

$\tan\dfrac{B-C}{2} = \dfrac{b-c}{b+c}\cot A/2$

3

(b) Show that $\cos 130° + \cos 110° + \cos 10° = 0$

2

13. (a) Find the value of $\tan 15°$.

2

(b) Show that in $\triangle ABC$,

a $(\sin B - \sin C) + b (\sin C - \sin A) +$ c $(\sin A - \sin B) = 0$.

3

14. (a) Solve for θ $(0 \le \theta \le 360°)$ $2 \sin^2\theta + 3\cos\theta = 0$

3

(b) If $\sin (A–B) = \dfrac{1}{2}$ and $\cos (A+B) = \dfrac{1}{2}$ find A and B.

2

GROUP–C

Answer any *three* questions. $3×5=15$

15. A regular hexagon is inscribed in a circle of radius 5 cm. Find the area of the circle which is outside the hexagon.

5

16. A river is 80 feet wide. Its depth at a distance of x feet from one bank is d feet and is given by the following table :

x:	0	10	20	30	40	50	60	70	80
d:	0	40	75	94	121	153	142	86	31

Find the cross sectional area of the river. 5

17. Find the amount of concrete required to erect a concrete pillar whose circular base will have a perimeter 8·8m and whose curved surface is 17·6 m. 5

18. The volume of a right circular cylinder and right circular cone standing on the same base are in the ratio 3:2. Show that the height of the cone is double the height of the cylinder. 5