SELECTED TOPICS IN NUMBER THEORY 1 - 106926
SPRING 2024

Instructor
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Office hours: by appointment

Course topics
The course will be about *analytic and probabilistic number theory*. We shall learn about some of the major tools and results in the area.
Among the tools we'll see are *sieve theory*, which is helpful in estimating the number of primes in a given set, and the *circle method*, which is helpful in counting solutions to equations in prime variables.
Some of the results we'll see are *Dirichlet's theorem* on the infinitude of primes in arithmetic progressions, and the *Erdős–Kac theorem* which says that the number of prime factors of a random integer follows a Gaussian distribution.
We'll also concern ourselves with the analogy between integers, polynomials over finite fields, and permutations. For ease of presentation, we shall sometimes prove results for polynomials instead of integers, and occasionally assume the generalized Riemann hypothesis.

Syllabus
(1) Mertens' and Chebyshev' theorem on prime sums.
(2) Multiplicative functions: Möbius function, nonnegative multiplicative functions, Dirichlet hyperbola method.
(3) Dirichlet's theorem on primes in arithmetic progressions.
(4) Additive functions: Hardy–Ramanujan's theorem, Turan–Kubilius's inequality, Erdős–Kac theorem.
(5) Sieve theory: Pure Brun sieve, Selberg sieve. Applications to prime number theory.
(6) The circle method. Applications to Vinogradov's three-prime theorem and other problems.
(7) The Poisson–Dirichlet distribution in number theory.

Resources
"Multiplicative Number Theory I" by Montgomery and Vaughan
On-line lecture notes:
- Kevin Ford's lecture notes on "Sieve methods" and on "Anatomy of integers and random permutations"
- Steve Lester and Zeev Rudnick's notes on "Sieve Theory and its applications"
- Adam Harper's lecture notes on "Probabilistic Number Theory" and "Elementary Methods in Analytic Number Theory"
- Terry Tao's lecture notes on "Analytic prime number theory"

Remarks
The course will be given in English.

Schedule
Monday 14:30–15:30
Thursday 9:30–11:30

Grading policy
In the end of the course, a home exam will be given. A few days will be given for solving it.

Homework
Non-mandatory homework assignments will be given during the semester.