ORTA DOĞU TEKNİK ÜNİVERSİTESİ
MIDDLE EAST TECHNICAL UNIVERSITY

Aerospace Engineering Department

AEE486: Spacecraft Dynamics

Homework 2

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Contents

1 Question 1

In this part, we are asked to obtain magnetic field vector in orbital frame by using given magnetic field vector in the ECI frame. For orbit frame calculations, we are given positions and velocity vectors in ECI frame. To obtain orbit frame, we need,

$$A_{OI} = \begin{bmatrix} \mathbf{O}_1 & \mathbf{O}_2 & \mathbf{O}_3 \end{bmatrix} \tag{1.1}$$

I used given velocity and position vector and got the orbit frame elements. We know,

$$\mathbf{O}_3 = \frac{-\mathbf{r}_i}{||\mathbf{r}_i||} \tag{1.2a}$$

$$\mathbf{O}_2 = \frac{-\mathbf{r}_i \times \mathbf{v}_i}{||\mathbf{r}_i \times \mathbf{v}_i||} \tag{1.2b}$$

For first vector, I took cross product of second and third one and completed the triad.

$$\mathbf{O}_1 = \mathbf{O}_2 \times \mathbf{O}_3 \tag{1.2c}$$

I have the attitude matrix now, hence I have all the three elements.

Now, we are asked to calculate magnetic vector field using dipole model. We know,

$$B_1(t) = \frac{M_e}{r_0^3}\left\{cos(\omega_0 t)[cos(\epsilon)sin(i) - sin(\epsilon)cos(i)cos(\omega_e t)] - sin(\omega_0 t)sin(\epsilon)sin(\omega_e t)\right\} \tag{1.3a}$$

$$B_2(t) = \frac{M_e}{r_0^3}\left[cos(\epsilon)cos(i) + sin(\epsilon)sin(i)cos\omega_e t\right) \tag{1.3b}$$

$$B_3(t) = \frac{2M_e}{r_0^3}\left\{sin(\omega_0 t)[cos(\epsilon)sin(i) - sin(\epsilon)cos(i)cos(\omega_e t)] - 2sin(\omega_0 t)sin(\epsilon)sin(\omega_e t)\right\} \tag{1.3c}$$

Now, I have magnetic field vector from dipole model.

$$\mathbf{M}_o' = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix} \tag{1.4}$$

- $M_e = 7.71 \times 10^{15}\ Wb.m$ (The magnetic dipole moment of the Earth)
- $i = 98.1245\ deg$ (The inclination of Orbit)
- $\mu = 3.986004418 \times 10^{14}\ m^3 s^{-2}$

- $\omega_e = 7.29 \times 10^{-5}\ rad/sec$ (The spin rate of the Earth)
- $r = 7056198.5\ m$ (Distance)
- $\omega_0 = \sqrt{\frac{\mu}{r^3}}$

I plotted the two magnetic field vectors of IGRF data and dipole model. Also, I shifted the time of dipole model approach with 4360 second further to match the plots.

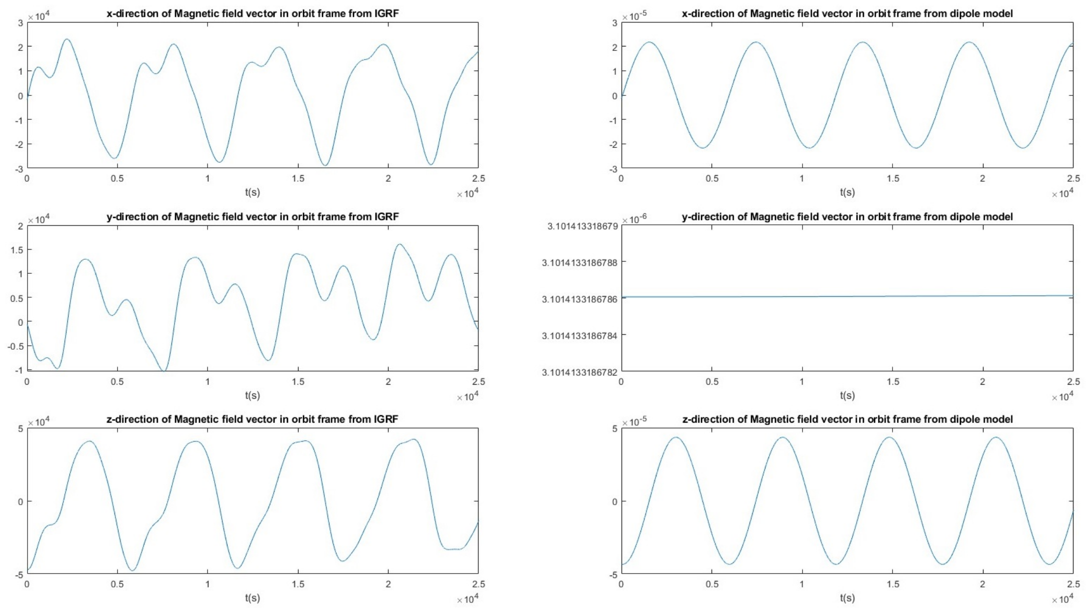

Figure 1.1: Comparison of Magnetic Field Vectors of IGRF data and Dipole Model

As we can see in Figure 1.1, both x and z directions, results are very similar to each other but in y direction, magnetic field vector from dipole model has an error. I think this because of the approach of dipole model.

2 Question 2

In this part, we are asked to obtain sun direction vector in orbit frame. Since we have transformation matrix for inertial frame to orbit frame, we can easily get sun direction vector in inertial frame by multiply.

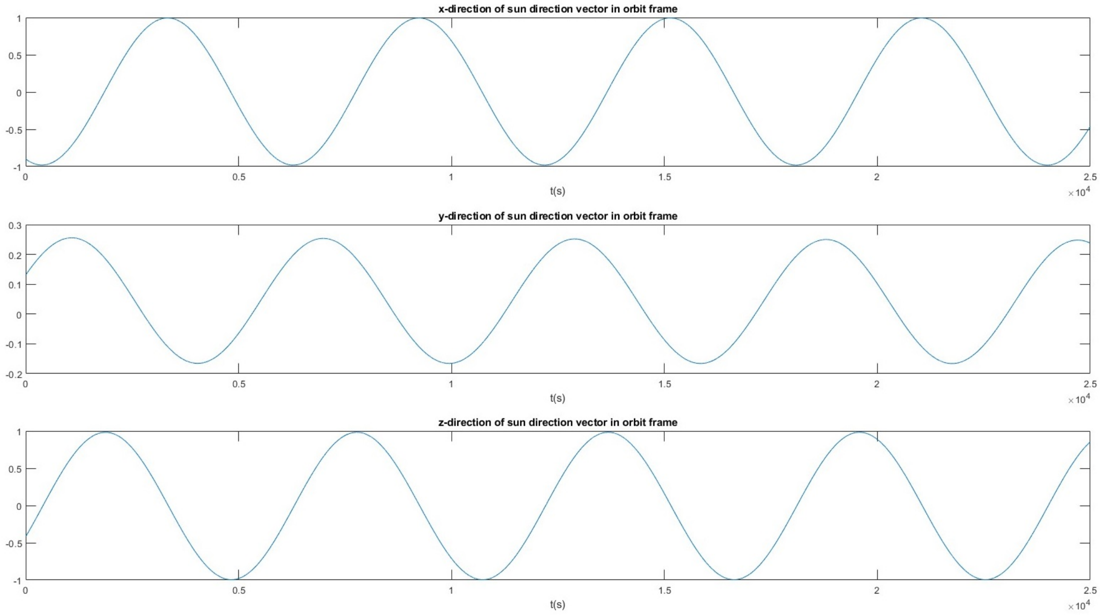

Figure 2.1: Sun Direction Vector in Orbit Frame

As we can see in Figure 2.1, all three elements are changing in time.

3 Question 3

In this question, we are asked to model small oscillations in orbit frame using Euler angles. I use sinusoidal oscillations with period of 50 seconds in pitch angle. I plotted all the three angles.

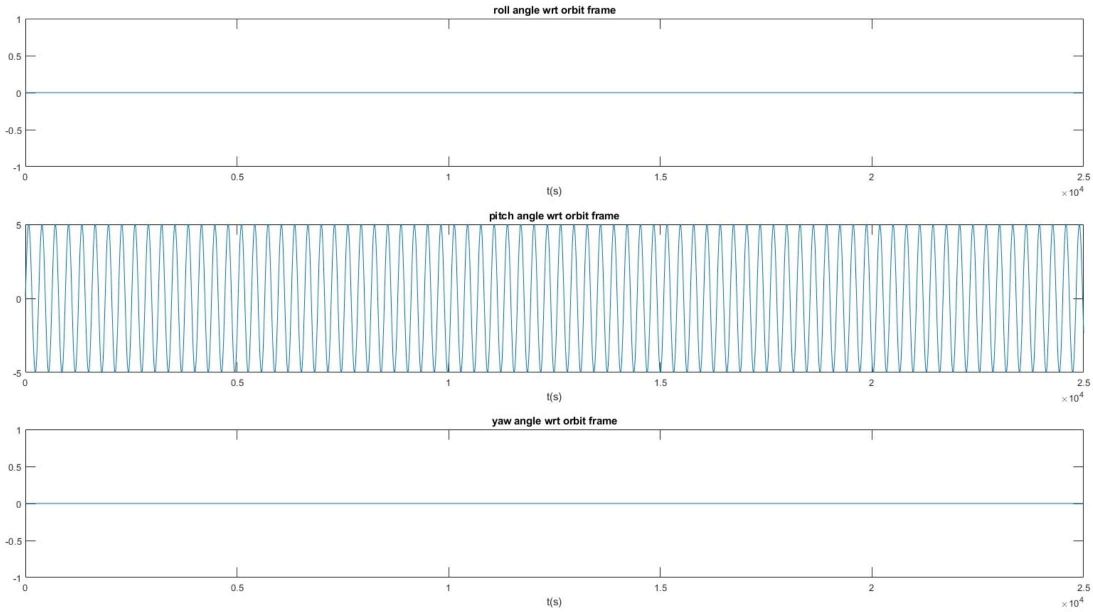

Figure 3.1: Euler Angles with respect to Orbit Frame

As we can see in Figure 3.1, only disturbance of body frame with respect to orbital frame is pitch angle. Other two angles remain same all the time.

4 Question 4

In this question, we are asked to obtain calculated magnetometer vector and sun direction vector in body frame with noise. Noise interval is given to us. Firstly, I obtained the transformation matrix from previous Euler angles with using angle2dcm function. Then we know,

We know,

$$\mathbf{S}_b = A\mathbf{S}_o + \nu \tag{4.1}$$

By transforming true orbit frame vectors into body frame vectors with transformation matrix I obtained, and adding noise by the desired value, I obtained body frame vectors with noise. So, we actually modelled real measurements and calculations by doing this.

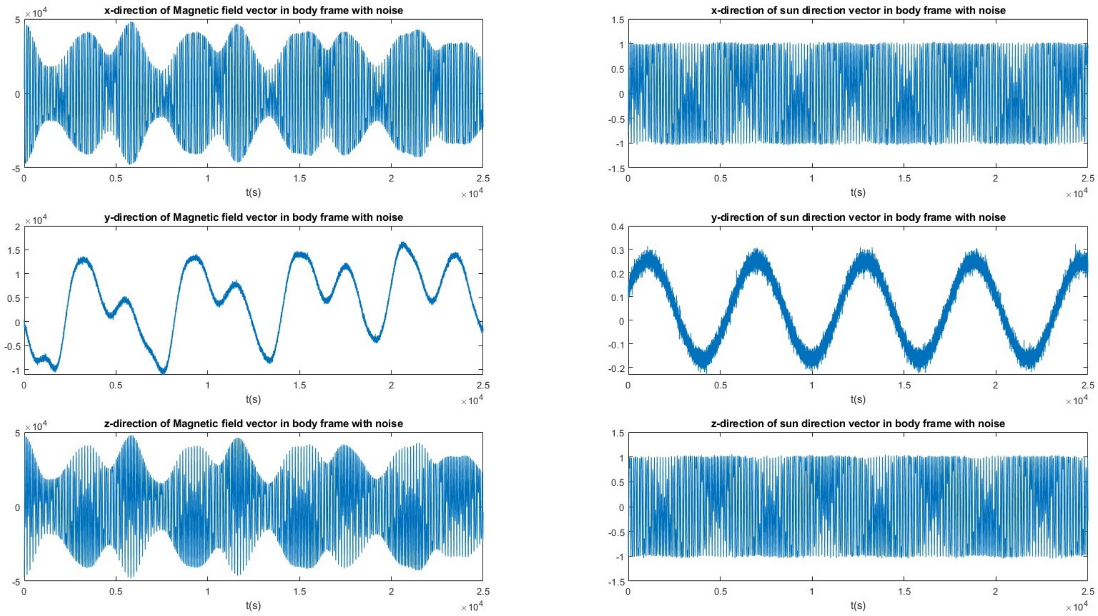

Figure 4.1: Magnetic Field and Sun Direction Vectors in Body Frame with Noise

As we see in Figure 4.1, vectors have noise and plots are looking crowded because of my period choosing for body frame oscillations.

If I had selected period 500 second, the plots would look like below.

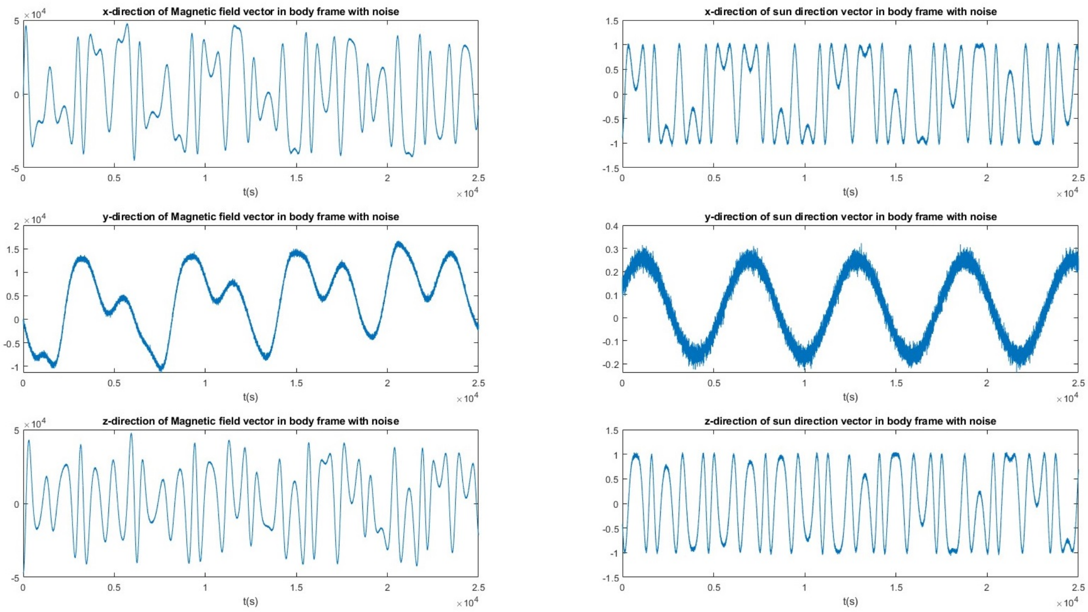

Figure 4.2: Magnetic Field and Sun Direction Vectors in Body Frame with Noise with period 500s

5 Question 5

In this part, we are asked to write a Quest algorithm and use it to find spacecraft attitude with our magnetometer and sun direction vectors. Firstly, we have to understand Wahba's Problem. In quest method, we obtain maximum eigenvalue by solving a quartic function for λ to minimize the cost function in Wahba's problem. Function is,

$$0 = \gamma(\lambda_{max} - trB) - Z^T(\alpha I + \beta S + S^2)Z \tag{5.1}$$

where,

- $\beta = \lambda_{max} - trB$
- $\alpha = \lambda_{max}^2 - (trB)^2 + tr(adjS)$
- $\gamma = \alpha(\lambda_{max} - (trB)^2) - detS)$

We know the maximum eigenvalue is very close to,

$$\lambda_0 = \sum_{i=1}^{n} w_i \tag{5.2}$$

If the optimized loss function is small. So, by starting λ_0, we can get λ_{max}, by using Newton-Raphson method. Finally, optimal quaternion estimate is,

$$\mathbf{q}_{opt} = \frac{1}{\sqrt{\gamma^2 + |\mathbf{x}|}} \begin{bmatrix} \mathbf{x} \\ \gamma \end{bmatrix} \tag{5.3}$$

I couldn't go further and write the algorithm.