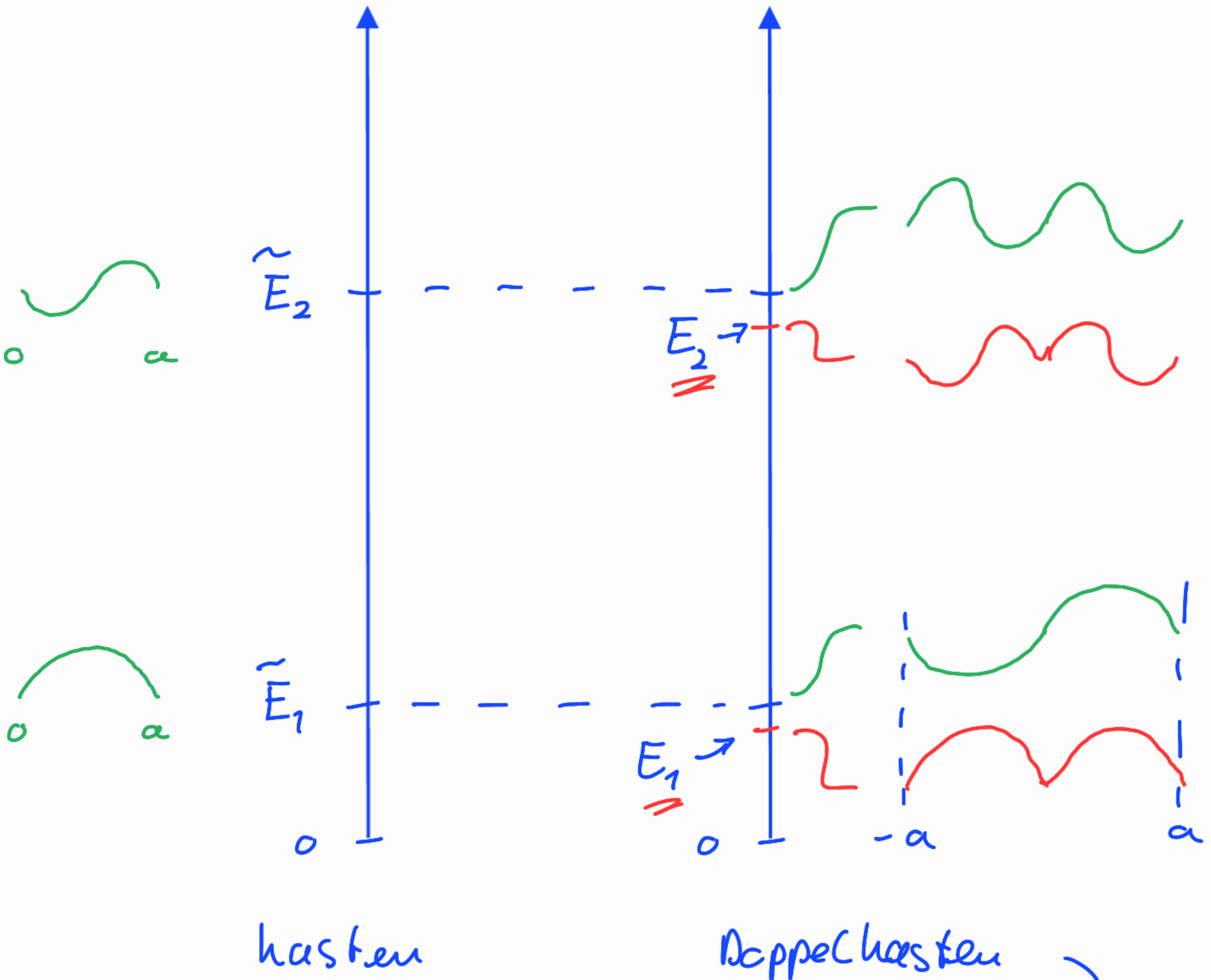
Letzte Vrlsg.: Teilchen im Doppelkasten



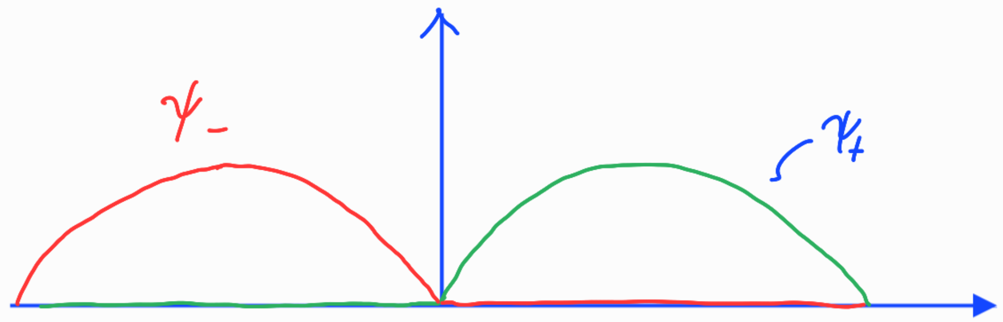
Grundzustand: $\psi_1(x) = \frac{1}{\sqrt{a}} \sin\left(\frac{\pi}{a+b}\left(|x|+b\right)\right)$

1. ang. Zustand: $\tilde\psi_1(x) = \frac{1}{\sqrt{a}} \sin\left(\frac{\pi}{a} x\right)$ $\qquad b \sim 1/u$

$$\frac{\Delta E}{E_1} = \frac{\tilde E_1 - E_1}{E_1} = \frac{2b}{a} \ll 1$$

$$\frac{\Delta E}{\hbar} = \boxed{\omega \ll \Omega} = \frac{E_1}{\hbar}$$

$\Rightarrow \quad |\psi_\pm\rangle = \frac{1}{\sqrt{2}} \left(|\psi_1\rangle \pm |\tilde{\psi}_1\rangle \right)$

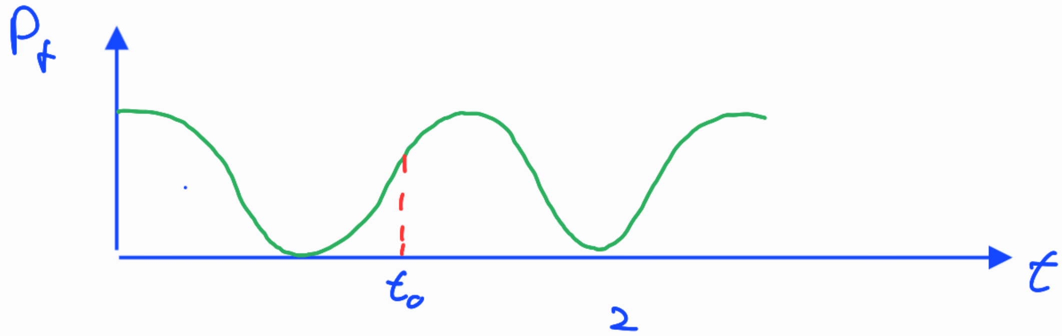


$$|\psi(0)\rangle = |\psi_+\rangle$$

$$t \quad |\psi(t)\rangle = \frac{e^{i\Omega t}}{\sqrt{2}} \left(\cos\frac{\omega t}{2} |\psi_+\rangle + i \sin\frac{\omega t}{2} |\psi_-\rangle \right)$$

Oszillation des Teilchen mit $\frac{\omega}{2}$ $(\ll \Omega)$

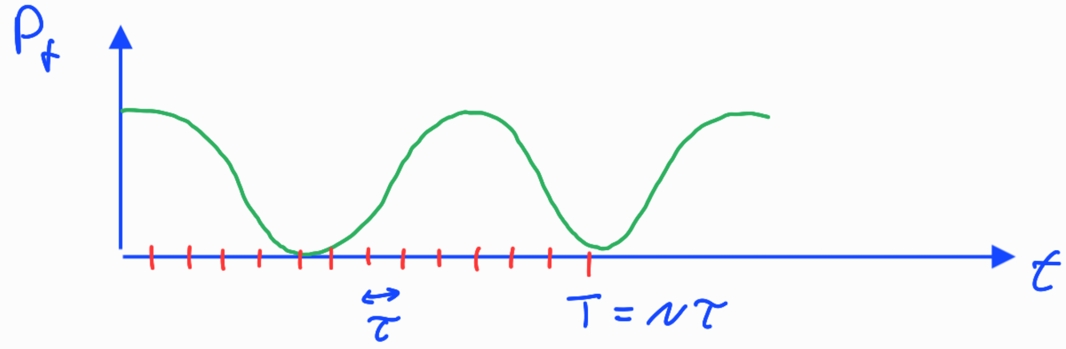
$\Rightarrow$ Maser



$$P_+(t_0) = \cos^2 \frac{\omega t_0}{2}$$

N ideale Messungen zu Zeiten

$$t_\ell = \frac{T}{N} \cdot \ell$$



Wkt., dass alle N Messungen ($|\psi_f\rangle\langle\psi_f|$) positiv.

$$P_N = |\langle\psi_f|\psi(\tau)\rangle|^{2N} = \left(\tfrac{1}{2}(1+\cos\omega\tau)\right)^N$$

$$\simeq e^{-\frac{\omega^2\tau^2}{N}} \xrightarrow[N\to\infty]{} 1 \quad !$$

hinreichend häufiges Messen hindert

Teilchen an q.m. Oszillation:

Teilchen bleibt im rechten Kasten !

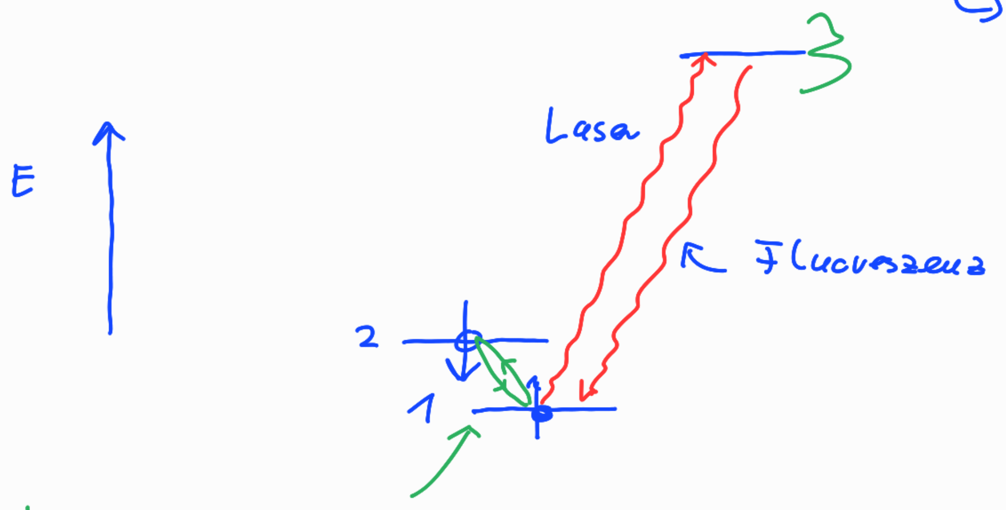
allg.: „Quanten-Zeno-Effekt"

experimenteller Nachweis 1990 durch

Itano, Heinzen, Bollinger, Wineland :

Messung an ca. 5000 Be^+ Ionen

↳ ^2S [He]

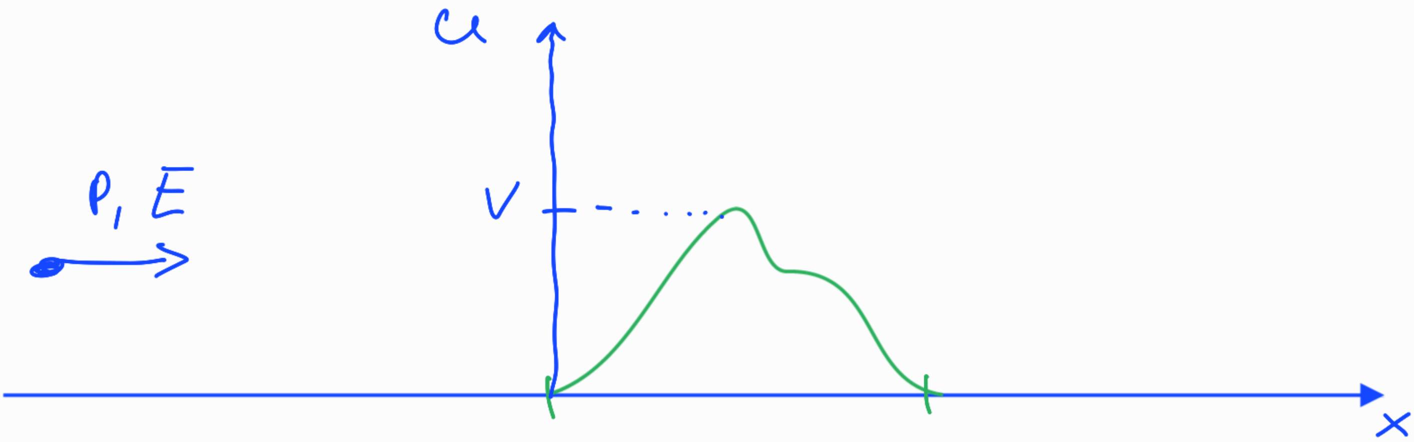


hochfreq. el. mag. Wechselfeld

Reflexion und Transmission an Potentialschwelle:

a) Tunneleffekt

b) Streuexperiment



klassisch: $E > V$: Transmission mit Wkt. 1

$E < V$: Reflexion mit Wkt. 1

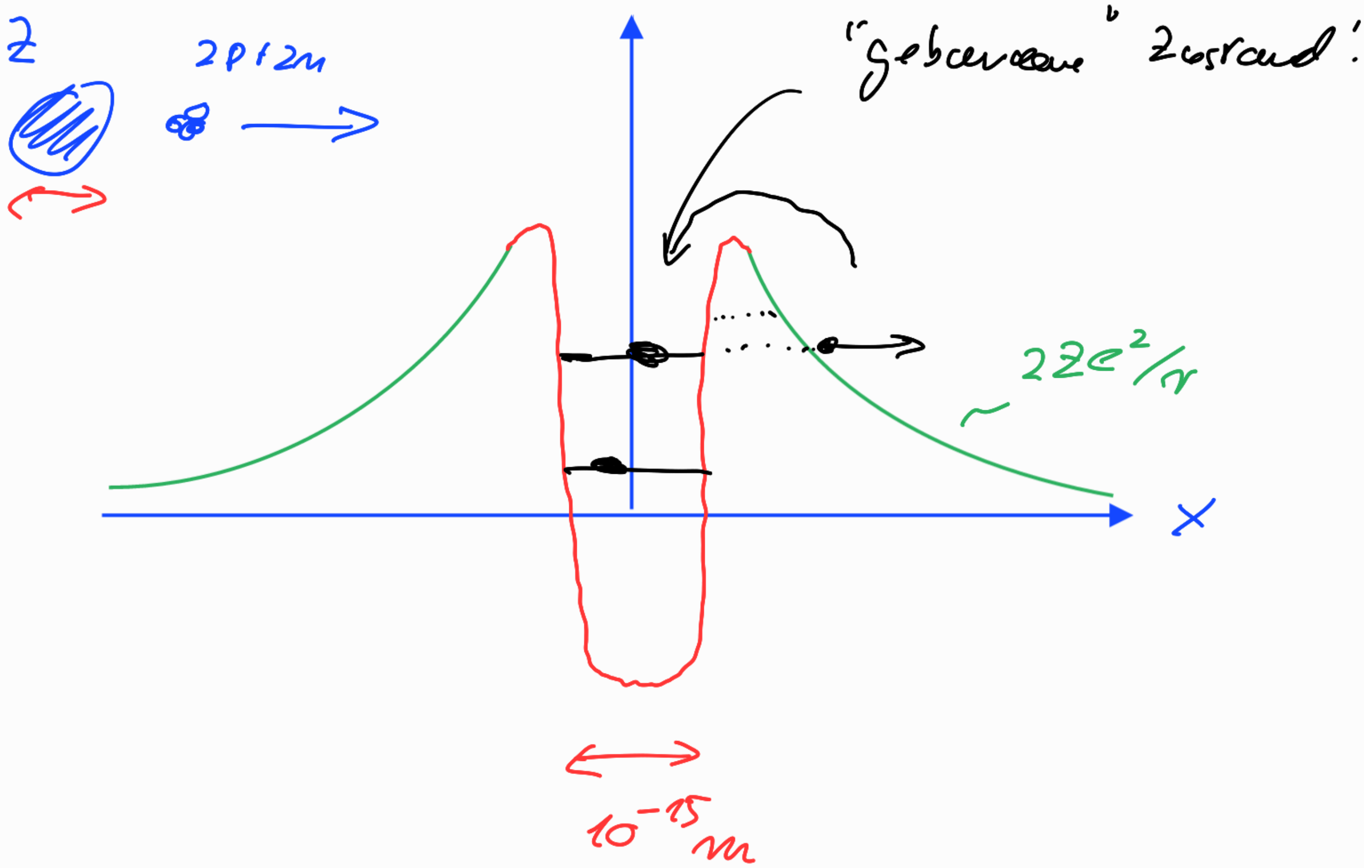
QM: : $E > V$: Transmission mit Wkt < 1

$E < V$: Transmission mit Wkt > 0 !

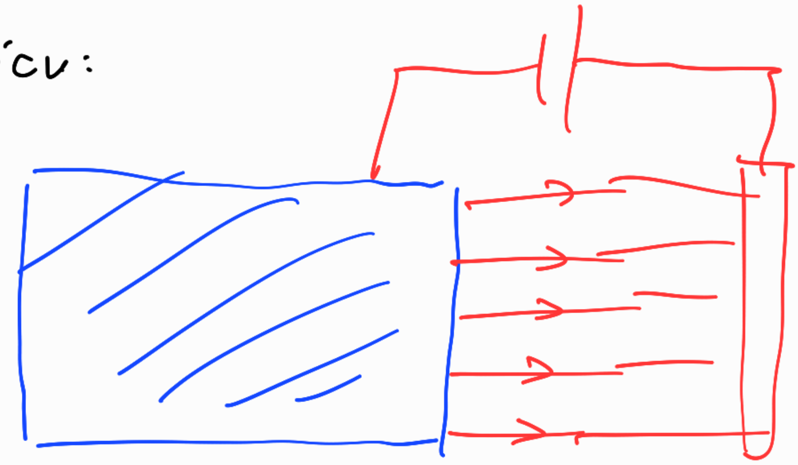
Tunneleffekt!

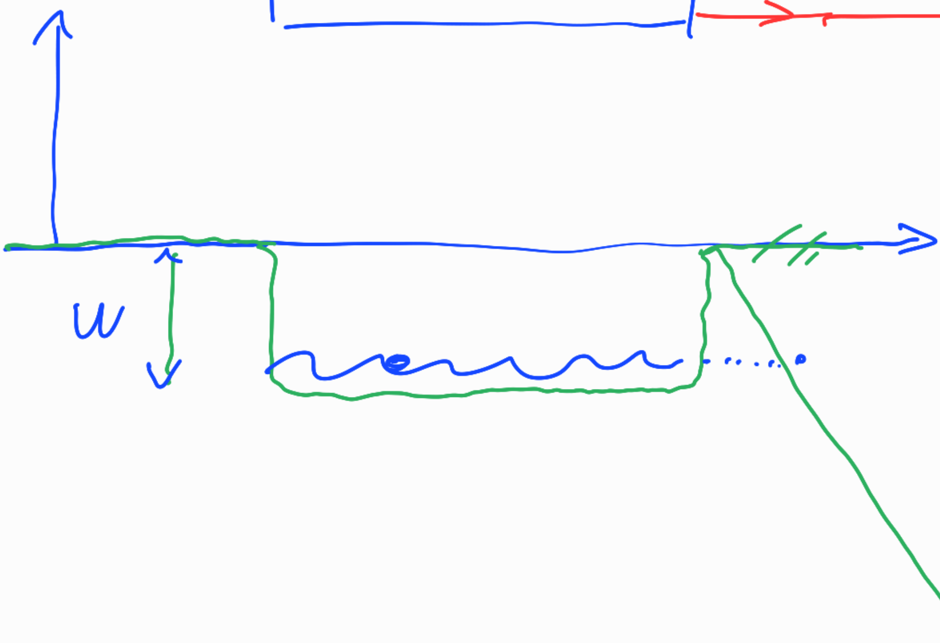
relevant z.B für: α-Zerfall, Kernfusion, ...

...., NH_3, ..., Feldemission, ...

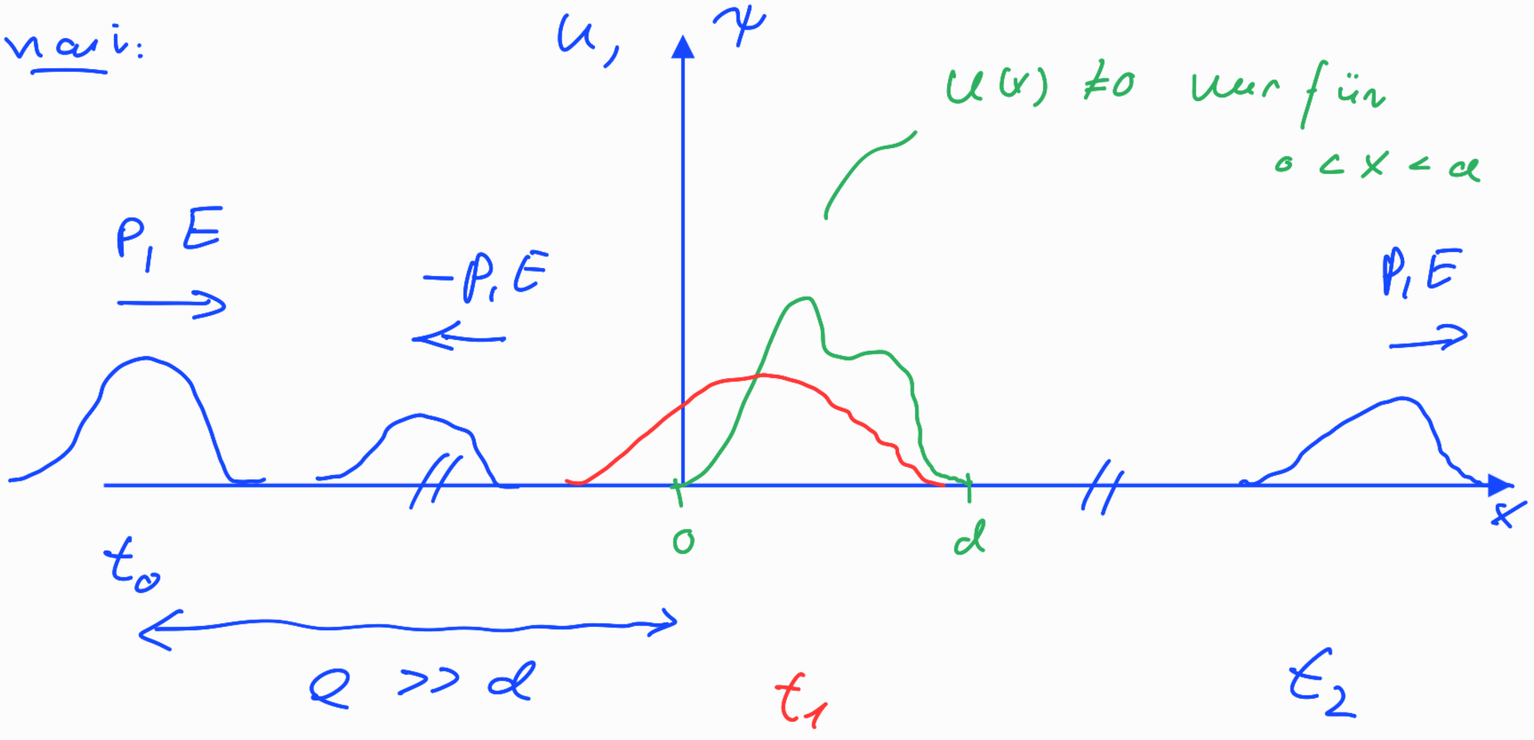


Feldemission:





naiv:



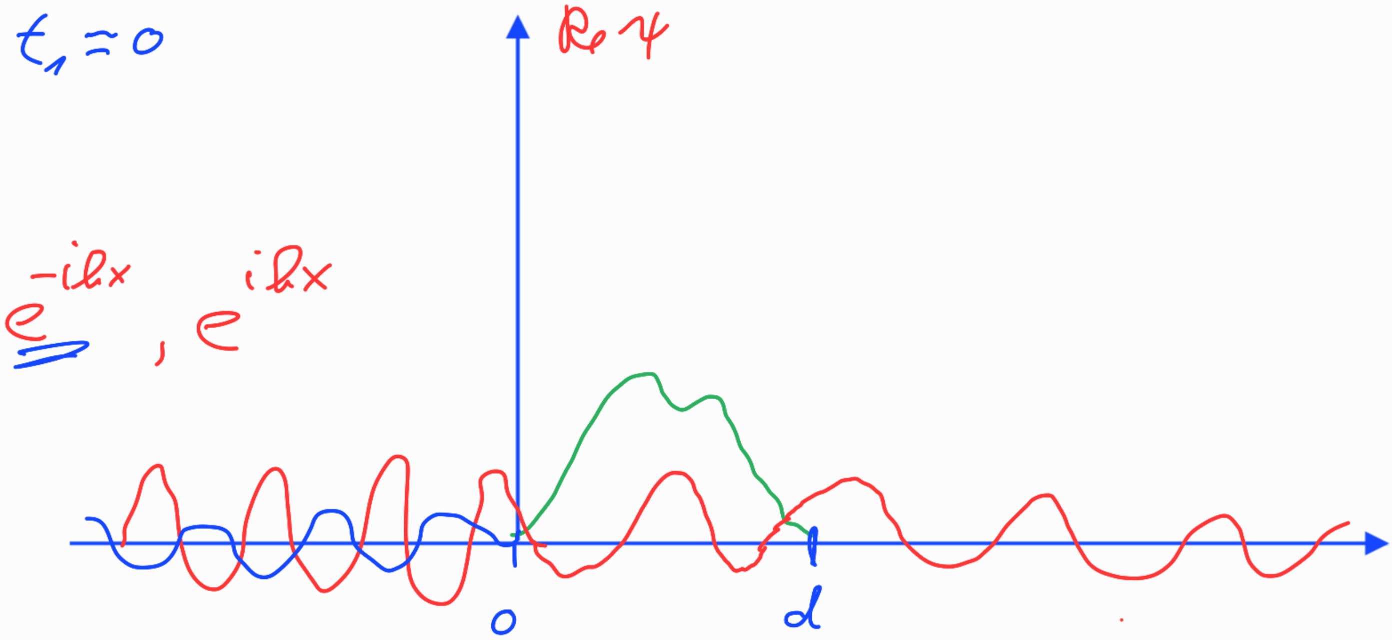
t_0 : $|\psi(t_0)\rangle$: $\cdot$ $\langle x \rangle_{t_0} = -\ell$, $\ell >> d$

$t_0 < 0$ $\cdot$ $\langle \Delta x \rangle_{t_0} = b$:

$d << b << \ell$

$\cdot$ $\langle p \rangle = \underset{\hbar k}{p}$, $\rightarrow \langle H \rangle = p^2/2m$

$$\psi(x, t_0) = \frac{1}{(2\pi b^2)^{1/4}} e^{-\frac{(x+\ell)^2}{4b^2}} e^{ikx}$$



<u>stationäres</u> Problem!

<u>Streuansatz:</u> Wellenfkt. mit 3 Anteilen:

$$\psi(x) = \begin{cases} 1\, e^{ikx} + \underline{\underline{r}}\, e^{-ikx} & : \quad x < 0 \\ \psi_0(x) & : \quad 0 < x < d \\ \underline{\underline{t}}\, e^{ikx} & : \quad x > d \end{cases}$$

$\psi_0(x)$: Lsg. der. stat. SGL:

$$\psi_0''(x) = \overbrace{\frac{2m}{\hbar^2}\left(U(x) - E\right)}^{\text{freie Param. s, u!}} \psi_0(x)$$

<u>Anschlussbedingungen:</u> $\psi(x)$ und $\psi'(x)$ stetig!

insbesondere bei <u>$x=0$ und $x=d$</u> !

p, E vorgegeben;

4 Anschlussbedingungen (Stetigkeit von ψ, ψ' in $x=0$, $x=d$)

$\to$ determinieren r, t, s, u !

physikalische Bedeutung von r, t:

$$R = |r|^2 \quad : \quad \text{Reflexionswkt.}$$

$$T = |t|^2 \quad : \quad \text{Transmissionswkt.}$$

Wahrscheinlichkeitsstromdichte:

$|\psi(x,t)|^2$ Wkt.sdichte, $\int_{\mathbb{R}} |\psi(x,t)|^2 \overset{!}{=} 1$

$$\frac{d}{dt}|\psi(x,t)|^2 = \frac{d}{dt}(\psi^* \psi) = \dot{\psi}^* \psi + \psi^* \dot{\psi}$$

$$= +\frac{i\hbar}{2m} \psi^{*''} \psi + \cancel{\frac{i}{\hbar} u(x) \psi^*} \psi - \frac{i\hbar}{2m} \psi^* \psi'' - \cancel{\frac{i}{\hbar} u(x) \psi^*} \psi$$

$$\dot{\psi} \overset{!}{=} -\frac{i}{\hbar}\left(\frac{\hbar^2}{2m} \psi'' + u(x) \psi\right)$$

$$\overset{!}{=} \frac{i\hbar}{2m} \frac{\partial}{\partial x} \left(\psi^{*'} \psi - \psi^* \psi'\right)$$

$$= -\frac{\partial}{\partial x} \boxed{\frac{\hbar}{m} \operatorname{Im}\left(\psi^* \frac{\partial \psi}{\partial x}\right)} =: j(x,t)$$

$$\rightarrow \quad \frac{d}{dt} |\psi(x,t)|^2 + \frac{\partial}{\partial x} j(x,t) \overset{!}{=} 0$$

mit Whk.stromdichte

$$\boxed{j(x,t) = \frac{\hbar}{m} \operatorname{Im} \psi^*(x,t) \frac{\partial}{\partial x} \psi(x,t)}$$

[3D:

$$\frac{d}{dt} |\psi(\vec{r},t)|^2 + \operatorname{div} \vec{j}(\vec{r},t) = 0$$

$$\vec{j}(\vec{r},t) = \frac{\hbar}{m} \operatorname{Im} \psi^*(\vec{r},t) \vec{\nabla} \psi(\vec{r},t) \quad]$$

$$\psi_<(x) = e^{ikx} + r\, e^{-ikx} \quad \rightarrow \quad j(x) = 1 - |r|^2$$

$$\psi_>(x) = t\, e^{ikx} \quad \rightarrow \quad j(x) = |t|^2$$

Whkserhaltung $\rightarrow \quad 1 - |r|^2 = |t|^2$

$\rightarrow \quad |t|^2 = T \quad$ Transmissionskoeff.