Homework 1

Instructions

1) You are free to discuss the questions with your classmates, but if I notice the same answer is copied all the copiers will get "0" for that question.
2) I prefer typed reports (MS Word, LaTeX etc.) Please be consistent in notations. You can use the following notations:
    - lower case bold for vector ($\boldsymbol{q} = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T$)
    - lower case regular for scalar (q_1),
    - capital letter for matrix (C)
    - lower case right subscript/superscript for the reference frame notations (C_{ab}, $\boldsymbol{\omega}_{ab}^{b}$).
3) Provide the algorithms that you coded in the submitted package.
4) Due date is 6th November (Sunday) until the midnight (23:59).
5) Submit your reports on ODTUClass.
6) Title of your report file (or zipped package) should be: AE486_2022_Name_Surname_HW1

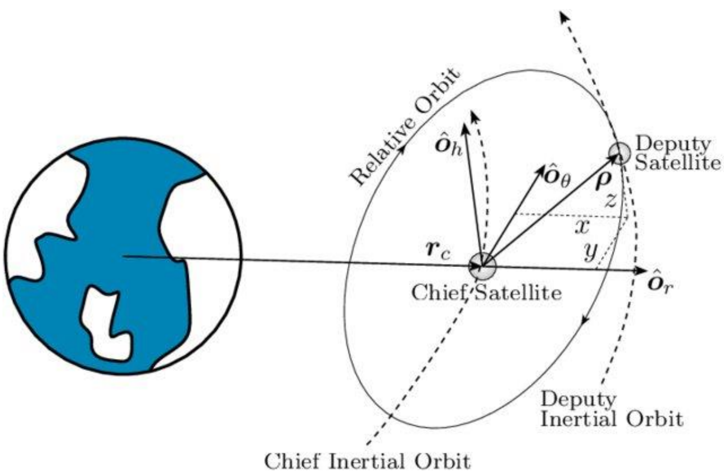
Preliminaries

The "hw1_data.mat" file shared within the HW package includes:

W_act: Actual angular velocity vector of the spacecraft, showing the body frame's angular velocity with respect to the inertial frame and resolved in the body frame. ($\boldsymbol{\omega}_{bi}^{b}$). Data is sampled at every second and there is a total of 25000s data.

Questions

1) (**25pts**) A useful frame for formation flying applications is the "Hill frame" shown in below figure. The frame is given by $\{o_r, o_\theta, o_h\}$, where o_r points in the chief spacecraft's radial direction, o_h is along the chief orbit momentum vector and o_θ completes the right-handed coordinate system. Using the dot product approach fully derive the attitude matrix that rotates vectors from the Hill frame to an inertial frame. Also, determine the relationship between the Hill frame and the orbit frame.



2) **(25pts)** Write a MATLAB function which calculates the quaternion vector from the DCM. Test your function by generating 3 random DCMs and compare with MATLAB's own build-in function of *dcm2quat*. Be careful that *dcm2quat* outputs quaternion vector such that the first term is the scalar one. For calculations you can refer to the lecture slides or "Markley and Crassidis, Fundamentals of Spacecraft Attitude Determination and Control, page 48, Eq. 2.135".

   Propose a second way for calculating the same quaternion vector and code another function. Hint: Quaternion relationship with Euler Axis/Angle representation can be a nice starting point. Compare your results with the first code.

3) (**20pts**) Starting from an identity attitude matrix, which is showing the attitude of the body frame with respect to the inertial frame ($A_{bi} = I_3$), propagate the DCM matrix using the DCM kinematics equation. Obtain quaternion terms at each second using a DCM-to-quaternion function that you wrote. Plot each components of the quaternion vector.

4) **(30pts)** This time use the quaternion kinematics equation to directly propagate the quaternion vector starting from the same initial attitude in Q1. Use the same discrete integration method in Q1 (e.g. Euler method, Runge Kutta 4th Order). However, normalize the quaternion vector after each iteration. Compare your results with those you obtained in Q1.

   Obviously because of the quaternion normalization, there will be some difference in between Q1 and Q2 results. Have a look at the angular velocity terms. Would you expect larger difference if the angular velocities are higher? Why? Explain in your own words. How can you apply the constraint on DCM? Check your results once more after implementing your constraint.

Good luck!
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