Total No. of printed pages = 6

MATHEMATICS – II

Full Marks – 70

Pass Marks – 21

Time – Three hours

The figures in the margin indicate full marks
for the questions.

GROUP – A

1. (a) A function $f(x)$ is defined as follows :   3

$$f(x) = 2x^2 + 3, \qquad 0 \le x < 2$$
$$= 3x - 5, \qquad 2 < x \le 4$$

find $f(0)$, $f(1)$, $f(3)$.

(b) Find the Domain of the function :   2

$$f(Z) = \frac{3z^3 + 4z - 1}{(z-2)(z-1)}$$

[Turn over

2. Find the limit (any *two*) : $2 \times 3 = 6$

   (i) $\displaystyle\lim_{x \to 0} \log(1 + x)$

   (ii) $\displaystyle\lim_{x \to \infty} \frac{2x(x+1)}{(x+4)(x-1)}$

   (iii) $\displaystyle\lim_{x \to 0} \frac{8^x - 2^x}{x}$

   (iv) $\displaystyle\lim_{x \to 2} \frac{x^2 - 5x + 6}{x^2 - 7x + 10}$

3. Find $\dfrac{dy}{dx}$ of $y = a^x$ using first principle.   3

4. Find $\dfrac{dy}{dx}$ (any *two*) : $2 \times 3 = 6$

   (i) $y = e^{x \sin x}$

   (ii) $y = (\sec x)^{\tan x}$

   (iii) $x = a \cos^3 \theta$, $y = b \sin^3 \theta$

5. (a) Find the points on the curve $y = x^3 - 2x^2 + x$ at which the tangents are parallel to X-axis. Also find the tangents at these points. 4

(b) If $y = (\tan^{-1}x)^2$ show that $(1+x^2)^2 y_2 + 2x(1+x^2) y_1 - 2 = 0$. 3

(c) Show that $f(x) = x^3 - 6x^2 + 24x + 4$ has neither maxima nor minima. 3

GROUP – B

6. Integrate any *three* of the following : $2 \times 3 = 6$

(i) $\int \dfrac{dx}{a + bx^2}$

(ii) $\int \sin mx \sin nx \, dx$

(iii) $\int \dfrac{e^{5x} + e^{3x}}{e^x + e^{-x}} dx$

(iv) $\int \sqrt{1 + \sin 2x \, dx}$

7. Integrate any *three* of the following : $3 \times 3 = 9$

(i) $\int_0^1 \dfrac{dx}{\sqrt{x+1} - \sqrt{x}}$

(ii) $\int_0^{\pi/2} \sin^2 x \cos^2 x \, dx$

(iii) $\int_0^{\pi/2} \dfrac{dx}{2 + \cos x}$

(iv) $\int x^3 \cos x \, dx$

(v) $\int \dfrac{x - 13}{x^2 - 5x + 6} \, dx$

8. (a) Find the circumference of the circle   2
$x^2 + y^2 = a^2$

(b) Evaluate : $\int_0^{\pi/2} \dfrac{\sin x}{\sin x + \cos x} \, dx$ 4

Or

$$\lim_{x \to \infty} \left[\frac{1}{n} + \frac{n^2}{(n+1)^3} + \frac{n^2}{(n+2)^3} + \dots\dots + \frac{1}{8n} \right]$$

GROUP – C

9. Answer any *five* questions : $5 \times 2 = 10$

(i) Write the locus of a point equidistant from a fixed point (h, k), where the distance is 'k'.

(ii) Change $r = a \sin 2\theta$ to Cartesian form.

(iii) Show that the points $(0, 0)$, $(0, \pi/3)$ and $(0, 2\pi/3)$ are the vertices of an equilateral triangle.

(iv) In what ratio the line segment joining $A(3, 4)$ and $B(5, -7)$ will be divided by Y-axis ?

(v) Find the area of the quadrilateral formed by the points $(0, 3)$, $(2, -1)$, $(5, 0)$ and $(4, 6)$.

(vi) Show that the lines $2x - y + 8 = 0$, $3x + y + 2 = 0$ and $4x + 3y - 4 = 0$ are concurrent.

(vii) Find the eccentricity, focii, length of the L. R and the equation of the directrix of the hyperbola $16x^2 - 9y^2 = 144$.

10. Answer any *three* questions : $3 \times 3 = 9$

(i) Find the equation of the straight line which passes through the point $(2, 3)$ and whose sum of the intercepts on X-axis and Y-axis equals 10.

(ii) Find the equation of the tangent to the circle $x^2 + y^2 - 6x - 3y - 2 = 0$ at $(2, -2)$.

(iii) Express $x + \sqrt{3}y = 8$ in the perpendicular form. Also find 'α'.

(iv) Find the equation of the parabola with focus at $(1, -3)$ and directrix $x - 2y + 3 = 0$.

(v) The latus rectum of an ellipse is half of its major axis. Find the value of the two eccentricities.