



## Appendix E: Percentage Points $\chi^2_{\alpha,\nu}$ of the Chi-Square Distribution

Let  $X_1, \dots, X_n$  be a random sample from a normal distribution with mean  $\mu$  and variance  $\sigma^2$ , and let  $S^2$  be the sample variance. Then the random variable

$$\chi^2 = \frac{(n-1)S^2}{\sigma^2}$$

has a chi-square ( $\chi^2$ ) distribution with  $n - 1$  degrees of freedom.

The probability density function of a  $\chi^2$  random variable is

$$f(x) = \frac{1}{2^{k/2} \Gamma(k/2)} x^{(k/2)-1} e^{-x/2}, \quad x > 0,$$

where  $k$  is the number of degrees of freedom. The mean and variance of the  $\chi^2$  distribution are  $k$  and  $2k$ , respectively. The limiting form of  $\chi^2$  distribution as  $k \rightarrow \infty$  is the normal distribution.

The percentage points of the  $\chi^2$  distribution are given in the following table. We define  $\chi^2_{\alpha,\nu}$  as the percent point or value of the chi-square random variable with  $\nu$  degrees of freedom such that the probability that  $\chi^2$  exceeds this value is  $\alpha$ . We can write it as

$$P(\chi^2 > \chi^2_{\alpha,\nu}) = \alpha.$$

The  $\chi^2$  distribution is skewed, and hence we need to find separate value for  $\chi^2_{1-\alpha,\nu}$  from the table.

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$\chi^2_{\alpha,\nu}$					
$\nu \backslash \alpha$	0.100	0.050	0.025	0.010	0.001
1	2.706	3.841	5.024	6.635	10.828
2	4.605	5.991	7.378	9.210	13.816
3	6.251	7.815	9.348	11.345	16.266
4	7.779	9.488	11.143	13.277	18.467
5	9.236	11.070	12.833	15.086	20.515
6	10.645	12.592	14.449	16.812	22.458
7	12.017	14.067	16.013	18.475	24.322
8	13.362	15.507	17.535	20.090	26.125
9	14.684	16.919	19.023	21.666	27.877
10	15.987	18.307	20.483	23.209	29.588
11	17.275	19.675	21.920	24.725	31.264
12	18.549	21.026	23.337	26.217	32.910
13	19.812	22.362	24.736	27.688	34.528
14	21.064	23.685	26.119	29.141	36.123
15	22.307	24.996	27.488	30.578	37.697
16	23.542	26.296	28.845	32.000	39.252
17	24.769	27.587	30.191	33.409	40.790
18	25.989	28.869	31.526	34.805	42.312
19	27.204	30.144	32.852	36.191	43.820
20	28.412	31.410	34.170	37.566	45.315
21	29.615	32.671	35.479	38.932	46.797
22	30.813	33.924	36.781	40.289	48.268
23	32.007	35.172	38.076	41.638	49.728
24	33.196	36.415	39.364	42.980	51.179
25	34.382	37.652	40.646	44.314	52.620
26	35.563	38.885	41.923	45.642	54.052
27	36.741	40.113	43.195	46.963	55.476
28	37.916	41.337	44.461	48.278	56.892
29	39.087	42.557	45.722	49.588	58.301
30	40.256	43.773	46.979	50.892	59.703
31	41.422	44.985	48.232	52.191	61.098
32	42.585	46.194	49.480	53.486	62.487
33	43.745	47.400	50.725	54.776	63.870
34	44.903	48.602	51.966	56.061	65.247
35	46.059	49.802	53.203	57.342	66.619
36	47.212	50.998	54.437	58.619	67.985
37	48.363	52.192	55.668	59.893	69.347
38	49.513	53.384	56.896	61.162	70.703
39	50.660	54.572	58.120	62.428	72.055
40	51.805	55.758	59.342	63.691	73.402
41	52.949	56.942	60.561	64.950	74.745
42	54.090	58.124	61.777	66.206	76.084
43	55.230	59.304	62.990	67.459	77.419
44	56.369	60.481	64.201	68.710	78.750
45	57.505	61.656	65.410	69.957	80.077
46	58.641	62.830	66.617	71.201	81.400

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$\chi^2_{\alpha,\nu}$					
$\nu \backslash \alpha$	0.100	0.050	0.025	0.010	0.001
47	59.774	64.001	67.821	72.443	82.720
48	60.907	65.171	69.023	73.683	84.037
49	62.038	66.339	70.222	74.919	85.351
50	63.167	67.505	71.420	76.154	86.661
51	64.295	68.669	72.616	77.386	87.968
52	65.422	69.832	73.810	78.616	89.272
53	66.548	70.993	75.002	79.843	90.573
54	67.673	72.153	76.192	81.069	91.872
55	68.796	73.311	77.380	82.292	93.168
56	69.919	74.468	78.567	83.513	94.461
57	71.040	75.624	79.752	84.733	95.751
58	72.160	76.778	80.936	85.950	97.039
59	73.279	77.931	82.117	87.166	98.324
60	74.397	79.082	83.298	88.379	99.607
61	75.514	80.232	84.476	89.591	100.888
62	76.630	81.381	85.654	90.802	102.166
63	77.745	82.529	86.830	92.010	103.442
64	78.860	83.675	88.004	93.217	104.716
65	79.973	84.821	89.177	94.422	105.988
66	81.085	85.965	90.349	95.626	107.258
67	82.197	87.108	91.519	96.828	108.526
68	83.308	88.250	92.689	98.028	109.791
69	84.418	89.391	93.856	99.228	111.055
70	85.527	90.531	95.023	100.425	112.317
71	86.635	91.670	96.189	101.621	113.577
72	87.743	92.808	97.353	102.816	114.835
73	88.850	93.945	98.516	104.010	116.092
74	89.956	95.081	99.678	105.202	117.346
75	91.061	96.217	100.839	106.393	118.599
76	92.166	97.351	101.999	107.583	119.850
77	93.270	98.484	103.158	108.771	121.100
78	94.374	99.617	104.316	109.958	122.348
79	95.476	100.749	105.473	111.144	123.594
80	96.578	101.879	106.629	112.329	124.839
81	97.680	103.010	107.783	113.512	126.083
82	98.780	104.139	108.937	114.695	127.324
83	99.880	105.267	110.090	115.876	128.565
84	100.980	106.395	111.242	117.057	129.804
85	102.079	107.522	112.393	118.236	131.041
86	103.177	108.648	113.544	119.414	132.277
87	104.275	109.773	114.693	120.591	133.512
88	105.372	110.898	115.841	121.767	134.746
89	106.469	112.022	116.989	122.942	135.978
90	107.565	113.145	118.136	124.116	137.208
100	118.498	124.342	129.561	135.807	149.449

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$\chi^2_{\alpha,\nu}$					
$\nu \backslash \alpha$	0.900	0.950	0.975	0.990	0.999
1	0.016	0.004	0.001	0.000	0.000
2	0.211	0.103	0.051	0.020	0.002
3	0.584	0.352	0.216	0.115	0.024
4	1.064	0.711	0.484	0.297	0.091
5	1.610	1.145	0.831	0.554	0.210
6	2.204	1.635	1.237	0.872	0.381
7	2.833	2.167	1.690	1.239	0.598
8	3.490	2.733	2.180	1.646	0.857
9	4.168	3.325	2.700	2.088	1.152
10	4.865	3.940	3.247	2.558	1.479
11	5.578	4.575	3.816	3.053	1.834
12	6.304	5.226	4.404	3.571	2.214
13	7.042	5.892	5.009	4.107	2.617
14	7.790	6.571	5.629	4.660	3.041
15	8.547	7.261	6.262	5.229	3.483
16	9.312	7.962	6.908	5.812	3.942
17	10.085	8.672	7.564	6.408	4.416
18	10.865	9.390	8.231	7.015	4.905
19	11.651	10.117	8.907	7.633	5.407
20	12.443	10.851	9.591	8.260	5.921
21	13.240	11.591	10.283	8.897	6.447
22	14.041	12.338	10.982	9.542	6.983
23	14.848	13.091	11.689	10.196	7.529
24	15.659	13.848	12.401	10.856	8.085
25	16.473	14.611	13.120	11.524	8.649
26	17.292	15.379	13.844	12.198	9.222
27	18.114	16.151	14.573	12.879	9.803
28	18.939	16.928	15.308	13.565	10.391
29	19.768	17.708	16.047	14.256	10.986
30	20.599	18.493	16.791	14.953	11.588
31	21.434	19.281	17.539	15.655	12.196
32	22.271	20.072	18.291	16.362	12.811
33	23.110	20.867	19.047	17.074	13.431
34	23.952	21.664	19.806	17.789	14.057
35	24.797	22.465	20.569	18.509	14.688
36	25.643	23.269	21.336	19.233	15.324
37	26.492	24.075	22.106	19.960	15.965
38	27.343	24.884	22.878	20.691	16.611
39	28.196	25.695	23.654	21.426	17.262
40	29.051	26.509	24.433	22.164	17.916
41	29.907	27.326	25.215	22.906	18.575
42	30.765	28.144	25.999	23.650	19.239
43	31.625	28.965	26.785	24.398	19.906
44	32.487	29.787	27.575	25.148	20.576
45	33.350	30.612	28.366	25.901	21.251
46	34.215	31.439	29.160	26.657	21.929

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$\chi^2_{\alpha,\nu}$					
$\nu \backslash \alpha$	0.900	0.950	0.975	0.990	0.999
47	35.081	32.268	29.956	27.416	22.610
48	35.949	33.098	30.755	28.177	23.295
49	36.818	33.930	31.555	28.941	23.983
50	37.689	34.764	32.357	29.707	24.674
51	38.560	35.600	33.162	30.475	25.368
52	39.433	36.437	33.968	31.246	26.065
53	40.308	37.276	34.776	32.018	26.765
54	41.183	38.116	35.586	32.793	27.468
55	42.060	38.958	36.398	33.570	28.173
56	42.937	39.801	37.212	34.350	28.881
57	43.816	40.646	38.027	35.131	29.592
58	44.696	41.492	38.844	35.913	30.305
59	45.577	42.339	39.662	36.698	31.020
60	46.459	43.188	40.482	37.485	31.738
61	47.342	44.038	41.303	38.273	32.459
62	48.226	44.889	42.126	39.063	33.181
63	49.111	45.741	42.950	39.855	33.906
64	49.996	46.595	43.776	40.649	34.633
65	50.883	47.450	44.603	41.444	35.362
66	51.770	48.305	45.431	42.240	36.093
67	52.659	49.162	46.261	43.038	36.826
68	53.548	50.020	47.092	43.838	37.561
69	54.438	50.879	47.924	44.639	38.298
70	55.329	51.739	48.758	45.442	39.036
71	56.221	52.600	49.592	46.246	39.777
72	57.113	53.462	50.428	47.051	40.519
73	58.006	54.325	51.265	47.858	41.264
74	58.900	55.189	52.103	48.666	42.010
75	59.795	56.054	52.942	49.475	42.757
76	60.690	56.920	53.782	50.286	43.507
77	61.586	57.786	54.623	51.097	44.258
78	62.483	58.654	55.466	51.910	45.010
79	63.380	59.522	56.309	52.725	45.764
80	64.278	60.391	57.153	53.540	46.520
81	65.176	61.261	57.998	54.357	47.277
82	66.076	62.132	58.845	55.174	48.036
83	66.976	63.004	59.692	55.993	48.796
84	67.876	63.876	60.540	56.813	49.557
85	68.777	64.749	61.389	57.634	50.320
86	69.679	65.623	62.239	58.456	51.085
87	70.581	66.498	63.089	59.279	51.850
88	71.484	67.373	36.941	60.103	52.617
89	72.387	68.249	64.793	60.928	53.386
90	73.291	69.126	65.647	61.754	54.155
100	82.358	77.929	74.222	70.065	61.918