

Attempt 01

February 15, 2015

Assume that the assertion  $2n = P_1 + P_2$  is false  $\forall n$ , for  $n \in \mathbb{N}$ ,  $P_1, P_2$  – prime. Then we can say that there exists 4 prime numbers  $P_{11}, P_{12}, P_{22}, P_{21}$  such that

$$P_{11} + P_{12} + D_{11} = 2n = P_{21} + P_{22} - D_{12} \quad (1)$$

with  $D_{11}, D_{12} > 0$  the smallest number satisfying the equalities 1. We note that  $P_{21} + P_{22} - (P_{11} + P_{12}) > 0$ . We will attempt to prove the converse, namely  $P_{21} + P_{22} - (P_{11} + P_{12}) < 0$ .

Subtracting 2, we can say that there exists  $P_{31}, P_{32}, P_{41}, P_{42}$  primes, such that

$$P_{31} + P_{32} + D_{21} = 2n - 2 = P_{41} + P_{42} - D_{22} \quad (2)$$

equating, we get

$$P_{31} + P_{32} + D_{21} + 2 = P_{11} + P_{12} + D_{11} \quad (3)$$

$$P_{41} + P_{42} - D_{22} + 2 = P_{21} + P_{22} - D_{12} \quad (4)$$

subtracting the equations

$$P_{21} + P_{22} - D_{12} - D_{11} - (P_{11} + P_{12}) = P_{41} + P_{42} - D_{22} + 2 - 2 - D_{21} - (P_{31} + P_{32}) \quad (5)$$

$$P_{21} + P_{22} - (P_{11} + P_{12}) = P_{41} + P_{42} - (P_{31} + P_{32}) - D_{22} + D_{11} + D_{12} - D_{21} \quad (6)$$

There are two cases to consider (1)  $P_{31} + P_{32} < P_{11} + P_{12}$ , in this case the difference  $D_{11}$  must equal 1, and  $P_{41} + P_{42} = P_{21} + P_{22}$  and hence  $D_{22} = 1$  as well. substituting in the equation

$$P_{21} + P_{22} - (P_{11} + P_{12}) = P_{41} + P_{42} - (P_{31} + P_{32}) - 1 + 1 + D_{12} - D_{21} \quad (7)$$

replacing the terms

$$P_{21} + P_{22} - (P_{11} + P_{12}) = 2n - 2 + D_{22} - (2n - 2 - D_{21}) + D_{12} - D_{21} = D_{22} + D_{12} = 1 + D_{12}$$