Question 2

Let $X_{3\times3}$ be a single channel image and $K_{2\times1}$ some kernel. Denote $Y = X * K$ where $*$ is the convolution operator and let C be some cost function where $\frac{\partial C}{\partial y_{i,j}}$ has been already computed.

We can see that if

$$X = \begin{pmatrix} x_{1,1} & x_{1,2} & x_{1,3} \\ x_{2,1} & x_{2,2} & x_{2,3} \\ x_{3,1} & x_{3,2} & x_{3,3} \end{pmatrix}, \qquad K = \begin{pmatrix} k_1 \\ k_2 \end{pmatrix}$$

then

$$Y = \begin{pmatrix} k_1x_{1,1} + k_2x_{2,1} & k_1x_{1,2} + k_2x_{2,2} & k_1x_{1,3} + k_2x_{2,3} \\ k_1x_{2,1} + k_2x_{3,1} & k_1x_{2,2} + k_2x_{3,2} & k_1x_{2,3} + k_2x_{3,3} \end{pmatrix}$$

$$\Rightarrow y_{i,j} = k_1x_{i,j} + k_2x_{(i+1),j}$$

so

$$\frac{\partial y_{i,j}}{\partial k_p} = x_{(i+p-1),j} \tag{$*$}$$

and

$$\frac{\partial y_{i,j}}{\partial x_{p,q}} = \begin{cases} k_1, & p = i \wedge q = j \\ k_2, & p = i+1 \wedge q = j \\ 0, & otherwise \end{cases} \tag{$**$}$$

So, using the chain rule, we get that

$$\frac{\partial C}{\partial k_p} = \sum_{i=1}^{2}\sum_{j=1}^{3}\frac{\partial C}{\partial y_{i,j}}\cdot\frac{\partial y_{i,j}}{\partial k_p}, \qquad \frac{\partial C}{\partial x_{i,j}} = \sum_{i=1}^{2}\sum_{j=1}^{3}\frac{\partial C}{\partial y_{i,j}}\cdot\frac{\partial y_{i,j}}{\partial x_{i,j}}$$

substituting $(*)$, $(**)$ in the last two equations, we get our solution.