

Total No. of printed pages = 5

Sc-202/Maths-II/2nd Sem/Comm/2017/M

MATHEMATICS – II

Full Marks – 70

Pass Marks – 21

Time – Three hours

The figures in the margin indicate full marks for the questions.

GROUP – A

1. (a) If $f(x) = 1 + e^x$, find $f(f(x))$. 2

(b) Find the domain of the function $f(x) = \sqrt{x^2 - 1}$. 2

(c) Examine the continuity of $f(x)$ where

$$f(x) = \frac{|x-1|}{x-1} \quad \text{if } x \neq 1$$

$$= 0 \quad \text{if } x = 1$$

at $x = 1$.

3

[Turn over

2. Find the limit (any two) :

2×2=4

(a) $\lim_{x \rightarrow 9} \frac{\sqrt{x} - 3}{x - 9}$

(b) $\lim_{x \rightarrow 0} \frac{\tan \frac{x}{3}}{x}$

(c) $\lim_{x \rightarrow \infty} \frac{4x^2 - 5x + 1}{5x^2 + 2x + 3}$

3. Find $\frac{dy}{dx}$ (any three) :

3×3=9

(a) $y = e^{f(x)}$

(b) $y = \frac{e^x}{2+x}$

(c) $x^y = y^x$

(d) $x = a(t + \sin t), y = b \cos t$

4. Find $\frac{d^2y}{dx^2}$ (any two) :

2×3=6

(a) $y = e^x \tan x$

(b) $y = \cos^{-1} x$

(c) $y = \sin^5 x \cos x$

5. Find the equation of the tangent to the curve

$$\sqrt{x} + \sqrt{y} = 3 \text{ at } (4, 1). \quad 3$$

6. Find the extreme values of the function

$$f(x) = 2x^3 - 9x^2 + 12x + 5. \quad 3$$

GROUP - B

7. Integrate any *three* :

$$3 \times 2 = 6$$

(a) $\int (\cos x)^2 dx$

(b) $\int \left(x^2 + \frac{1}{x^2} \right)^3 dx$

(c) $\int x^2 \log x dx$

(d) $\int \frac{\cos x}{1 + \sin^2 x} dx$

8. Evaluate any *two* :

$$3 \times 2 = 6$$

(a) $\int_0^1 xe^x dx$

(b) $\int_0^1 \frac{\tan^{-1} x}{1+x^2} dx$

$$(c) \int_0^{\frac{\pi}{2}} \frac{\sin x}{\sin x + \cos x} dx$$

9. Find by the method of integration the area of the region bounded by the parabola $y^2 = 8x$ and its latus rectum. 3

10. Find the sum : 3

$$\lim_{n \rightarrow \infty} n \left[\frac{1}{n^2 + 1^2} + \frac{1}{n^2 + 2^2} + \dots + \frac{1}{n^2 + n^2} \right]$$

GROUP - C

11. Answer any *seven* questions : $7 \times 2 = 14$

- Find the centroid of the triangle with vertices $(0, 0)$, $(2, 4)$, $(4, 0)$.
- Show that the points $(4, 4)$, $(6, 2)$ and $(7, 1)$ are collinear.
- Find the equation of the straight line parallel to $x = 2y$ and passing through $(1, 1)$.
- Find intercepts on axes by the straight line $2x + 3y - 5 = 0$.

(e) What is the equation of directrix of the parabola $y^2 = 16x$?

M

(f) Express $\frac{x}{2} + \frac{y}{3} = 1$ in perpendicular form.

(g) Write down the equation of tangent to the circle $x^2 + y^2 = a^2$ at (x_1, y_1) .

(h) What are the lengths of major axis and minor axis of the ellipse $9x^2 + 16y^2 = 144$.

12. Find the equation of circle passing through the set of points $(0, 0)$, $(a, 0)$ and $(0, b)$. 3

13. Find the co-ordinates of the centre, vertices, foci and the equation of the directrices of the hyperbola $9x^2 - 16y^2 = 144$. 3

2

1.

2

3