

1.1 Effect of Depth on Expressiveness

Three message passing layers are needed to distinguish the two red nodes.

Layer 1 aggregates information from identical nodes 5. Layer 2 reaches identical nodes 4 and 6. Layer 3 reaches different nodes in each graph, making them distinguishable.

Thus, at least 3-layer GNN is necessary.

1.2 Random Walk Matrix

1

The random walk transition matrix is $M = \begin{bmatrix} 0 & 1/2 & 1/2 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 1/3 & 1/3 & 0 & 1/3 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

2

The limiting distribution r , such that $r = Mr$, is $r = (1/4, 1/4, 1/4, 1/4)$

1.3 Relation to Random Walk (i)