Lösungshinweise Blatt 9

33 a) $\varphi_0(x) = \dfrac{e^{-x^2/2}}{\pi^{1/4}}$ $\qquad (\hbar = 1, \quad \ell = 1)$

b) wegen $D(u+iv) = e^{is}\, \tilde{T}(\sqrt{2}\,v)\, T(\sqrt{2}\,u)$

$$\varphi_{u+iv}(x) \equiv \langle x \,|\, \alpha(u+iv) \rangle$$
$$= e^{is}\; e^{i\sqrt{2}\,v\,x}\; \frac{e^{-(x-\sqrt{2}\,u)^2/2}}{\pi^{1/4}}$$

34 a)

$$\langle \alpha(\alpha) \,|\, \alpha(\beta) \rangle \overset{32c)}{=} e^{-\frac{|\alpha|^2+|\beta|^2}{2}} \sum_{mn} \frac{\alpha^{*m}\,\beta^n}{\sqrt{m!\,n!}} \underbrace{\langle m \,|\, n \rangle}_{= \delta_{mn}}$$
$$= e^{-\frac{|\alpha|^2+|\beta|^2}{2}}\; e^{\alpha^*\beta}$$

$$\rightarrow\; |\langle \alpha(\alpha) \,|\, \alpha(\beta) \rangle|^2 = e^{-|\alpha-\beta|^2}.$$

34 b)

$$A := \frac{1}{\pi} \int du \int dv\; |\alpha(u+iv)\rangle\langle \alpha(u+iv)|$$

z. z.: $\langle m \,|\, A \,|\, n \rangle \overset{!}{=} \langle m | n \rangle = \delta_{mn}$,

$$\langle m \,|\, A \,|\, n \rangle \overset{32c)}{=} \frac{1}{\pi} \int du \int dv\; \frac{(u+iv)^m}{\sqrt{m!}}\, \frac{(u-iv)^n}{\sqrt{n!}}\, e^{-|u+iv|^2}$$

$$= \frac{1}{\pi} \int_0^\infty dr \int_0^{2\pi} d\varphi\; r\; \frac{r^{m+n}}{\sqrt{m!\,n!}}\; e^{i\varphi(m-n)}\; e^{-r^2}$$

$u + iv \rightarrow r\, e^{i\varphi}$

mit $\int_0^{2\pi} e^{i\varphi(m-n)} d\varphi = 2\pi\,\delta_{mn}$ folgt

$$\langle m|A|n\rangle = \delta_{mn}\,\frac{1}{n!}\underbrace{\int_0^\infty dr\; 2r\,(r^2)^n e^{-r^2}}_{\substack{\| \quad r=\sqrt{t},\; dr=\frac{1}{2\sqrt{t}}dt \\ \int_0^\infty dt\; t^n e^{-t} = \Gamma(n+1) = n!}} = \delta_{mn}$$

35a) $E_n(\lambda) = \hbar\omega\,(n+\tfrac{1}{2}) + \lambda\,\frac{\hbar\omega}{\ell}\,\overbrace{\langle n|x|n\rangle}^{=0}$

$$+\lambda^2(\hbar\omega)^2\sum_{m\neq n}\frac{|\langle m|x/\ell|n\rangle|^2}{E_n-E_m}$$

mit $x/\ell = (a^\dagger + a)/\sqrt{2}$ folgt

$$\langle m|\,x/\ell\,|n\rangle = \frac{1}{\sqrt{2}}\underbrace{\langle m|a^\dagger|n\rangle}_{\substack{\| \\ \delta_{n+1,m}\sqrt{n+1}}} + \frac{1}{\sqrt{2}}\underbrace{\langle m|a|n\rangle}_{\substack{\| \\ \delta_{n-1,m}\sqrt{n}}}$$

$$\rightarrow |\langle m|x/\ell|n\rangle|^2 = \frac{n+1}{2}\,\delta_{n+1,m} + \frac{n}{2}\,\delta_{n-1,m}$$

$$\rightarrow \hbar\omega\sum_{m\neq n}\frac{|\langle m|x/\ell|n\rangle|^2}{\underbrace{\hbar\omega(n-m)}_{=E_n-E_m}} = \frac{1}{2}\left(\frac{n+1}{-1} + \frac{n}{+1}\right) = -\frac{1}{2} \qquad \rightarrow$$

d. h. $E_n(\lambda) = \hbar\omega\left(n+\frac{1}{2}\right) - \lambda^2 \frac{\hbar\omega}{2}$

35 b) $\frac{m\omega^2}{2} x^2 + \lambda \hbar\omega \frac{x}{\ell}$

$$= \frac{\hbar\omega}{2}\Big(\underbrace{\frac{m\omega}{\hbar}}_{= 1/\ell^2} x^2 + 2\lambda \frac{x}{\ell} \Big)$$

$$= \frac{\hbar\omega}{2\ell^2}\left(x^2 + 2\lambda\ell x\right) = \frac{\hbar\omega}{2\ell^2}\left((x+\lambda\ell)^2 - \lambda^2\ell \right)$$

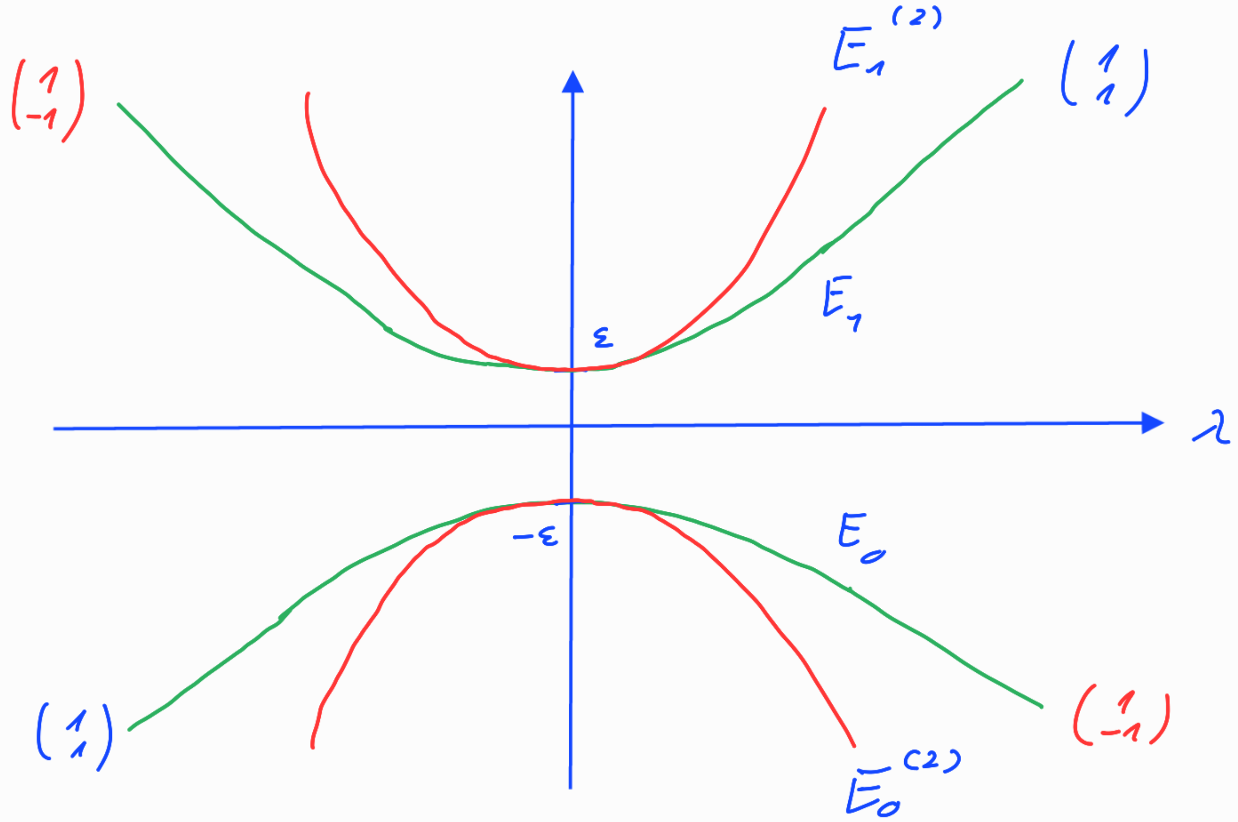
$$= \frac{m\omega^2}{2}(x+\lambda\ell)^2 - \lambda^2 \frac{\hbar\omega}{2}$$

d. h. $$H(\lambda) = \underbrace{p^2/2m + \frac{m\omega^2}{2} \underbrace{(x+\lambda\ell)^2}_{= x'}}_{\Rightarrow\ \hbar\omega(n+1/2)} \underbrace{- \lambda^2 \frac{\hbar\omega}{2}}_{\Delta E(\lambda)}$$

36 a) $H = \begin{pmatrix} \varepsilon & \lambda d \\ \lambda d^* & \varepsilon \end{pmatrix}$

$$0 \overset{!}{=} \det\begin{pmatrix} \varepsilon - E & \lambda d \\ \lambda d^* & -\varepsilon - E \end{pmatrix} = E^2 - \varepsilon^2 - \lambda^2 |d|^2$$

$$\rightarrow \quad E_{1/0} = \pm\left(\varepsilon^2 + \lambda^2 |d|^2\right)^{1/2}$$



36 b)

$$E_0(\lambda) = -\varepsilon + \lambda \underbrace{\begin{pmatrix}0\\1\end{pmatrix}^{\dagger}\begin{pmatrix}0 & \alpha\\ \alpha^* & 0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}}_{=0} + \lambda^2 \underbrace{\frac{\left|\begin{pmatrix}1\\0\end{pmatrix}^{\dagger}\begin{pmatrix}0 & \alpha\\ \alpha^* & 0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}\right|^2}{-2\varepsilon}}_{= -\frac{|\alpha|^2}{2\varepsilon}}$$

$$\rightarrow \quad E_0(\lambda) = -\varepsilon - \frac{\lambda^2 |\alpha|^2}{2\varepsilon}$$

analog: $E_1(\lambda) = -\varepsilon + \dfrac{\lambda^2 |\alpha|^2}{2\varepsilon}$

36 c) $\varphi_0(0) = \begin{pmatrix}0\\1\end{pmatrix}$, $\varphi_1(0) = \begin{pmatrix}1\\0\end{pmatrix}$

$\lambda \to +\infty$: $\varphi_0(\lambda) \to \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$, $\varphi_1(\lambda) \to \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$

$\lambda \to -\infty$: $\varphi_0(\lambda) \to \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}$, $\varphi_1(\lambda) \to \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix}$