

### Example 1:

rights = [1,2,4] # different rights  
valuations = [[11, 11, 22, 33, 44], [11, 22, 44, 55, 66], [11, 33, 22, 11, 66]] # different objects  
y = 0.5

#### Iteration 1:

1. Initialization:  
players\_chosen\_objects = [0,0,0]  
remaining\_objects = [0,1,2,3,4]
2. Computing: player's right / his current number of objects + y:
  - player 0:  $1 / (0 + 0.5) = 2$
  - player 1:  $2 / (0 + 0.5) = 4$
  - player 2:  $4 / (0 + 0.5) = 8 \leftarrow$  The player with the highest portion
3. The player with highest portion is the one that will choose now:  
His valuations: [11, 33, 22, 11, 66] so object 4 is the most object he wants.  
Now we will remove object 4: remaining\_objects = [0,1,2,3]  
Update the valuations: [[11, 11, 22, 33, 0], [11, 22, 44, 55, 0], [11, 33, 22, 11, 0]]  
Also update the number of objects player 2 took: players\_chosen\_objects = [0,0,1]

#### Iteration 2:

1. Initialization:  
remaining\_objects = [0,1,2,3]  
valuations = [[11, 11, 22, 33, 0], [11, 22, 44, 55, 0], [11, 33, 22, 11, 0]]  
players\_chosen\_objects = [0,0,1]  
rights = [1,2,4]
2. Computing: player's right / number of objects he chosen + y
  - player 0:  $1 / (0 + 0.5) = 2$
  - player 1:  $2 / (0 + 0.5) = 4 \leftarrow$  The player with the highest portion
  - player 2:  $4 / (1 + 0.5) = 2.667$
3. The player with highest portion is the one that will choose now:  
His valuations: [11,22,44,55,0] so object 3 is the most object he wants.  
Now we will remove object 3: remaining\_objects = [0,1,2]  
Update the valuations: [[11, 11, 22, 0, 0], [11, 22, 44, 0, 0], [11, 33, 22, 0, 0]]  
Also update the number of objects player 1 took: players\_chosen\_objects = [0,1,1]

### Iteration 3:

1. **Initialization:**  
remaining\_objects = [0,1,2]  
valuations = [[11, 11, 22, 0, 0], [11, 22, 44, 0, 0], [11, 33, 22, 0, 0]]  
players\_chosen\_objects = [0,1,1]  
rights = [1,2,4]
2. **Computing: player's right / number of objects he chosen + y**
  - player 0:  $1 / (0 + 0.5) = 2$
  - player 1:  $2 / (1 + 0.5) = 1.333$
  - player 2:  $4 / (1 + 0.5) = 2.667 \leftarrow \text{The player with the highest portion}$
3. **The player with highest portion is the one that will choose now:**  
His valuations: [11, 33, 22, 0, 0] so object 1 is the most object he wants.  
Now we will remove object 1: remaining\_objects = [0,2]  
Update the valuations: [[11, 0, 22, 0, 0], [11, 0, 44, 0, 0], [11, 0, 22, 0, 0]]  
Also update the number of objects player 2 took: players\_chosen\_objects = [0,1,2]

### Iteration 4:

1. **Initialization:**  
remaining\_objects = [0,2]  
valuations = [[11, 0, 22, 0, 0], [11, 0, 44, 0, 0], [11, 0, 22, 0, 0]]  
players\_chosen\_objects = [0,1,2]  
rights = [1,2,4]
2. **Computing: player's right / number of objects he chosen + y**
  - player 0:  $1 / (0 + 0.5) = 2 \leftarrow \text{The player with the highest portion}$
  - player 1:  $2 / (1 + 0.5) = 1.333$
  - player 2:  $4 / (2 + 0.5) = 1.6$
3. **The player with highest portion is the one that will choose now:**  
His valuations: [11, 0, 22, 0, 0] so object 2 is the most object he wants.  
Now we will remove object 2: remaining\_objects = [0]  
Update the valuations: [[11, 0, 0, 0, 0], [11, 0, 0, 0, 0], [11, 0, 0, 0, 0]]  
Also update the number of objects player 0 took: players\_chosen\_objects = [1,1,2]

### Iteration 5:

1. **Initialization:**  
remaining\_objects = [0]  
valuations = [[11, 0, 0, 0, 0], [11, 0, 0, 0, 0], [11, 0, 0, 0, 0]]  
players\_chosen\_objects = [1,1,2]  
rights = [1,2,4]

2. **Computing: player's right / number of objects he chosen + y**
  - player 0:  $1 / (1 + 0.5) = 0.666$
  - player 1:  $2 / (1 + 0.5) = 1.333$
  - player 2:  $4 / (2 + 0.5) = 1.6 \leftarrow \text{The player with the highest portion}$
3. **The player with highest portion is the one that will choose now:**  
 His valuations: [11, 0, 0, 0, 0] so object 0 is the most object he wants.  
 Now we will remove object 0: remaining\_objects = []  
 Update the valuations: [[0, 0, 0, 0, 0], [0, 0, 0, 0, 0], [0, 0, 0, 0, 0]]  
 Also update the number of objects player 2 took: players\_chosen\_objects = [1,1,3]

Once there are no remaining objects the algorithm stops.

#### Results:

Player 0 takes:

- item 2 with value 22

Player 1 takes:

- item 3 with value 55

Player 2 takes:

- item 4 with value 66
- item 1 with value 33
- item 0 with value 11

#### Example 2:

rights = [1,1,1] *# equal rights*

valuations = [[10, 10, 10], [10, 10, 10], [10, 10, 10]] *# same objects*

y = 1

#### Iteration 1:

1. **Initialization:**  
 players\_chosen\_objects = [0,0,0]  
 remaining\_objects = [0,1,2]
2. **Computing: player's right / number of objects he chosen + y**
  - player 0:  $1 / (0 + 1) = 1 \leftarrow \text{The player with the highest portion}$
  - player 1:  $1 / (0 + 1) = 1$
  - player 2:  $1 / (0 + 1) = 1$
3. **The player with highest portion is the one that will choose now:**  
 His valuations: [10, 10, 10] so object 0 is the most object he wants.  
 Now we will remove object 0: remaining\_objects = [1,2]  
 Update the valuations: [[0, 10, 10], [0, 10, 10], [0, 10, 10]]  
 Also update the number of objects player 0 took: players\_chosen\_objects = [1,0,0]

### Iteration 2:

#### 1. Initialization:

remaining\_objects = [1,2]  
valuations = [[0, 10, 10], [0, 10, 10], [0, 10, 10]]  
players\_chosen\_objects = [1,0,0]  
rights = [1,1,1]

#### 2. Computing: player's right / number of objects he chosen + y:

- player 0:  $1 / (1+1) = 0.5$
- player 1:  $1 / (0+1) = 1 \leftarrow$  *The player with the highest portion*
- player 2:  $1 / (0+1) = 1$

#### 3. The player with highest portion is the one that will choose now:

His valuations: [0, 10, 10] so object 1 is the most object he wants.  
Now we will remove object 1: remaining\_objects = [2]  
Update the valuations: [[0, 0, 10], [0, 0, 10], [0, 0, 10]]  
Also update the number of objects player 1 took: players\_chosen\_objects = [1,1,0]

### Iteration 3:

#### 1. Initialization:

remaining\_objects = [2]  
valuations = [[0, 0, 10], [0, 0, 10], [0, 0, 10]]  
players\_chosen\_objects = [1,1,0]  
rights = [1,1,1]

#### 2. Computing: player's right / number of objects he chosen + y:

- player 0:  $1 / (1+1) = 0.5$
- player 1:  $1 / (1+1) = 0.5$
- player 2:  $1 / (0+1) = 1 \leftarrow$  *The player with the highest portion*

#### 3. The player with highest portion is the one that will choose now:

His valuations: [0, 0, 10] so object 0 is the most object he wants.  
Now we will remove object 0: remaining\_objects = []  
Update the valuations: [[0, 0, 0], [0, 0, 0], [0, 0, 0]]  
Also update the number of objects player 2 took: players\_chosen\_objects = [1,1,1]

Once there are no remaining objects the algorithm stops.

### Results:

Player 0 takes:

- item 0 with value 10

Player 1 takes:

- item 1 with value 10

Player 2 takes:

- item 2 with value 10

### Example 3:

rights = [1,2,3] *# different rights*  
valuations = [[10, 10, 10, 10, 10], [10, 10, 10, 10, 10], [10, 10, 10, 10, 10]] *# same objects*  
y = 0.5

#### Iteration 1:

1. **Initialization:**  
players\_chosen\_objects = [0,0,0]  
remaining\_objects = [0,1,2,3,4]
2. **Computing: player's right / number of objects he chosen + y:**
  - player 0:  $1 / (0 + 0.5) = 2$
  - player 1:  $2 / (0 + 0.5) = 4$
  - player 2:  $3 / (0 + 0.5) = 6 \leftarrow$  *The player with the highest portion*
3. **The player with highest portion is the one that will choose now:**  
His valuations: [10, 10, 10, 10, 10] so object 0 is the most object he wants.  
Now we will remove object 0: remaining\_objects = [1,2,3,4]  
Update the valuations: [[0, 10, 10, 10, 10], [0, 10, 10, 10, 10], [0, 10, 10, 10, 10]]  
Also update the number of objects player 2 took: players\_chosen\_objects = [0,0,1]

#### Iteration 2:

1. **Initialization:**  
remaining\_objects = [1,2,3,4]  
valuations = [[0, 10, 10, 10, 10], [0, 10, 10, 10, 10], [0, 10, 10, 10, 10]]  
players\_chosen\_objects = [0,0,1]  
rights = [1,2,3]
2. **Computing: player's right / number of objects he chosen + y:**
  - player 0:  $1 / (0 + 0.5) = 2$
  - player 1:  $2 / (0 + 0.5) = 4 \leftarrow$  *The player with the highest portion*
  - player 2:  $3 / (1 + 0.5) = 2$
3. **The player with highest portion is the one that will choose now:**  
His valuations: [0, 10, 10, 10, 10] so object 1 is the most object he wants.  
Now we will remove object 0: remaining\_objects = [2,3,4]  
Update the valuations: [[0, 0, 10, 10, 10], [0, 0, 10, 10, 10], [0, 0, 10, 10, 10]]  
Also update the number of objects player 1 took: players\_chosen\_objects = [0,1,1]

#### Iteration 3:

1. **Initialization:**  
remaining\_objects = [2,3,4]  
valuations = [[0, 0, 10, 10, 10], [0, 0, 10, 10, 10], [0, 0, 10, 10, 10]]  
players\_chosen\_objects = [0,1,1]  
rights = [1,2,3]

2. **Computing: player's right / number of objects he chosen + y:**
  - player 0:  $1 / (0 + 0.5) = 2 \leftarrow$  *The player with the highest portion*
  - player 1:  $2 / (1 + 0.5) = 1.333$
  - player 2:  $3 / (1 + 0.5) = 2$
3. **The player with highest portion is the one that will choose now:**  
 His valuations: [0, 0, 10, 10, 10] so object 2 is the most object he wants.  
 Now we will remove object 2: remaining\_objects = [3,4]  
 Update the valuations: [[0, 0, 0, 10, 10], [0, 0, 0, 10, 10], [0, 0, 0, 10, 10]]  
 Also update the number of objects player 0 took: players\_chosen\_objects = [1,1,1]

#### Iteration 4:

1. **Initialization:**  
 remaining\_objects = [3,4]  
 valuations = [[0, 0, 0, 10, 10], [0, 0, 0, 10, 10], [0, 0, 0, 10, 10]]  
 players\_chosen\_objects = [1,1,1]  
 rights = [1,2,3]
2. **Computing: player's right / number of objects he chosen + y:**
  - player 0:  $1 / (1 + 0.5) = 0.666$
  - player 1:  $2 / (1 + 0.5) = 1.333$
  - player 2:  $3 / (1 + 0.5) = 2 \leftarrow$  *The player with the highest portion*
3. **The player with highest portion is the one that will choose now:**  
 His valuations: [0, 0, 0, 10, 10] so object 3 is the most object he wants.  
 Now we will remove object 3: remaining\_objects = [4]  
 Update the valuations: [[0, 0, 0, 0, 10], [0, 0, 0, 0, 10], [0, 0, 0, 0, 10]]  
 Also update the number of objects player 2 took: players\_chosen\_objects = [1,1,2]

#### Iteration 5:

1. **Initialization:**  
 remaining\_objects = [4]  
 valuations = [[0, 0, 0, 0, 10], [0, 0, 0, 0, 10], [0, 0, 0, 0, 10]]  
 players\_chosen\_objects = [1,1,2]  
 rights = [1,2,3]
2. **Computing: player's right / number of objects he chosen + y:**
  - player 0:  $1 / (1 + 0.5) = 0.666$
  - player 1:  $2 / (1 + 0.5) = 1.333 \leftarrow$  *The player with the highest portion*
  - player 2:  $3 / (2 + 0.5) = 1.2$
3. **The player with highest portion is the one that will choose now:**  
 His valuations: [0, 0, 0, 0, 10] so object 4 is the most object he wants.  
 Now we will remove object 4: remaining\_objects = []  
 Update the valuations: [[0, 0, 0, 0, 0], [0, 0, 0, 0, 0], [0, 0, 0, 0, 0]]  
 Also update the number of objects player 1 took: players\_chosen\_objects = [1,2,2]

Once there are no remaining objects the algorithm stops.

**Results:**

Player 0 takes:

- item 2 with value 10

Player 1 takes:

- item 1 with value 10
- item 4 with value 10

Player 2 takes:

- item 0 with value 10
- item 3 with value 10