FinalReport

Meghan Myles and Owen Fiore

Student Exam Excellence

Abstract

Standardized tests have been used to try and evaluate how much each student has learned in a classroom. However, it is possible that predetermined factors related to a student's socioeconomic status play a significant role in how student's learn and how well they perform on standardized tests. It is important to understand that although standardized tests are meant to level the playing field between students, there are many elements that can contribute to student performance on standardized tests. We created a number of models to best describe test scores in terms of a number of academic and socioeconomic variables. We found that the standardized test scores analyzed in this data set were highly dependent on socioeconomic factors. We believe that these tests are not necessarily good measures of student academic performance.

Introduction

It is important to be able to measure students' academic performance, as such measurements can help guide decisions about best educational practices at every level. They can help to identify individual students who are performing poorly in order to give them more individualized instruction. They can identify districts which are underperforming in order to better invest educational resources. They can identify subject areas in which students are consistently underperforming in order to tailor teaching methods. One method of evaluating students' academic performance is by investigating their scores on standardized exams.

It is desirable to explore factors influencing standardized test scores to determine if standardized tests are performing as they should be. Ideally we would like to see that standardized test scores are poorly correlated with socioeconomic factors and very strongly correlated with how much students study for them. This would be indicative that standardized test scores and thus learning are not impacted very much by prior factors and that students of all backgrounds have the chance to succeed.

There have been a number of investigations into the interactions between socioeconomic factors and student academic performance. According to the NIH, around half of people who grew up in a high-income family have a bachelor's degree by the age of 25, while only about 10% of people from low-income families have a bachelor's degree by 25. In addition, there tend to be fewer AP classes offered in schools which traditionally serve low-income and minority students (National Institutes of Health). In addition, on certain standardized tests, some groups tend to consistently outperform others. On the SAT II writing exam, the average white student outperformed the average student of color (Thomas 2004). A University of South Florida study showed that students' SAT and ACT scores were not predictive of their college academic performance (Micceri 2010). In a 2009 study, Black and latino students performed lower on the MCAT, on average, than white students (Davis 2013). It is clear that standardized test scores are not necessarily only indicative of academic success - it seems that they also capture variation introduced by socioeconomic factors.

We will examine a dataset detailing students' standardized test scores, as well as measures for several socioeconomic factors and student dedication to academics. In doing so, we will be able to determine whether or not this standardized test is appropriate for measuring true student achievement.

Data Description:

Here is a link to the kaggla data: https://www.kaggle.com/datasets/desalegngeb/students-exam-scores

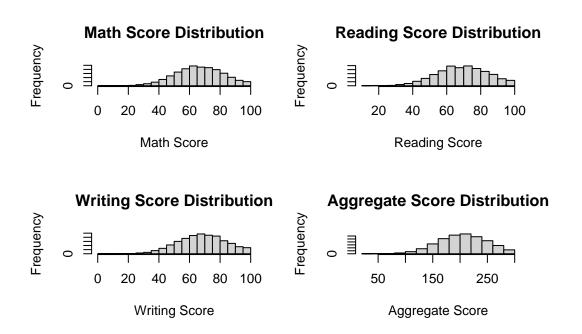
- Gender: Gender of the student (male or female)
- EthnicGroup: Ethnic group of the student (nominal groups from A to E)
- ParentEduc: Parent education background: (ordinal groups from some high school to master's)
- LunchType: Cost of school lunch (free/reduced or standard)
- TestPrep: Whether the student completed a test preparation course (none or completed)
- ParentMaritalStatus: Student's parent's marital status(nominal groups: married, single, widowed, divorced)
- PracticeSport: How often a student plays a sport (ordinal groups from never, sometimes, regularly)
- IsFirstChild: If the student is the eldest child in their family (yes or no)
- NrSibling: number of siblings the student has
- TransportMeans: how the student gets to school (schoolbus or private)
- WklyStudyHours: how much the student self-studies (ordinal groups from 0-5, 5-10, and 10+ hours)

MathScore, ReadingScore, WritingScore are the student's math, reading, and writing scores respectively and serve as the response variable in this dataset. Scores range from 0 to 100 with 0 indicating low performance and 100 indicating perfect.

We will treat the variables Gender, EthnicGroup, ParentEduc, ParentMaritalStatus, Practice-Sport, NrSibling, and TransportMeans as indicators of students' socioeconomic status. We will treat the variables TestPrep and WklyStudyHours as indicators of students' academic dedication.

As this dataset is fictional, it is impossible to perform any additional research on the background of the town this data came from or provide more context.

Here are histograms showing the distribution of Math, Writing, Reading, and Aggregate Scores



Goal

Our goal is to determine whether or not standardized tests are performing well in measuring academic performance. Our research questions are as follows:

Do math, reading, and writing scores on this standardized test correlate with each other? Do performances on the tests correlate with socioeconomic factors, such as highest parental education level or receiving a free/reduced lunch? Do performances on the tests correlate with indicators of academic commitment, such as studying for many hours weekly and completing test preparation courses? Do test scores vary significantly across gender and ethnic groups? In all, does it appear that each of these tests are measuring true academic performance, or are they heavily influenced by other factors?

By regressing test scores on the other variables available in this dataset, we hope to be able to determine answers to these questions. We will determine whether standardized test performance is more highly correlated with socioeconomic factors or academic dedication. This will help us to determine whether or not the tests are appropriate measures of academic performance.

Methods

Generalized Linear Model

In this paper we will test out the use of several different models of varying complexities to try and best fit our data, starting with a linear model. Regression diagnostics showed that the model was over-predicting the achievement of students who should be doing really well. Thus we decided to implement a Generalized Linear Model to try and remedy this. A generalized linear model has a random component, systematic component, and a link function.

The random component is a probability density function from an exponential family:

 $f(y;\theta,\phi)=\exp\frac{y\theta-b(\theta)}{a(\phi)+c(y,\phi)}$ where θ and $\phi>0$ are scalar parameters, and $a,\,b,\,c$, are known functions. The form of a Gaussian distribution with an identity link is:

 $f(y\|\mu,\sigma^2)=\frac{1}{\sqrt{(2\pi\sigma^2)}}\exp(-\frac{(y-\mu)^2}{2\sigma^2})$. Where μ is the mean of the distribution and σ^2 is the variance

The systematic component is the main part of the model and is a generalized form of a linear model such that for the systematic component: η_i we have:

$$\eta_i = \beta_0 + \sum_{j=1}^p \beta_j X_{ij}$$

To obtain this part, we implemented stepwise selection through the step R function in both directions, allowing both forward and backwards selection. Forward or backwards selection is a variable selection algorithm that looks for the most significant predictors and returns a model when there cannot be any additional significant variables added. For each predictor, step selection tests $H_0: \beta=0$ versus $H_a: \beta\neq 0$ to determine whether the predictor is significant and should be added to the model. This process will continue until convergence, at which point adding or removing additional variables would either induce variance, bias, or multicollinearity. The algorithm returned that the following parameters should be included in the model: Male, ParentEduc, LunchDiscount, TestPrep, PracticeSport, IsFirstChild, WklyStudyHours, and all variables of the form EthnicGroup. Thus our model looks like:

$$\eta_i = \beta_0 + \beta_1 Male + \beta_2 ParentEduc + ... + \beta_{11} EthnicGroupE$$

The final part of a GLIM is the link function which has the form:

$$g(\mu_i) = \eta_i$$

Where g() is a continuous function that maps the mean response μ_i to its systematic component η_i . Several link functions were considered, but the one that showed the best results when utilizing a normal quantile-quantile plot was a Gaussian family with identity link, so that will be the GLIM presented throughout the rest of the paper. Additionally, a Gaussian family with identity link is highly interpretable, as the coefficients can be interpreted as how they appear in the model.

Random Forests

In addition to the linear model methods described above, we wanted to implement more complex methods through more sophisticated models. Random Forests are an ensemble learning method that use a collection of decision trees to create optimal predictions. Decision trees typically work using the CART algorithm, which is a greedy algorithm that minimizes the number of incorrectly classified cases, typically measured by the Gini index of a node. Eventually stopping conditions will be met based on a number of potential factors, but the one used in this paper is minimum node size. As random forests are made up of a number of decision trees, the number of trees is a hyper-parameter in the input of the random forest. As we have multiple hyperparameters that we want to control: number of trees and minimum node size, we will implement grid search cross validation in order to iterate through possible combinations of hyperparameters to find the optimal hyperpaperameters. After running the search, we concluded that the optimal number of trees was 900 and that we should have a minimum of 25 observations in each node. Additionally, we will need to analyze how important each variable was in building the random forests. Random forests are a collection of trees, and tree based methods work by splitting at the most decisive points to try and maximize the separation between children nodes. Thus, variables that are indicated to be important are significant predictors in AggregateScore, as the predicted score will change decently based on whether an observation has that particular feature or not. While we hope to see that socio-economic factors do not play a large role in performance, we know from prior reading that is not likely to be the case. Nonetheless, this analysis is important and aims to motivate further questions about the quality of standardized tests.

Results

MathScore ReadingScore WritingScore AggregateScore

```
MathScore
               1.0000000
                             0.8189865
                                           0.8085328
                                                          0.9205491
ReadingScore
               0.8189865
                             1.0000000
                                           0.9526215
                                                          0.9694976
WritingScore
               0.8085328
                             0.9526215
                                           1.0000000
                                                          0.9665601
AggregateScore 0.9205491
                                                          1.0000000
                             0.9694976
                                           0.9665601
```

We have four predictor variables in our dataset: MathScore, ReadingScore, WritingScore, and AggregateScore which is the sum of the previous three. The correlation results show that the scores are generally very highly correlated and thus it seems reasonable for this section to only use AggregateScore, as the results should generalize well to the other three scores. This will help to keep the results concise as any results that can be applied to AggregateScore should be able to be applied to the other variables. Logically, it also does not seem reasonable that many of the predictors such as LunchType would be good at predicting MathScore but not ReadingScore.

Now that we have chose AggregateScore as our predictor, we can split our data into 80% training and 20% testing.

Linear Model Results

Now, we are ready to implement a linear model. Several simpler models were considered without exponential families, but were excluded for the purpose of keeping the Results section concise, but extended work and methodology can be found in the Supplementary Files folder under "Models.Rmd".

We can see how the fit is with all predictor in the model.

Call:

```
glm(formula = AggregateScore ~ Male + ParentEduc + LunchDiscount +
    TestPrep + PracticeSport + IsFirstChild + NrSiblings + PrivateTransport +
    WklyStudyHours + EthnicGroupA + EthnicGroupB + EthnicGroupC +
    EthnicGroupD + EthnicGroupE + ParentDivorced + ParentMarried +
    ParentSingle + ParentWidowed, family = gaussian(link = "identity"),
    data = df.train)
```

```
Coefficients: (2 not defined because of singularities)
                 Estimate Std. Error t value Pr(>|t|)
(Intercept)
                               2.7631 73.321 < 2e-16 ***
                 202.5949
Male
                 -11.4011
                               0.5950 - 19.163
                                               < 2e-16 ***
ParentEduc
                               0.1994 33.329
                    6.6454
                                               < 2e-16 ***
LunchDiscount
                 -29.4773
                               0.6234 - 47.288
                                               < 2e-16 ***
TestPrep
                  20.3328
                               0.6248 32.543
                                               < 2e-16 ***
PracticeSport
                               0.4463
                   3.5921
                                        8.049 8.96e-16 ***
IsFirstChild
                    0.9390
                               0.6254
                                        1.501
                                                 0.133
NrSiblings
                               0.2073
                                        0.724
                                                 0.469
                   0.1501
PrivateTransport
                               0.6039 -0.180
                  -0.1088
                                                 0.857
WklyStudyHours
                               0.3885
                                                 0.141
                    0.5714
                                        1.471
EthnicGroupA
                 -26.3222
                               1.3425 - 19.606
                                               < 2e-16 ***
EthnicGroupB
                 -25.8734
                               1.0288 -25.148
                                               < 2e-16 ***
                                               < 2e-16 ***
EthnicGroupC
                 -22.6545
                               0.9491 -23.869
EthnicGroupD
                 -13.0246
                               0.9831 -13.249
                                               < 2e-16 ***
EthnicGroupE
                                                     NA
                       NA
                                   NA
                                           NA
ParentDivorced
                  -0.4883
                               2.2864
                                      -0.214
                                                  0.831
ParentMarried
                  -1.0581
                               2.2052
                                       -0.480
                                                 0.631
ParentSingle
                  -1.5822
                               2.2535
                                       -0.702
                                                  0.483
ParentWidowed
                       NA
                                   NA
                                           NA
                                                     NA
```

Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1

(Dispersion parameter for gaussian family taken to be 1350.116)

Null deviance: 28262312 on 15283 degrees of freedom Residual deviance: 20612220 on 15267 degrees of freedom

AIC: 153559

Number of Fisher Scoring iterations: 2

We see there are too many parameters in the model, as several such as IsFirstChild and NrSiblings are not significant. Additionally, we do not to include some variables that were one-hot encoded such as EthnicGroupE and ParentWidowed, as if an observation is not in Ethnic Groups A-D then it is known that person must have come from Ethnic Group E. There is almost certainly multicollinearity, which will have to be resolved later.

Next, we will use step selection to remove poor indicating variables, while hoping to attain similar residual plots.

```
step_model <- step(all, direction = "both", trace = 0)
summary(step_model)</pre>
```

Call:

```
glm(formula = AggregateScore ~ Male + ParentEduc + LunchDiscount +
    TestPrep + PracticeSport + IsFirstChild + WklyStudyHours +
    EthnicGroupA + EthnicGroupB + EthnicGroupC + EthnicGroupD,
    family = gaussian(link = "identity"), data = df.train)
```

Coefficients:

	${\tt Estimate}$	Std. Error	t value	Pr(> t)	
(Intercept)	201.8452	1.6488	122.422	< 2e-16	***
Male	-11.4038	0.5946	-19.178	< 2e-16	***
ParentEduc	6.6478	0.1993	33.351	< 2e-16	***
LunchDiscount	-29.4815	0.6233	-47.301	< 2e-16	***
TestPrep	20.3223	0.6246	32.536	< 2e-16	***
PracticeSport	3.5873	0.4462	8.040	9.66e-16	***
IsFirstChild	0.8918	0.6207	1.437	0.151	
WklyStudyHours	0.5765	0.3883	1.484	0.138	
EthnicGroupA	-26.3258	1.3420	-19.616	< 2e-16	***
EthnicGroupB	-25.8893	1.0286	-25.169	< 2e-16	***
EthnicGroupC	-22.6609	0.9488	-23.883	< 2e-16	***
${\tt EthnicGroupD}$	-13.0288	0.9828	-13.257	< 2e-16	***
Signif. codes:	0 '***'	0.001 '**'	0.01 '*	0.05 '.'	0.1 ' ' 1

(Dispersion parameter for gaussian family taken to be 1349.867)

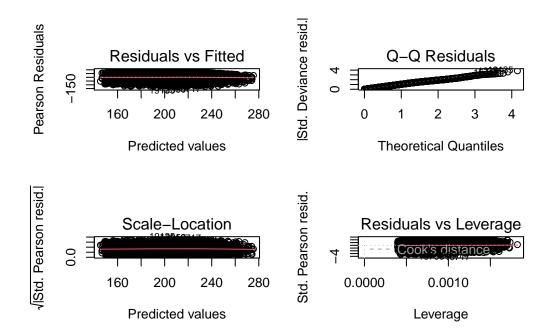
Null deviance: 28262312 on 15283 degrees of freedom Residual deviance: 20615168 on 15272 degrees of freedom

AIC: 153552

Number of Fisher Scoring iterations: 2

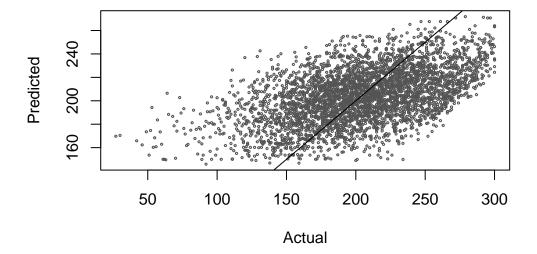
The step algorithm was able to remove many of the insignificant predictors and able to cut down the list of predictors to only the most important. It is worth noting that although there are some variables that are technically not significant at $\alpha=0.05$, the coefficient estimates and standard errors show that the variables are still adding value.

```
par(mfrow = c(2,2))
plot(step_model)
```



As we hoped to see, the residual plots all still appear to be strong, indicating that there are no outlier points exerting undue influence on the model, and no particularly high leverage points as well. Although the normal q-q plot shows that some values are underfitted, it is reasonable given that we are working with standardized test data where there is a maximum score (300 as this is AggregateScore we are modeling) and thus even the students who have the highest probability to score extremely high are bounded by an upper score and thus the observed values are lower than the theoretical.

We can now see how well we did by validating on the test data set.



While there was error, it appears to be normally distributed around the y=x line, and helps to show that the model fit is good. Now we need to see if the step selection reduced multicollinearity and if there were any issues with variables having a high VIF.

```
car::vif(step_model)
```

Male	ParentEduc	LunchDiscount	TestPrep	PracticeSport
1.000506	1.000538	1.000517	1.000465	1.000544
IsFirstChild	WklyStudyHours	${\tt EthnicGroupA}$	${\tt EthnicGroupB}$	${\tt EthnicGroupC}$
1.000412	1.000772	1.416668	1.944399	2.220065
${\tt EthnicGroupD}$				
2.099234				

There is no evidence to suggest there is multicollinearity, as many values are very close to 1, with the exception of the Ethnic Groups which are one-hot encoded and thus dependent on each other. These results show that the step function did a good job of selecting the correct variables to be used.

```
mse <- mean((test_results$Actual - test_results$Predicted)^2)
mse</pre>
```

[1] 1501.613

The mean squared error of the GLIM is 1501.613. We can now compare these results to what we get when implementing a random forest.

Random Forest Results

Ranger result

Call:

ranger(AggregateScore ~ Male + ParentEduc + LunchDiscount + TestPrep + PracticeSport +

Type: Regression

Number of trees: 500
Sample size: 15284
Number of independent variables: 12
Mtry: 3
Target node size: 5

Variable importance mode: impurity
Splitrule: variance
00B prediction error (MSE): 1372.105
R squared (00B): 0.2580269

We can run a random forest with many default parameters that use 500 trees and a target node size of 5, but we also expect that we can improve results by changing hyperparameters. Using the methods described above we can use a brute force algorithm to find the best combinations of hyperparameters in hopes of finding something with a lower Out Of Bag prediction error, which is represented by mean squared error. This model is reported as best_model below as to save space we have hidden the code chunk from the rendered pdf version, but the code will appear in the qmd file.

```
# The best model is stored in 'best_model' with the lowest OOB error
print(best_model)
```

Ranger result

Call:

ranger(AggregateScore ~ Male + ParentEduc + LunchDiscount + TestPrep + PracticeSport +

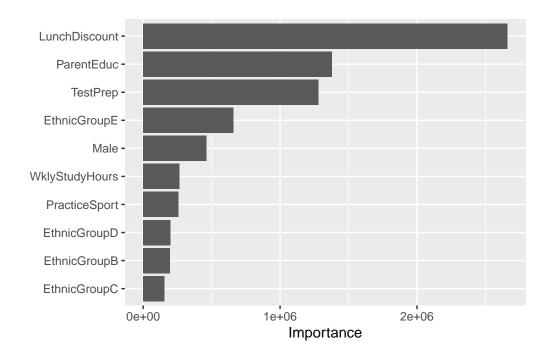
Type: Regression

Number of trees: 900
Sample size: 15284
Number of independent variables: 12
Mtry: 3
Target node size: 25

Variable importance mode: impurity
Splitrule: variance
00B prediction error (MSE): 1368.377
R squared (00B): 0.2600428

We see that the best model has 900 trees and has a target node size of 25 rather than 5. Despite these changes, we unfortunately do not see a huge change in the mean squared error, as the improvement was quite small. Nonetheless, the mean squared error from this model was 1368.4 which is an improvement over the error produced by the GLIM, which was 1501.6. In order to get a better understanding for the effectiveness of the random forest, it is helpful to look at variable importance. The graph below summarizes the importance of each variable that was included in the random forest.

```
importance <- vi(best_model)
vip(importance)</pre>
```



We see that LunchDiscount is by far the most important variable, followed by ParentEduc and then TestPrep, indicating that the largest factor in predicting student performance in a random forest is based on whether or not a school qualifies for free or reduced lunch. Having reduced or free lunch is a measure of the income of the student's family, and thus it is reasonable to conclude that family income is a major factor in determining AggregateScore. We can confirm these results by viewing the output of the GLIM, and after doing so we see that the coefficient on LunchDiscount is -29.5, which means that we expect to see, on average a student that gets free/reduced lunch score 29.5 points lower than somebody who does not.

Summary and Conclusion

We were able to achieve our goals in our analysis of this data set. Our first research question concerned whether or not math, reading, and writing scores on this standardized test correlate with each other. We found that the correlation between each of these variables is greater than 0.8, and that the correlation between each of these variables and the constructed AggregateScore variable was above 0.9. Therefore, it seemed reasonable to us to assume that AggregateScore is a good predictor of math, reading, and writing scores on this standardized test.

Our second research question concerned whether test scores were influenced by socioeconomic factors and employed a GLIM and random forest to do so. The coefficients on the GLIM were informative to our understanding of the variables that impact AggregateScore and are

highly interpretable. The coefficient on Male was -11.4, indicating that on average, males are expected to score 11.4 points lower than females. The coefficient on LunchDiscount was -29.5, such that students who were on free or reduced lunch would be expected to score 29.5 points lower. Students on free or reduced lunch come from poor families, meaning that students who are poorer are expected to score lower on standardized tests. This leads us to question the validity of standardized exams, if students coming from marginalized communities are less likely to score high than their more affluent peers.

This is a massive decrease and a very important conclusion from our data. ParentEduc was encoded as an ordinal variable with ranges from 1: some high school and 6: master's, and as the coefficient was 6.65, we expect an increase in AggregateScore of 6.65 points for every one level increase in parental education. This is a drastic difference and can indicate that a parent's education can have a dramatic impact on the child's education as well. The results from the random forests variable importance show that the two most important features were ParentEduc and LunchDiscount, both socioeconomic variables. Thus, we have answered our research question by concluding that socioeconomic variables are highly significant to predicting standardized test scores.

Our third research question concerned whether test scores were influenced by educational factors. The GLIM output shows that the coefficient on TestPrep was 20.32, indicating that those who completed the test prep course scored on average, 20.3 points higher on AggregateScore than those who did not. Additionally, TestPrep was ranked as the third most important feature by the random forest, showing that this factor is important. However, this variable may be considered as both a socioeconomic and educational factor if the test preparation course is prohibitively expensive. While the variable WklyStudyHours was not significant in the model at $\alpha=0.05$, the coefficient was 0.5765 which indicates that there may be a slight positive correlation between studying and AggregateScore. It would have been informative if we had more information about the student's academics or grades in school as a measure of their education level, as this data set really only has two variables that can measure whether standardized tests are affected by academic measures. It appears that the only measured academic variable with a significant effect on test scores is whether or not test preparation was completed.

Our fourth research question was to find the variables which have the greatest effect on test scores. According to our random forest model, the order of variable importance is as follows (we have placed each variable's classification as a primarily socioeconomic or primarily academic within parentheses):

- LunchDiscount (socioeconomic)
- ParentEduc (socioeconomic)
- TestPrep (academic)
- Being a member of EthnicGroupE (socioeconomic)
- Gender (socioeconomic)
- WklyStudyHours (academic)
- PracticeSport (socioeconomic)

- Being a member of EthnicGroupD (socioeconomic)
- Being a member of EthnicGroupB (socioeconomic)
- Being a member of EthnicGroupC (socioeconomic)
- IsFirstChild (socioeconomic)
- Being a member of EthnicGroupA (socioeconomic)

In this way, we achieved our goal of determining which variables were most important in determining test scores.

Our final research question was to make a determination about whether these tests are good measures of academic performance, or whether they are too heavily influenced by other factors. We can see that of the five most important variables, four are socioeconomic. Although there were more socioeconomic variables included in this study than academic ones, this seems to be an indication that socioeconomic factors were generally more important in predicting test scores than academic ones. For this reason, we believe that these standardized tests are not necessarily good measures of student academic success. Instead, we believe that they are too dependent on socioeconomic factors to be an effective measure.

Moving forward, these results could be more informative if the data set was more informational. While there were solid statistics about the student's background and home-life, there were no statistics about a student's grades or academic performance on non-standardized tests. These could have been extremely informative and could have keyed us into important insights to see if poorer students are learning less or scoring worse on exams. It is possible that standardized tests are not measuring learning well if questions are worded improperly or geared towards students from more affluent backgrounds, and somebody who had access to data that showed that poorer students perform well in class but not in standardized tests would be able to make these assumptions. As we do not have this data however, we cannot.

Overall we enjoyed the linear modeling more, as the results are more interpretable and easier to understand than the random forest model which only is computing how important factors are. However, random forests may give better predictions, so if we were more focused on trying to predict a student's exam scores based on their background, random forests would likely give better results.

A github repository for this project has been setup at https://github.com/ofiore/STAT5405Project which contains additional resources and visualizations.

References

Davis, Dwight MD; Dorsey, J. Kevin MD, PhD; Franks, Ronald D. MD; Sackett, Paul R. PhD; Searcy, Cynthia A. PhD; Zhao, Xiaohui PhD. Do Racial and Ethnic Group Differences in Performance on the MCAT Exam Reflect Test Bias?. Academic Medicine 88(5):p 593-602, May 2013. | DOI: 10.1097/ACM.0b013e318286803a

"Individuals from Disadvantaged Backgrounds." National Institutes of Health, U.S. Department of Health and Human Services, extramural-diversity.nih.gov/diversity-matters/disadvantaged-backgrounds.

Micceri, Theodore. "Assessing the Usefulness of SAT and Act Tests in Minority Admissions." Online Submission, Institute of Educational Sciences, 31 Dec. 2009, eric.ed.gov/?id=ED510111.

Thomas, M.K. (2004), The SAT II: Minority/Majority Test-Score Gaps and What They Could Mean for College Admissions. Social Science Quarterly, 85: 1318-1334. https://doi.org/10.1111/j.0038-4941.2004.00278.x