Attempt 01

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Assume that the assertion $2n = P_1 + P_2$ is false $\forall n$, for $n \in \mathbb{N}$, $P_1, P_2 - prime$. Then we can say that there exists 4 prime numbers $P_{11}, P_{12}, P_{22} P_{21}$ such that

$$P_{11} + P_{12} + D_{11} = 2n = P_{21} + P_{22} - D_{12} \tag{1}$$

with $D_{11}, D_{12} > 0$ the smallest number satisfying the equalities 1. We note that $P_{21} + P_{22} - (P_{11} + P_{12}) > 0$. We will attempt to prove the converse, namely $P_{21} + P_{22} - (P_{11} + P_{12}) < 0$.

Subtracting 2, we can say that there exists $P_{31}, P_{32}, P_{41}, P_{42}$ primes, such that

$$P_{31} + P_{32} + D_{21} = 2n - 2 = P_{41} + P_{42} - D_{22} \tag{2}$$

equating, we get

$$\begin{aligned} P_{31} + P_{32} + D_{21} + 2 &= P_{11} + P_{12} + D_{11} & (3) \\ P_{41} + P_{42} - D_{22} + 2 &= P_{21} + P_{22} - D_{12} & (4) \end{aligned}$$

subtracting the equations

$$P_{21}+P_{22}-D_{12}-D_{11}-(P_{11}+P_{12}) = P_{41}+P_{42}-D_{22}+2-2-D_{21}-(P_{31}+P_{32}) \tag{5}$$

$$P_{21} + P_{22} - (P_{11} + P_{12}) = P_{41} + P_{42} - (P_{31} + P_{32}) - D_{22} + D_{11} + D_{12} - D_{21} \tag{6}$$

There are two cases to consider (1)$P_{32} + P_{32} < P_{11} + P_{12}$, in this case the difference D_{11} must equal 1, and $P_{41} + P_{42} = P_{21} + P_{22}$ and hence $D_{22} = 1$ as well. substituting in the equation

$$P_{21} + P_{22} - (P_{11} + P_{12}) = P_{41} + P_{42} - (P_{31} + P_{32}) - 1 + 1 + D_{12} - D_{21} \tag{7}$$

replacing the terms

$$P_{21}+P_{22}-(P_{11}+P_{12}) = 2n-2+D_{22}-(2n-2-D_{21})+D_{12}-D_{21} = D_{22}+D_{12} = 1+D_{12}$$